Ex 6,7

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(6) $A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ DIAGONALIZZABILE?

TROVARE S t.c. $S^{-1}AS$ È DIAGONALE.

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 2 & 0 & 4 \\ 0 & 2-\lambda & 3 & 1 \\ 0 & 0 & 3-\lambda & 0 \\ 0 & 0 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)^2(2-\lambda)(1-\lambda).$$

→ AUTOVALORI $1, 2, 3$, $m_A(1)=1$, $m_A(2)=1$, $m_A(3)=2$

→ COME AL SOLITO (VEDI 4ⓐ) BASTA CONTROLLARE

$m_G(3) = 2$.

$$A-3I = \begin{pmatrix} -2 & 2 & 0 & 4 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow rg(A-3I) = 2$$

$\Rightarrow m_G(3) = 4-2 = 2 \Rightarrow A$ DIAG. BILE ✓.

S t.c. $S^{-1}AS$ DIAGONALE?

TROVARE S CORRISPONDE A TROVARE UNA BASE $\mathcal{B} = \{v_1, \dots, v_4\}$ DI $\mathbb{R}^n$ DI AUTOVETTORI DI A.

INFATTI, SE $\mathcal{C} = \{e_1, \dots, e_4\}$ È LA BASE CANONICA:

$A = M^{\mathcal{C}}_{\mathcal{C}}(L_A)$;

$$\underbrace{M^{\mathcal{B}}_{\mathcal{B}}(L_A)}_{\downarrow} = \underbrace{M^{\mathcal{B}}_{\mathcal{C}}(id_{\mathbb{R}^4})}_{=M^{\mathcal{C}}_{\mathcal{B}}(id_{\mathbb{R}^4})^{-1}} \underbrace{M^{\mathcal{C}}_{\mathcal{C}}(L_A)}_{=A} \underbrace{M^{\mathcal{C}}_{\mathcal{B}}(id_{\mathbb{R}^4})}_{=:S}$$

DIAGONALE, IN QUANTO $L_A(v_i) = \lambda_i v_i$, CON λ_i L'AUTOVALORE CORRISPONDENTE.

⇝ CERCHIAMO GLI AUTOSPAZI DI A.

- **Aut(1)?**

$Aut(1) = Ker(A-I)$. ! $\dim(Aut(1)) = m_G(1) = 1$

$$A-I = \begin{pmatrix} 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

1ª COLONNA NULLA → $e_1 \in A-I$
$\Rightarrow \langle e_1 \rangle \subseteq Aut(1)$.

$\Rightarrow Aut(1) = \langle e_1 \rangle$, $v_1 = e_1$.

- **Aut(2)?**

$$A-2I = \begin{pmatrix} -1 & 2 & 0 & 4 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \xrightarrow[\times \text{ RIGHE}]{\text{RIDUCO}} \begin{pmatrix} -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\leadsto \begin{cases} -x_1 + 2x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{cases} \Rightarrow Aut(2) = \{(2x_2, x_2, 0, 0) \mid x_2 \in \mathbb{R}\}$$

$\Rightarrow Aut(2) = \langle (2,1,0,0) \rangle$, $v_2 = (2,1,0,0)$.

- **Aut(3)**

$$A-3I = \begin{pmatrix} -2 & 2 & 0 & 4 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \leadsto \begin{cases} -2x_1 + 2x_2 + 4x_4 = 0 \\ -x_2 + 3x_3 + x_4 = 0 \end{cases}$$

$$\to \begin{cases} x_1 = x_2 + 2x_4 \\ x_2 = 3x_3 + x_4 \end{cases} \to \begin{cases} x_1 = 3x_3 + 3x_4 \\ x_2 = 3x_3 + x_4 \end{cases}$$

$\to Aut(3) = Ker(A-3I) = \{(3x_3+3x_4, 3x_3+x_4, x_3, x_4) \mid x_3, x_4 \in \mathbb{R}\}$

$= \{x_3(3,3,1,0) + x_4(3,1,0,1) \mid x_3, x_4 \in \mathbb{R}\}$

$= \langle \underbrace{(3,3,1,0)}_{v_3}, \underbrace{(3,1,0,1)}_{v_4} \rangle$

$\Rightarrow \mathcal{B} = \{v_1, v_2, v_3, v_4\} = \{(1000), (2100), (3310), (3101)\}$.

$$\Rightarrow S = M^{\mathcal{C}}_{\mathcal{B}}(id_{\mathbb{R}^4}) = \begin{pmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \checkmark//.$$

$$\left[\text{E } S^{-1}AS = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \right]$$

(7) $V = \mathbb{R}[t]_{\leq 2}$. $\begin{cases} T: V \to V \\ p(t) \mapsto p(t+1) \end{cases}$

$\mathcal{B} = (1, t, t^2 - \frac{2}{3})$. $M^{\mathcal{B}}_{\mathcal{B}}(T)$?

$T(1) = 1 = 1 + 0 \cdot t + 0 \cdot (t^2 - \frac{2}{3})$.

$T(t) = t+1 = 1 + 1 \cdot t + 0 \cdot (t^2 - \frac{2}{3})$

$T(t^2 - \frac{2}{3}) = (t+1)^2 - \frac{2}{3} = 1 + 2 \cdot t + 1 \cdot (t^2 - \frac{2}{3})$

$$\Rightarrow M^{\mathcal{B}}_{\mathcal{B}}(T) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

AUTOVALORI E AUTOSPAZI DI T?

$A := M^{\mathcal{B}}_{\mathcal{B}}(T)$.

$$P_A(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3.$$

(↑ SVILUPPO COLONNA × COLONNA)

$\Rightarrow T$ HA SOLO AUTOVALORE 1.

Aut(1)?

$$A - 1 \cdot I = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow Ker(A-I) = \{(c,0,0) \mid c \in \mathbb{R}\}$$

$\Rightarrow Ker(T - id_V) = \{c \cdot 1 + 0 \cdot t + 0 \cdot (t^2 - \frac{2}{3}) \mid x \in \mathbb{R}\}$
$= \{c \mid c \in \mathbb{R}\}$

$\Rightarrow m_G(1) = 1 < 3 = m_A(1)$

$\Rightarrow T$ NON DIAG. BILE ✗