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Recently, N. H. Kagan and E. Mendoza have criticized the historical approach to special relativity and outlined a new approach developed for the 12th grade.[1] In this article I take their part and describe a related attempt. First let me present some general ideas.

Some physicists and physics teachers think that a physical theory should be introduced in school as complete as possible, adapted, of course, to the level of the school. One can understand this attitude. Physicists and teachers know this theory well and appreciate it as a whole. They see its completeness, internal consistency, symmetry, usefulness, and its place in the framework of physics. Students, however, learning the theory part by part, may see things somewhat differently. They will overlook most of the parts of the theory until after they have grasped it as a whole, at least vaguely. One is tempted to compare them with mountaineers who do not see the whole countryside on their tiresome way until they reach the summit.

But mountaineers can enjoy, instead of a trip to the summit, a series of shorter, less tiresome trips in the foothills, overlooking at one time a lovely valley and at later times other ones. Instead of presenting a physical theory in school as a whole sometimes it may be convenient to split it into parts and present some of them separately. The idea is certainly not new, but one can say that it has not been used often in connection with the special theory of relativity.

Four steps

As a theory is split into parts a criterion would be welcome to decide which parts should be taught and in what order. Lacking a universal criterion we may use a possible simple criterion sorting the parts according to their significance in everyday life, e.g., by determining the frequency of corresponding concepts in mass media.

A hierarchy of concepts in special relativity is not difficult to propose. The most important are nuclear power plants and energy production in the sun, to say nothing about nuclear weapons. Then come great accelerators, storage rings, and other expensive pieces of equipment in high-energy physics. They are followed by atomic clocks, synchronization, and timekeeping, and last appears the structure of space and time.

The corresponding chapters, or steps, in the special theory of relativity contain:

1. rest energy, total energy, energy conservation, and energy balance in nuclear reactions,
2. relativistic mechanics,
3. time dilatation, decay of particles in flight, and
4. space-time.

As the first step is most important, according to our criterion, it should be taught first and to the widest audience. The following steps being less and less important should be taught to a smaller and smaller audience.

Step 1 can be presented as an extension of energy conservation to nuclear reactions. Rest energy is introduced studying a particular reaction. The most appropriate appear to be simple reactions that can be visualized by cloud-chamber photographs, e.g., ^{7}Li (p, 2α). The kinetic energy of α particles is deduced via the energy-range relation. It is found that an excess of energy is accompanied by a deficit of mass and vice versa. This leads to the idea that

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Table I

Reaction $^1H + {}^7Li \rightarrow {}^4He + {}^4He$

mass: $m_i = 1.0078\ u + 7.0160\ u, m_f = 2 \times 4.0036\ u$

$m_f - m_i = -0.0186\ u = -3.09 \times 10^{-29}$ kg

energy: $K_i \approx O, K_f = 2 \times 8.7$ MeV

$K_f - K_i = 17.4 \text{ MeV} = 2.78 \times 10^{-12}\ J$

coefficient: $(K_f - K_i)/(m_i - m_f) = 2.78 \times 10^{-12}\ J/3.09 \times 10^{-29}\ \text{kg} = 9.0 \times 10^{16}\ m^2/s^2 = (3.0 \times 10^8\ m/s)^2 = c^2$

old conservation laws $m_i = m_f$ and $K_i = K_f$ are not valid

new conservation law $K_i + m_ic^2 = K_f + m_fc^2$ if no change in internal (excitation) energy and no potential energy.

Reaction energy $Q = (m_f - m_i)c^2 = K_i - K_f$

exoergic $Q < 0 \quad m_f < m_i \quad K_i < K_f$

endoergic $Q > 0 \quad m_f > m_i \quad K_i > K_f$

The first nuclear reaction observed with artificially accelerated particles by means of the Cockroft-Walton accelerator is taken as an example in step 1. In the cloud-chamber photograph of P. I. Dee and E. T. S. Walton (1933) [W. Gentner, H. Maier-Leibnitz and W. Bothe, *An Atlas of Typical Expansion Chamber Photographs* (Pergamon Press, London 1954) p. 99] both α particles are colinear, thus the momentum and kinetic energy of the incoming proton can be neglected. From an effective range of about 8 cm in air at NTP a kinetic energy of 8.7 MeV for each α particle is estimated.

A short outline of the first part of step 1.

Table II

Total energy of a particle $W = K + mc^2$
For a slow particle, $K_o = 1/2\ mv^2$,
$W_o = mc^2\ [1 + 1/2\ (v^2/c^2)]$
According to the principle of ultimate speed $v/c \rightarrow 1$ as $W/mc^2 \rightarrow \infty$.

Conditions $W/mc^2 \rightarrow 1 + 1/2\,(v^2/c^2)$ as $v/c \rightarrow 0$
and $W/mc^2 \rightarrow \infty$ as $v/c \rightarrow 1$

are satisfied by

$$W = mc^2\ (1 - v^2/c^2)^{-1/2}$$

(to envisage this for the first one use the binomial theorem).

Relativistic kinetic energy

$$K = W - mc^2 = mc^2\ [(1 - v^2/c^2)^{-1/2} - 1]$$

$\Delta K = -eV$ is verified by Bertozzi's experiment (V is the acceleration voltage).

For Newtonian momentum, $p_o = mv$, the equation $p_o/W_o = p_o/mc^2 = v/c^2$ is valid. It is supposed to hold for any $v/c < 1$
Thus relativistic momentum is:

$$p = Wv/c^2 = mv\ (1 - v^2/c^2)^{-1/2}$$

Relation $W^2 = c^2p^2 + m^2c^4$, which follows, is indirectly verified: $dW/dp = dK/dp = v$ is true even as in Newtonian mechanics, $dK_o/dp_o = v$.

Equation of motion in one dimension:

relativistic $eE = dp/dt$ Newtonian $eE = dp_o/dt$

(E is the electric field strength). $-edV = eEdx = dK$ is verified by Bertozzi's experiment.

Equation of motion in three dimensions:

relativistic $e(\vec{E} + \vec{v} \times \vec{B}) = d\vec{p}/dt$

$\vec{p} = m\vec{v}\,(1 - \vec{v} \cdot \vec{v}/c^2)^{-1/2}$

(B is the magnetic induction). For radial acceleration $evB = m\ (1 - v^2/c^2)^{-1/2}v^2/r$ (r is the radius of curvature of the trajectory). Equation $erB = p$ is verified and is used to deduce momentum of fast charged particles from bubble-chamber photographs.

A short outline of the first part of step 2.

energy should be ascribed to mass, both quantities being proportional. The coefficient of proportionality equals c^2 as it is found from experimental data (Table I). The old conservation laws for mass and energy merge into a new conservation law for energy including rest energy.

With the new law reaction energies are calculated and exoergic and endoergic reactions distinguished. Further, binding energy and specific binding energy of nuclei is studied and fission and fusion recognized as the most promising exoergic reactions. Finally, nuclear reactors and fusion in the sun are mentioned.

In step 2 the total energy of a particle is introduced as the sum of its rest and kinetic energies. For slow particles the Newtonian kinetic energy is taken. On the basis of a principle of the ultimate speed, which was formulated by H. Poincaré in 1902, the total energy of fast particles is considered. Taking the Newtonian expression as the first two terms of a series expansion, the relativistic total energy can be guessed (Table II). The result is supported by a description of Bertozzi's experiment and the movie *The Ultimate Speed.*[2] Later on the relativistic mechanics of a particle are deduced as a generalization of Newtonian mechanics. Then independent systems of particles, which at the time of observation do not interact with other particles, are studied. Inelastic collisions are considered and the maximum available energy is obtained. Accelerators and storage rings are briefly discussed and the advantage of the latter is explained.

Step 3 starts with a thought experiment in which an idealized clock is observed in its proper inertial frame and in a moving inertial frame. The clock consists of a source with detector and a mirror. The source emits a light pulse that is reflected by the mirror and returns to the detector.[3] Calculating the travel time of the pulse in the proper frame and in the moving frame and taking into account that the speed of the pulse is equal to the ultimate speed in both frames, yields the time dilatation formula. Its predictions agree with the measurement of the decay time of muons in flight.[4] This step may include length contraction and the Lorentz transformation.

Step 4 derives the Lorentz transformation on the basis of the relativity principle and principle of light-speed invariance. The second principle is supported by the measured speed of photons produced in the decay of very fast neutral pions[5] and by an improved version of the de Sitter double-star argument.[6] Time dilatation is verified through a description of measurements of the decay time of fast charged pions in flight for approximately unaccelerated[7] and accelerated[8] motion. Space-time is introduced, proper time defined, and relativistic mechanics in four-dimensional form are built up.

Practical experience

The described scheme would remain a theoretical exercise were it not tested in school. In fact, the scheme was developed gradually as the author was a member of the committee for physics curricula at secondary school. In brief, in Slovenia a modification of secondary schools is planned in the next few years. The old secondary schools, differing appreciably in character, will be replaced by a common secondary school. The first year of it, i.e., grade 9, will be compulsory, whereas in grades 10 to 12, students will have to choose between a few orientations differing in emphasis of some subjects. Since the committee had to take into account a variety of demands, the curricula could not follow entirely the proposed scheme. After all, special relativity is only a small fraction of physics.

Step 1 is a part of the curriculum for the 9th grade. In primary school in grades 7 and 8, as in grade 9, physics is taught two hours per week or 70 hours per grade. The central quantity is energy and the first law of thermodynamics is obtained as a generalization of the work-energy theorem. The difference of internal energy is introduced as missing kinetic energy in an adiabatic system to which work against friction is supplied. Heat is introduced through the transfer of internal energy in a system in thermal contact with a hotter body and receiving no work. So the generalization of the law to nuclear reactions and the introduction of rest energy did not lead to difficulties. The curriculum was tested for two years in pilot classes and was found to be acceptable. The only possible amendment is that the discussion of nuclear binding energy is to be abandoned owing to lack of time.

A somewhat abridged step 2 may become a part of the curriculum in grade 12 for students interested in mathematics and science. These students have acquired a good background in Newtonian mechanics having had three hours of physics per week during grades 10 and 11, as they have in grade 12. In case that time may prove short, the discussion of inelastic collisions will be abridged.

Step 3 is not a part of the regular curriculum. It is intended for students of grade 12 who are particularly interested in special relativity and is studied on a voluntary basis. Step 4 is taught to physics sophomores.

A preliminary version of a textbook containing step 1 was published in 1978. Booklets containing the somewhat extended material for steps 2 and 3 appeared in 1979 in honor of the Einstein centennial. The textbook of modern physics contains a chapter on special relativity covering step 4.

Discussion

It is difficult to compare curricula of different educational systems. In general the physics curriculum in Slovenian secondary schools with regard to special relativity may be placed somewhere between two extremes. On the one side are curricula without relativity[9] and, on the other, curricula with almost complete special relativity.[10] Our proposal with steps 1 to 3 may be called "relativistic mechanics without relativity." The basic idea is similar to a French approach.[11] The student does not get an impression of the special theory of relativity as a whole. However, taking into account the limited time devoted to physics in general and to special relativity particular, this appears to be the only possible solution. This shortcoming will be at least partially overcome by special seminars. These should give an overview of parts of physics not covered by the curriculum. Secondary school students who are primarily interested in the humanities will participate.

It is regrettable that in special relativity there are no experiments that could be demonstrated in class. This has to be compensated by thorough descriptions of experiments, by movies, book illustrations, use of photographs, etc. Nuclear reactions can be studied by means of cloud-chamber photographs. Tracks of α particles can be, however, observed directly by demonstrating the operation of a cloud chamber. Owing to this the reaction $^7\mathrm{Li}(p, 2\alpha)$ is better suited as an example in step 1 rather than the reaction $^{12}\mathrm{C}(\gamma, 3\alpha)$, visualized in nuclear photographic emulsion, that was used at an earlier stage.

Let us summarize the essential novelty of the proposed approach: introduction of rest energy and energy conservation in nuclear reactions in grade 9 – this is not usual – and relativistic mechanics in grade 12 – as usual but without the Lorentz transformation. Steps 3 and 4 are not decisive for this proposal and are rather standard. Therefore they were not described in such detail as steps 1 and 2.

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