

THE ATOMIC ARRANGEMENT IN GLASS

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1. Introduction

It must be frankly admitted that we know practically nothing about the atomic arrangement in glasses. Glasses are described as supercooled liquids or as solids. The former term is justifiable from the point of view of physical chemistry, the latter from the theory of elasticity. It seems rather futile, however, to try to decide which of the two descriptions is the proper one to use, when we are ignorant about the characteristic properties of the atomic arrangement.

- network-forming systems ($\text{SiO}_2, \text{GeO}_2, \text{B}_2\text{O}_3$) → continuous random network
- close-packed systems (metalli, colloidì, polimeri) → random close packing
- frustrazione geometrica → struttura localmente preferita

Periodic Table of the Elements

Element symbol represents state at room temperature

Solid, Liquid or Gas

Mg Magnesium 28.6 [Ne]3s ²	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	Al Aluminum 28.3 [Ne]3s ² p ¹	Si Silicon 28.4 [Ne]3s ² p ²	P Phosphorus 28.5 [Ne]3s ² p ³	S Sulfur 28.6 [Ne]3s ² p ⁴	Cl Chlorine 28.7 [Ne]3s ² p ⁵	Ar Argon 28.8 [Ne]3s ² p ⁶
Ca Calcium 28.8 [Ar]3d ²	21 Sc Scandium 28.9 [Ar]3d ¹ s ²	22 Ti Titanium 28.10 [Ar]3d ² s ²	23 V Vanadium 28.11 [Ar]3d ³ s ²	24 Cr Chromium 28.13 [Ar]3d ⁵ s ¹	25 Mn Manganese 28.13 [Ar]3d ⁵ s ²	26 Fe Iron 28.14 [Ar]3d ⁶ s ²	27 Co Cobalt 28.15 [Ar]3d ⁷ s ²	28 Ni Nickel 28.16 [Ar]3d ⁸ s ²	29 Cu Copper 28.18 [Ar]3d ¹⁰ s ¹	30 Zn Zinc 28.18 [Ar]3d ¹⁰ s ²	31 Ga Gallium 28.18 [Ar]3d ¹⁰ s ² p ¹	32 Ge Germanium 28.18 [Ar]3d ¹⁰ s ² p ²	33 As Arsenic 28.18 [Ar]3d ¹⁰ s ² p ³	34 Se Selenium 28.18 [Ar]3d ¹⁰ s ² p ⁴	35 Br Bromine 28.18 [Ar]3d ¹⁰ s ² p ⁵	36 Kr Krypton 28.18 [Ar]3d ¹⁰ s ² p ⁶
Sr Strontium 28.18 [Kr]3s ²	39 Y Yttrium 28.18 [Kr]4d ¹ s ²	40 Zr Zirconium 28.18 [Kr]4d ² s ²	41 Nb Niobium 28.18 [Kr]4d ⁵ s ¹	42 Mo Molybdenum 28.18 [Kr]4d ⁵ s ¹	43 Tc Technetium 28.18 [Kr]4d ⁵ s ²	44 Ru Ruthenium 28.18 [Kr]4d ⁷ s ¹	45 Rh Rhodium 28.18 [Kr]4d ⁸ s ¹	46 Pd Palladium 28.18 [Kr]4d ¹⁰	47 Ag Silver 28.18 [Kr]4d ¹⁰ s ¹	48 Cd Cadmium 28.18 [Kr]4d ¹⁰ s ²	49 In Indium 28.18 [Kr]4d ¹⁰ s ² p ¹	50 Sn Tin 28.18 [Kr]4d ¹⁰ s ² p ²	51 Sb Antimony 28.18 [Kr]4d ¹⁰ s ² p ³	52 Te Tellurium 28.18 [Kr]4d ¹⁰ s ² p ⁴	53 I Iodine 28.18 [Kr]4d ¹⁰ s ² p ⁵	54 Xe Xenon 28.18 [Kr]4d ¹⁰ s ² p ⁶
Ba Barium 28.18 [Xe]4f ²	57-71 Hf Hafnium 28.18 [Xe]4f ¹ d ² s ²	72 Ta Tantalum 28.18 [Xe]4f ¹ 4s ² s ²	73 W Tungsten 28.18 [Xe]4f ¹ 4d ² s ²	74 Re Rhenium 28.18 [Xe]4f ¹ 4d ⁵ s ²	75 Os Osmium 28.18 [Xe]4f ¹ 4d ⁶ s ²	76 Ir Iridium 28.18 [Xe]4f ¹ 4d ⁷ s ²	77 Pt Platinum 28.18 [Xe]4f ¹ 4d ⁸ s ¹	78 Au Gold 28.18 [Xe]4f ¹ 4d ¹⁰ s ¹	79 Hg Mercury 28.18 [Xe]4f ¹ 4d ¹⁰ s ²	80 Tl Thallium 28.18 [Xe]4f ¹ 4d ¹⁰ s ³	81 Pb Lead 28.18 [Xe]4f ¹ 4d ¹⁰ s ⁴	82 Bi Bismuth 28.18 [Xe]4f ¹ 4d ¹⁰ s ⁵	83 Po Polonium 28.18 [Xe]4f ¹ 4d ¹⁰ s ⁶	84 At Astatine 28.18 [Xe]4f ¹ 4d ¹⁰ s ⁷	85 Rn Radon 28.18 [Xe]4f ¹ 4d ¹⁰ s ⁸	86 22.028
Ra Radium 28.18 [Rn]5f ²	89-103 Rf Rutherfordium 28.18 [Rn]5f ¹ 4d ² s ² *	104 Db Dubnium 28.18 [Rn]5f ¹ 4d ³ s ² *	105 Sg Seaborgium 28.18 [Rn]5f ¹ 4d ⁷ s ² *	106 Bh Bohrium 28.18 [Rn]5f ¹ 4d ¹²	107 [264] Hs Hassium 28.18 [Rn]5f ¹ 4d ¹²	108 [269] Mt Meitnerium 28.18 [Rn]5f ¹ 4d ¹²	109 [268] Ds Darmstadtium 28.18 [Rn]5f ¹ 4d ¹²	110 [269] Rg Roentgenium 28.18 [Rn]5f ¹ 4d ¹²	111 [272] Cn Copernicium 28.18 [Rn]5f ¹ 4d ¹²	112 [277] Uut Ununtrium 28.18 [Rn]5f ¹ 4d ¹²	113 unknown Ununtrium 28.18 [Rn]5f ¹ 4d ¹²	114 [289] Fl Flerovium 28.18 [Rn]5f ¹ 4d ¹²	115 unknown Ununpentium 28.18 [Rn]5f ¹ 4d ¹²	116 [298] Lv Livermorium 28.18 [Rn]5f ¹ 4d ¹²	117 unknown Ununseptium 28.18 [Rn]5f ¹ 4d ¹²	118 unknown Ununoctium 28.18 [Rn]5f ¹ 4d ¹²
Lanthanide Series																
57 La Lanthanum 28.18 [Xe]5d ¹ s ²	58 Ce Cerium 28.18 [Xe]4f ¹ 5d ¹ s ²	59 Pr Praseodymium 28.18 [Xe]4f ¹ 5d ² s ²	60 Nd Neodymium 28.18 [Xe]4f ¹ 5d ³ s ²	61 Pm Promethium 28.18 [Xe]4f ¹ 5d ⁶ s ²	62 Sm Samarium 28.18 [Xe]4f ¹ 5d ⁶ s ²	63 Eu Europium 28.18 [Xe]4f ¹ 5d ⁷ s ²	64 Gd Gadolinium 28.18 [Xe]4f ¹ 5d ⁷ s ²	65 Tb Terbium 28.18 [Xe]4f ¹ 5d ⁸ s ²	66 Dy Dysprosium 28.18 [Xe]4f ¹ 5d ⁹ s ²	67 Ho Holmium 28.18 [Xe]4f ¹ 5d ¹⁰ s ²	68 Er Erbium 28.18 [Xe]4f ¹ 5d ¹⁰ s ²	69 Tm Thulium 28.18 [Xe]4f ¹ 5d ¹⁰ s ²	70 Yb Ytterbium 28.18 [Xe]4f ¹ 5d ¹⁰ s ²	71 Lu Lutetium 28.18 [Xe]4f ¹ 5d ¹⁰ s ²		
Actinide Series																
89 Ac Actinium 28.18 [Rn]6d ¹ s ²	90 Th Thorium 28.18 [Rn]6d ² s ²	91 Pa Protactinium 28.18 [Rn]5f ² d ¹ s ²	92 U Uranium 28.18 [Rn]5f ³ d ¹ s ²	93 Np Neptunium 28.18 [Rn]5f ⁴ d ¹ s ²	94 Pu Plutonium 28.18 [Rn]5f ⁵ d ² s ²	95 Am Americium 28.18 [Rn]5f ⁶ d ³ s ²	96 Cm Curium 28.18 [Rn]5f ⁷ d ³ s ²	97 Bk Berkelium 28.18 [Rn]5f ⁸ d ³ s ²	98 Cf Californium 28.18 [Rn]5f ⁹ d ³ s ²	99 Es Einsteinium 28.18 [Rn]5f ¹⁰ d ³ s ²	100 Fm Fermium 28.18 [Rn]5f ¹¹ d ³ s ²	101 Md Mendelevium 28.18 [Rn]5f ¹² d ³ s ²	102 No Nobelium 28.18 [Rn]5f ¹³ d ³ s ²	103 Lr Lawrencium 28.18 [Rn]5f ¹⁴ d ³ s ²		

Lanthanide Series

Actinide Series

Alkali Metal

Alkaline Earth

Transitio
Metal

Basic Metal

Metalloids

Nonmet

Halogens

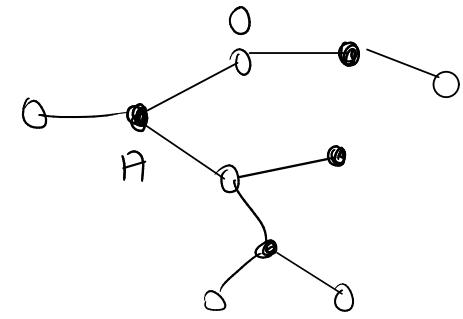
Noble
Gas

Lanthani

Actinide

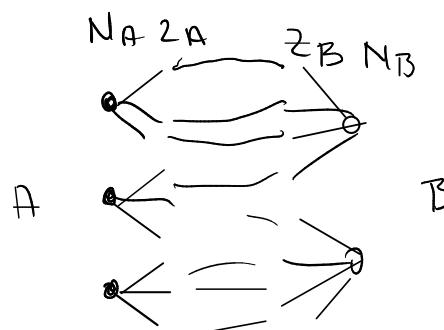
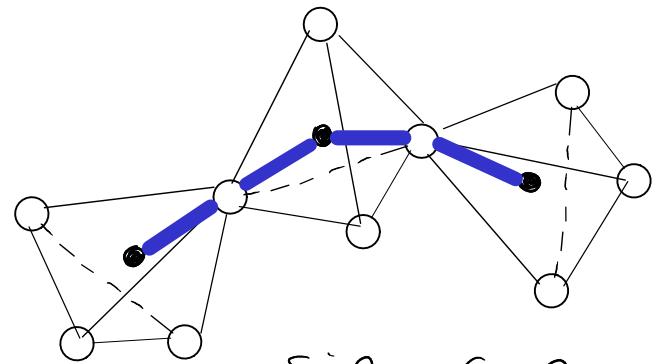
Network-forming glasses

1932 Zachariasen $\rightarrow A_n O_m$



$$\left\{ \begin{array}{l} x_O \\ x_A \end{array} \right. \text{ concentrazione} \quad 1 = x_A + x_O$$

$$\left\{ \begin{array}{l} z_O \\ z_A \end{array} \right. \text{n. coordinazione}$$



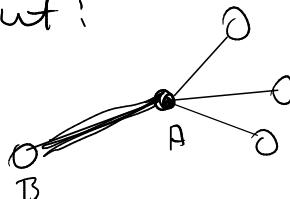
$$z_{Si} = 4, z_O = 2$$

$$4 \times z_i = (1 - x_i) 2 \rightarrow 6 \times z_i = 2 \rightarrow x_{Si} = \frac{1}{3} \rightarrow x_O = \frac{2}{3}$$

ES: z_A, z_B fissati
 $x_A, x_B = 1 - x_A$ tali
che network connesse?

$$x_A z_A = x_B z_B$$

Hint:



$$N_b \frac{1}{z_A} = N_B$$

$$N_b \frac{1}{z_B} = N_A$$

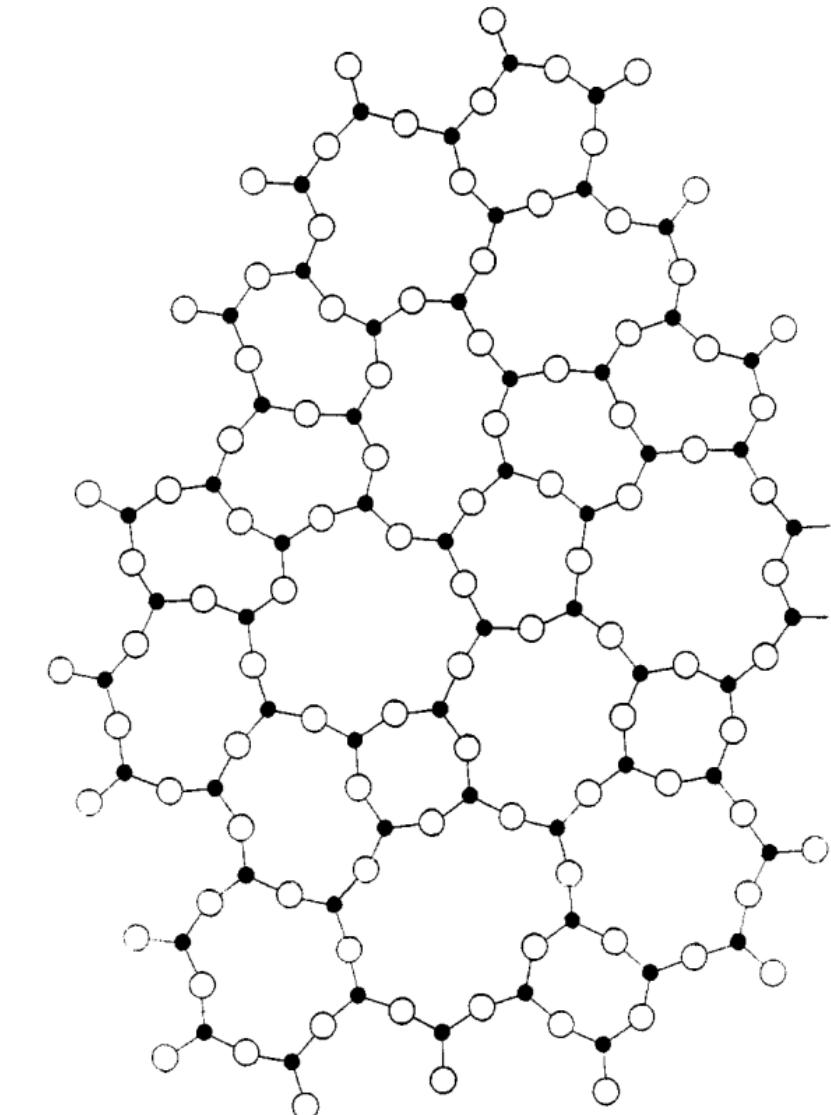
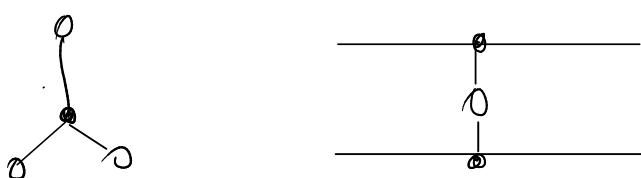


Fig. 1b.

Zachariasen 1932

ES: $Ge + Se$
 $As + Se$

Silice 2d $\Rightarrow x_h = ? \quad x_0 = ?$

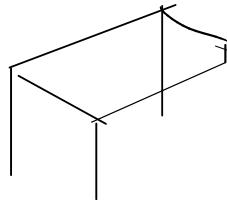


$$\left\{ \begin{array}{l} z_{Si} = 3 \\ z_O = 2 \end{array} \right.$$

Teoria della rigidità

- 1) Gupta - Cooper (1978) \sim Zachariasen
- 2) Phillips - Thorpe (1979) \sim bonds

Idea: rigidità "minimale" \rightarrow glass-forming ability
isostaticità



$n=2$ instabile

$n=3$ stabile marginale / isostaticità

$n=4$ stabile

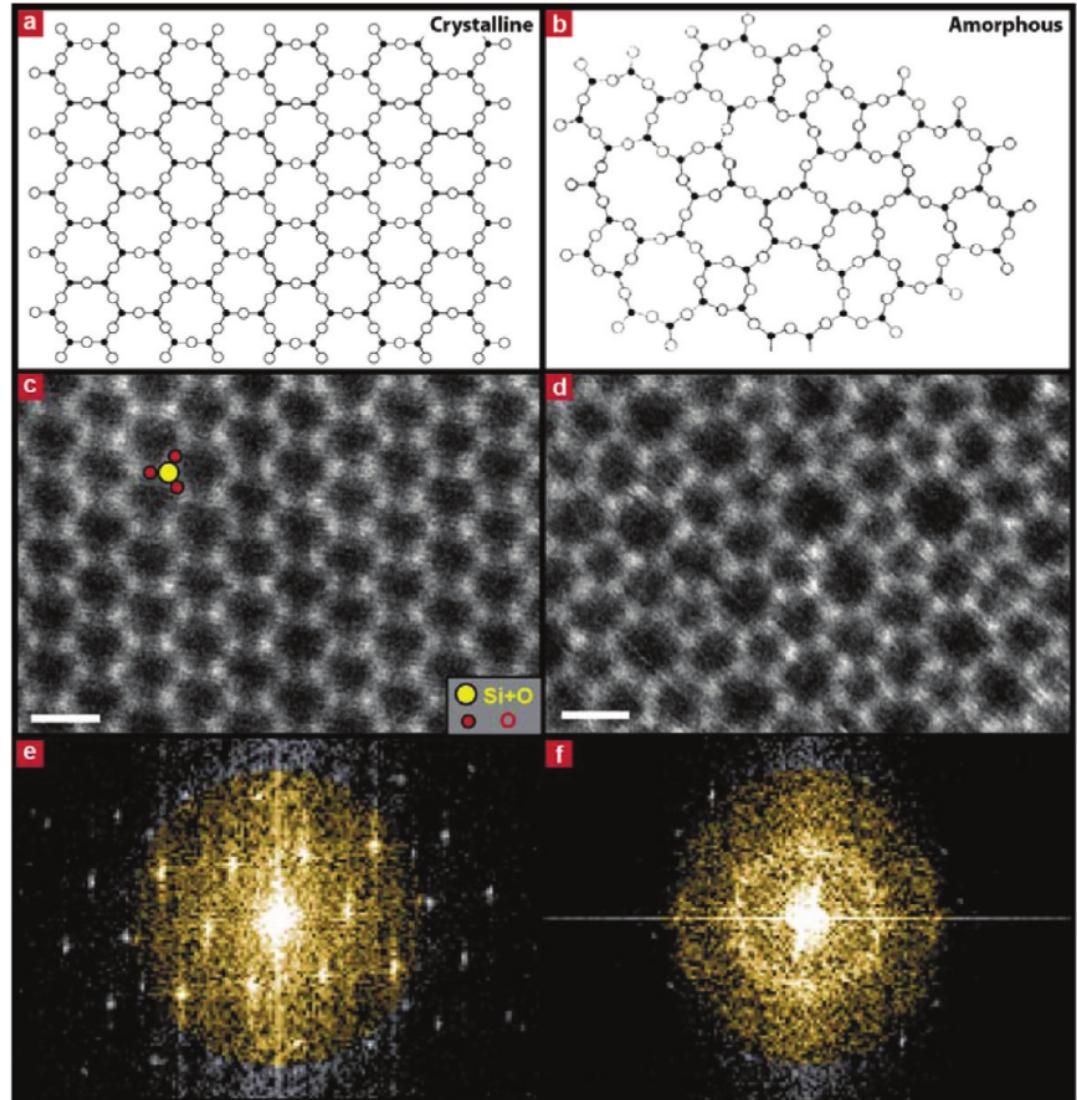
Criterio di Maxwell: isostaticità se

$$n. \text{ gradi libertà} = n. \text{ vincoli}$$

Direct Imaging of a Two-Dimensional Silica Glass on Graphene

Pinshane Y. Huang,^{†,■} Simon Kurasch,^{‡,■} Anchal Srivastava,^{§,○} Viera Skakalova,^{§,||} Jani Kotakoski,^{||,⊥} Arkady V. Krasheninnikov,^{⊥,¶} Robert Hovden,[†] Qingyun Mao,[†] Jannik C. Meyer,^{‡,||} Jurgen Smet,[§] David A. Muller,^{*,†,□} and Ute Kaiser^{*,‡}

Nano Lett. 2012, 12, 1081–1086



$T=0$ fisso distanza legame + angoli tra z legami $\geq \alpha \rightarrow$ legami spezie α

$$N_v = \sum_{\alpha} N_{\alpha} \frac{z_{\alpha}}{2} + N_{\infty} (2z_{\infty} - 3) \rightarrow n_v = \sum_{\alpha} x_{\alpha} \left(\frac{z_{\alpha}}{2} + 2z_{\infty} - 3 \right)$$

$$\langle z \rangle = \sum_{\alpha} x_{\alpha} z_{\alpha} \quad n_v = \frac{\langle z \rangle}{2} + 2 \langle z \rangle - 3$$

Condizione Maxwell $N \rightarrow \infty$

$$3N = N n_v + 6 \approx N n_v \Rightarrow n_v = 3 \quad (3d)$$

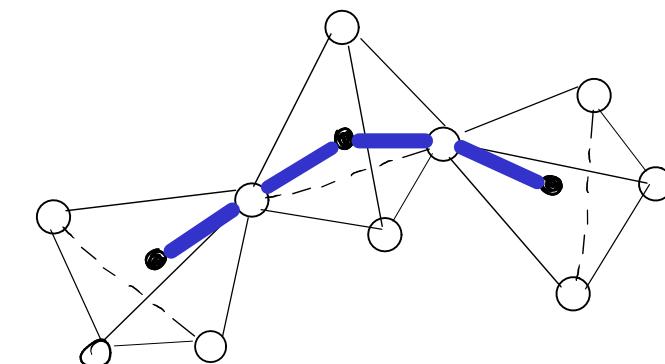
$$\frac{\langle z \rangle}{2} + 2 \langle z \rangle = 6$$

$$\frac{5}{2} \langle z \rangle = 6$$

$$\langle z \rangle = \frac{12}{5} = 2.4 \quad (\text{MF}) \rightarrow \text{SiO}_2, \text{GeO}_2$$

Ese: SiO_2

$$\langle z \rangle = \frac{1}{3} 4 + \frac{2}{3} 2 = \frac{8}{3} \gtrsim 2.4$$



Close-packed glasses

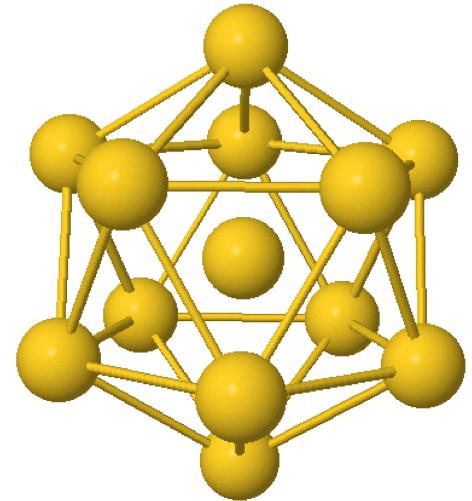
SUPERCOOLING OF LIQUIDS

1952

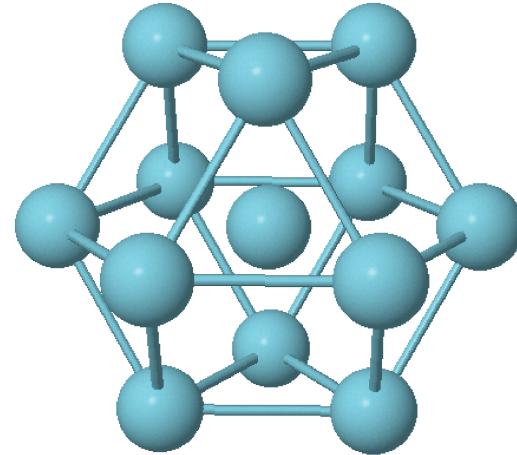
By F. C. FRANK

H. H. Wills Physics Laboratory, Bristol University

I shall concentrate upon reviewing the important recent change in our appreciation of the facts of supercooling which has been brought about particularly by the work of Turnbull at the General Electric Research Laboratory in Schenectady. I suppose that most of us, talking about supercooling a couple of years ago, would have divided substances into two classes, one with simple crystal structures like gold, and all the other 'good' metals on the one hand, and those with complex crystal structures, such as glycerol and the silicates on the other; saying that whereas the latter class can be very much supercooled, and will form glasses, the former class can only be supercooled a very few degrees. Then we would have added that there are some 'bad' metals, with moderately complex crystal structures, such as antimony or bismuth, which can be supercooled some tens of degrees, forming an intermediate class.

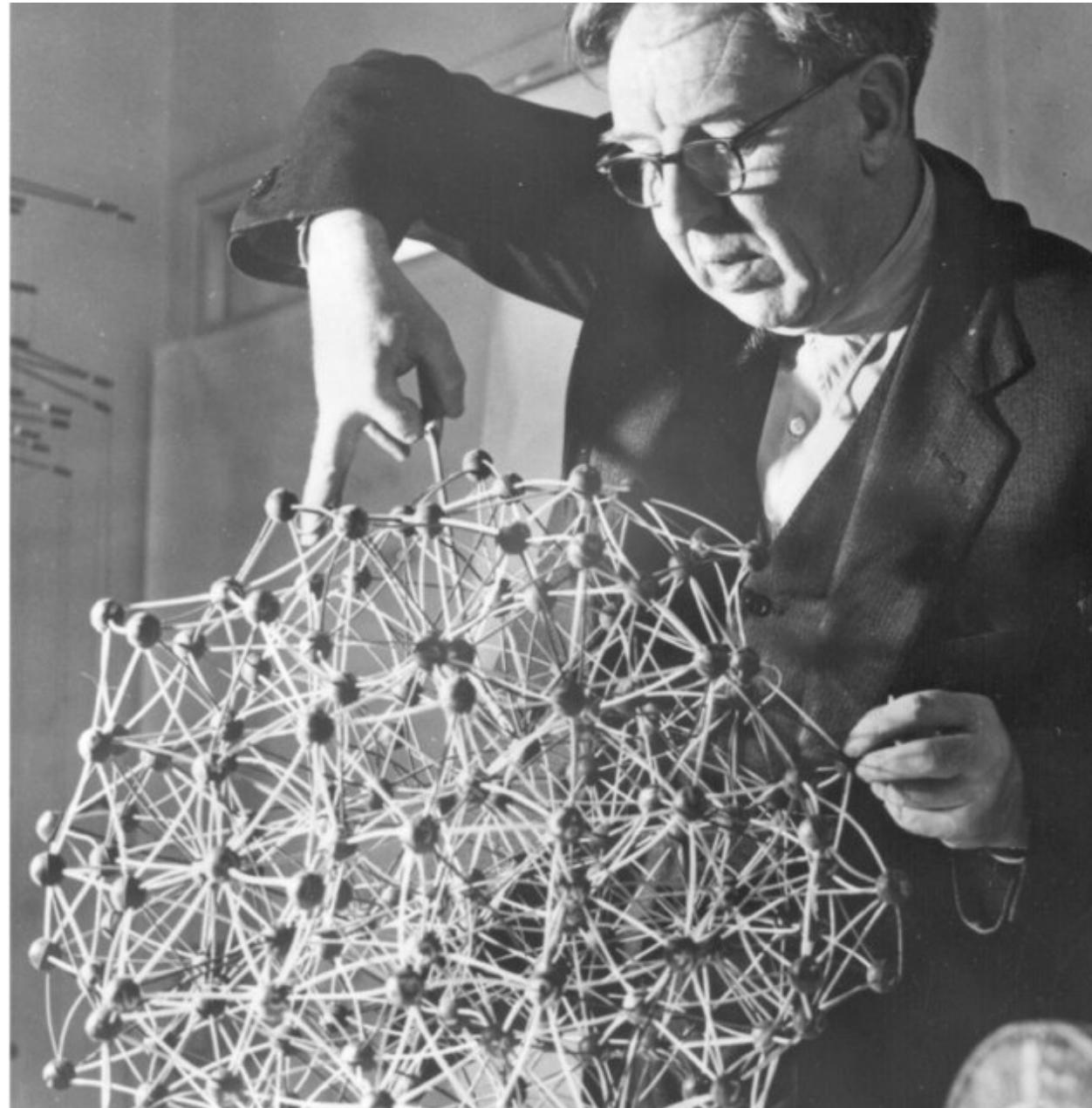


icosaedro



FCC

→ frattura geometrica



Bernal 1964

Tassellazione di Voronoi

→ Wigner-Seitz

cella 3d = poliedro

f : n. facce di un poliedro

p : n. vertici di ogni faccia

$$\bar{f} = \frac{12}{6 - \bar{p}}$$

(f_3, f_4, f_5, \dots)

Random close packing

Congettura Eulera : $\phi_{FCC} = 0.74$

$\phi_{RCP} \approx 0.64 - 0.68$

Packing isotatici : $dN = \frac{1}{2} Nz$

$$z = 2d$$

→

JAMMING

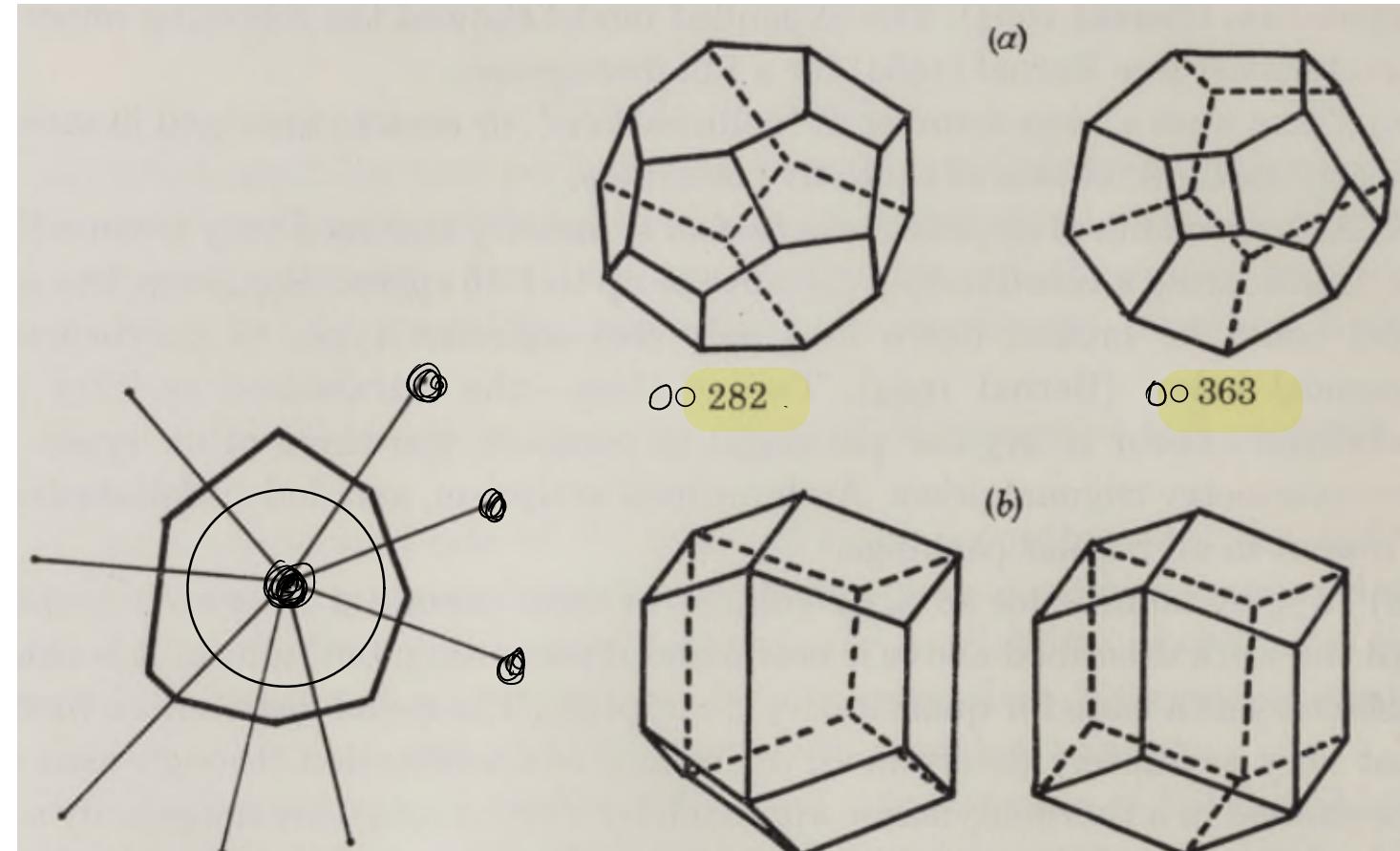
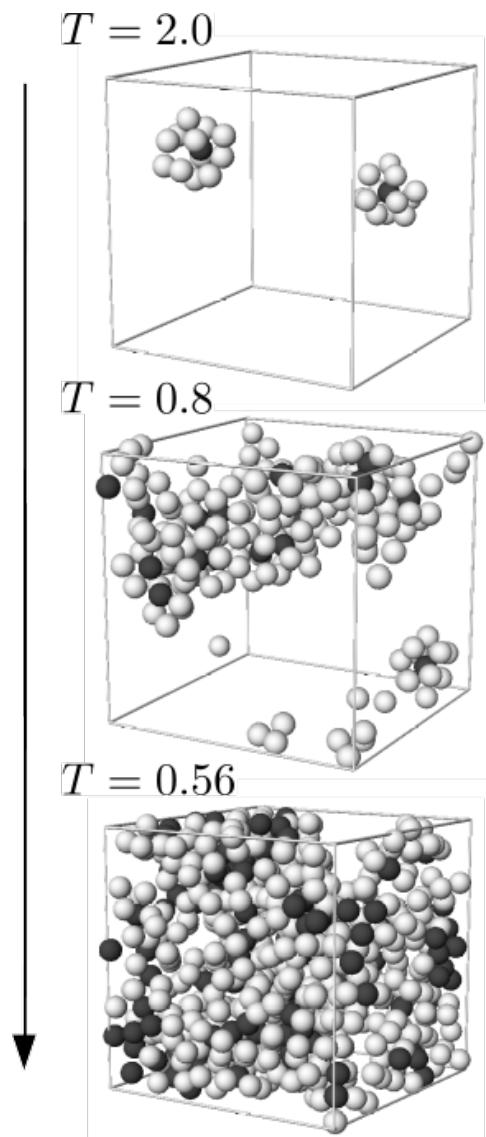


FIGURE 1. Construction of a two-dimensional Voronoi polygon.

FIGURE 2. (a) Two typical irregular Voronoi polyhedra. (b) Two regular Voronoi polyhedra, derived from hexagonal and cubic close-packing respectively.

Finnerty & Bernal 1972

Strutture localmente favorite (LFS)



Hansen, Barrat, Pastore

$$u_{\alpha\beta}(r) = \epsilon_{\alpha\beta} \left(\frac{\sigma_{\alpha\beta}}{r} \right)^{12}$$

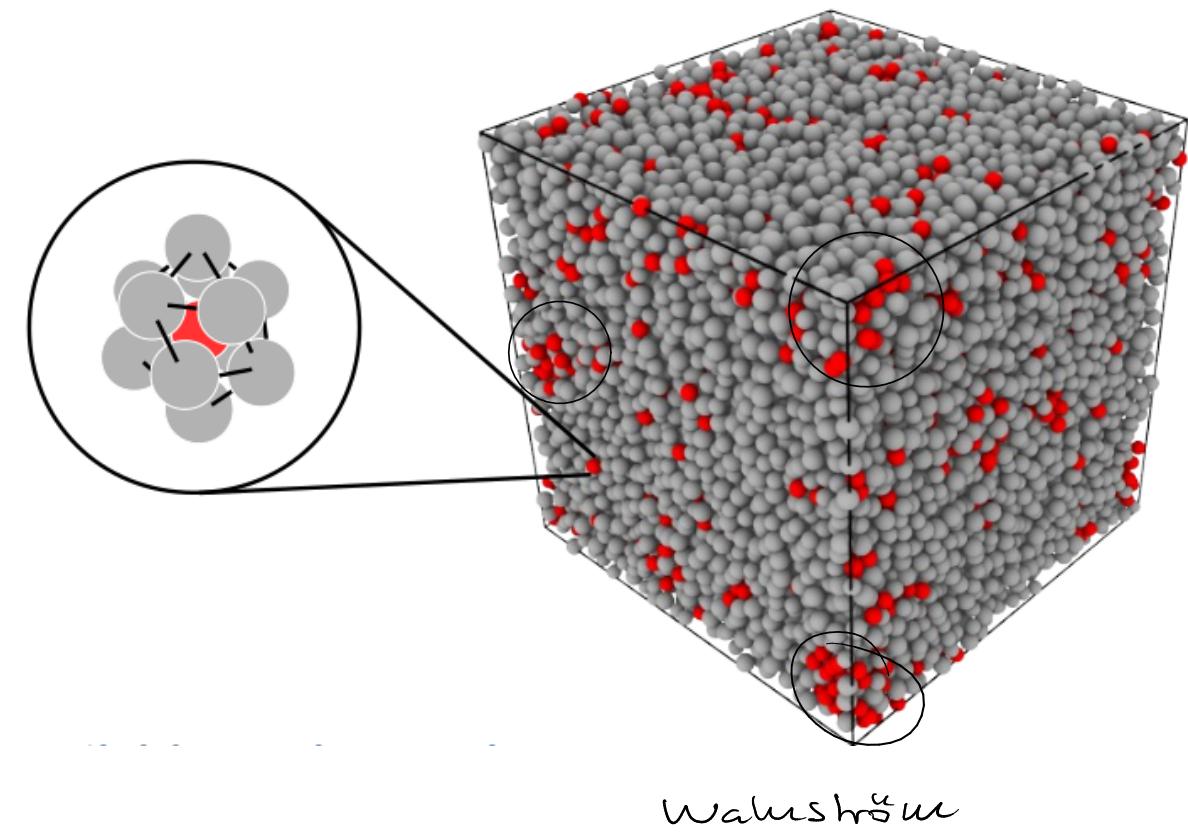
$$\frac{\sigma_{AA}}{\sigma_{BB}} = 1.4 \quad \epsilon_{AA} = \epsilon_{BB} = \epsilon_{AB}$$

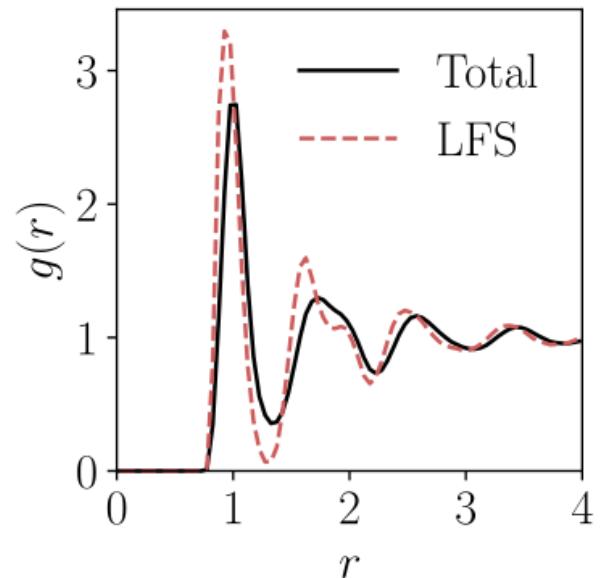
$$\frac{\sigma_{AA}}{\sigma_{BB}} = 1.2 \quad \text{Wahnström}$$

$$T_{\text{ouset}} \approx 1.0$$

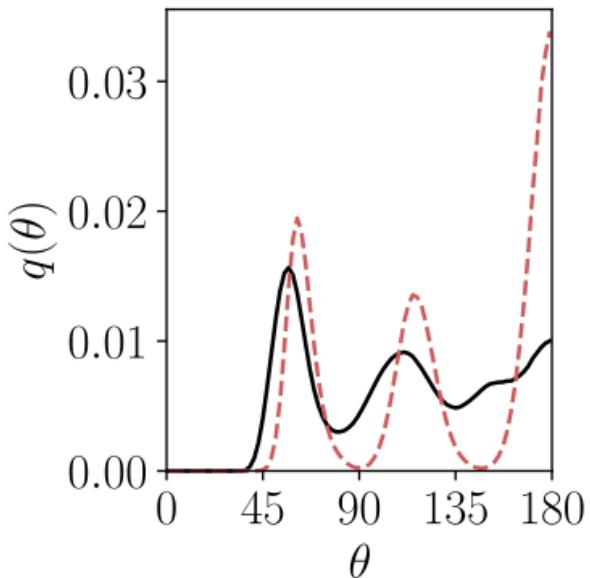
$$T_c \approx 0.55$$

$$\Delta \text{Mg zu z}$$



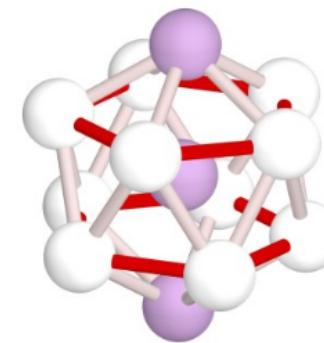


→ community inference



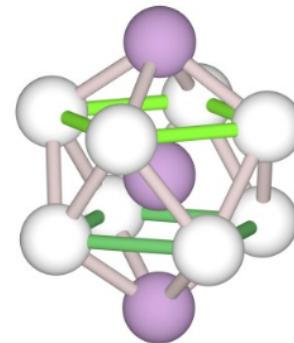
JCP 2019

(6,0,12)



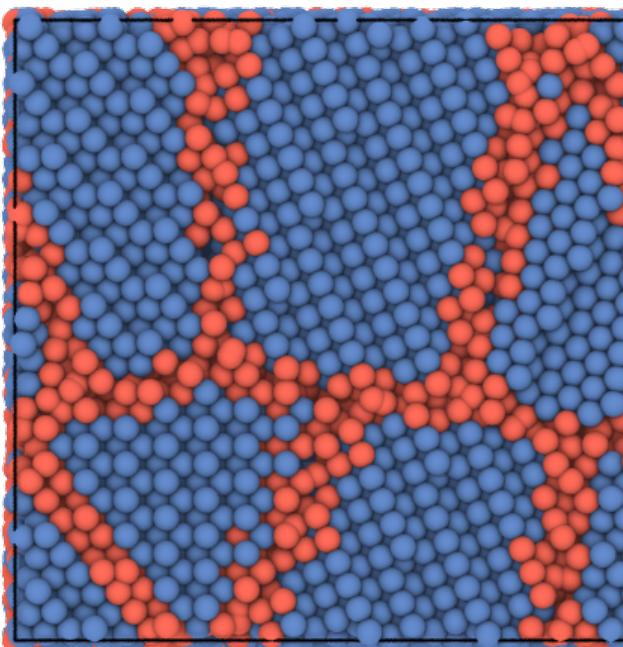
Mahnström
mixture

(0,2,8)



Kob - Andersen
mixture

```
from partycls import Trajectory, Workflow
traj = Trajectory('grains.xyz')
wf = Workflow(traj, descriptor='ba', clustering='kmeans')
wf.run()
traj[0].show(color='label', backend='ovito')
```



<https://github.com/jorisparet/partycls>