Correzione Pre Appello A

Es. 2

$$(A+B)(A-B)=A^2-B^2$$

a) $$\lim_{n\to+\infty}\left[\sqrt{n(n+1)}-\sqrt{n(n-1)}\right]$$

FORMA INDETERMINATA $+\infty-\infty$

$$\lim_{n\to+\infty}\left[\sqrt{n(n+1)}-\sqrt{n(n-1)}\right]\cdot\frac{\left[\sqrt{n(n+1)}+\sqrt{n(n-1)}\right]}{\left[\sqrt{n(n+1)}+\sqrt{n(n-1)}\right]}=$$

$$=\lim_{n\to+\infty}\frac{n(n+1)-n(n-1)}{\sqrt{n(n+1)}+\sqrt{n(n-1)}}=\lim_{n\to+\infty}\frac{\cancel{n^2}+n-\cancel{n^2}+n}{\underbrace{\sqrt{n^2+n}}+\underbrace{\sqrt{n^2-n}}}=$$

raccolgo n^2

$$=\lim_{n\to+\infty}\frac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)}=\frac{2}{2}=1$$

$\frac{1}{n}\to 0$ $\qquad$ $\frac{1}{n}\to 0$

b) $\lim_{x \to 0} \dfrac{\boxed{\sqrt{1+\arctan^3 x} - 1}}{x\,\boxed{(1-\cos x)}}$

FORMA INDETERMINATA $\dfrac{0}{0}$

$\lim_{x\to 0} \dfrac{x^2}{\boxed{1-\cos x}} \cdot \dfrac{1}{x^2} \cdot \dfrac{\boxed{(\sqrt{1+\arctan^3 x} - 1)}}{\arctan^3 x} \cdot$

$\cdot \arctan^3 x \cdot \dfrac{1}{x} =$

$\to 2$ $\quad \to \frac{1}{2}$

$= \lim_{x\to 0} \cancel{2} \cdot \dfrac{1}{\cancel{2}} \cdot \dfrac{\arctan^3 x}{x^3} = 1$

$\to 1^3$

$\lim_{x \to 0} \dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim_{x\to 0} \dfrac{\tan x}{x} = 1$

$\lim_{x\to +\infty} \left(1+\dfrac{1}{x}\right)^x = e$

$\lim_{x\to +\infty} \left(1+\dfrac{n}{x}\right)^x = e^n$

$\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$

$\lim_{x\to 0} \dfrac{\ln(x+1)}{x} = 1$

$\lim_{x\to 0} \dfrac{\arctan x}{x} = 1$

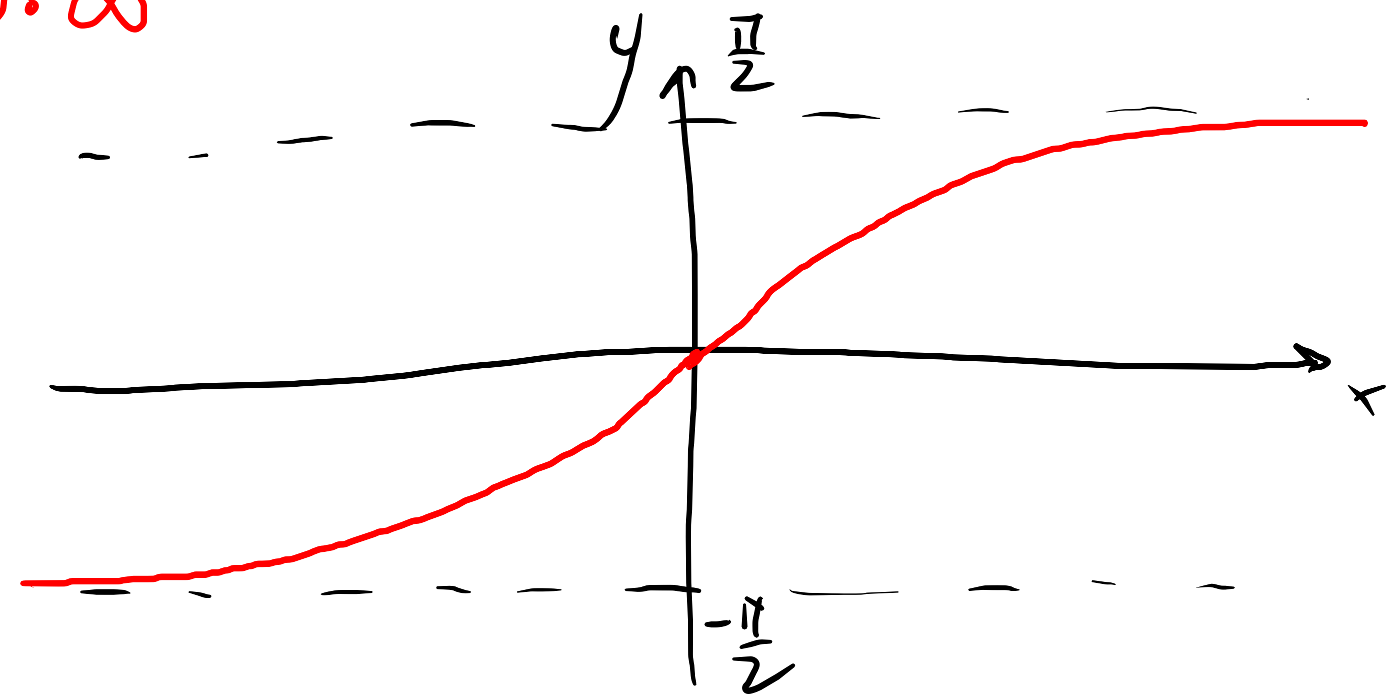
$\lim_{x\to 0} \dfrac{(1+x)^\alpha - 1}{x} = \alpha$

$$\lim_{\boxed{x\to+\infty}} x\left(\arctan x-\frac{\pi}{2}\right)=$$

$$\lim_{x\to+\infty}\arctan x=\frac{\pi}{2}$$

$$\lim_{x\to-\infty}\arctan x=-\frac{\pi}{2}$$

FORMA INDETERMINATA $0\cdot\infty$



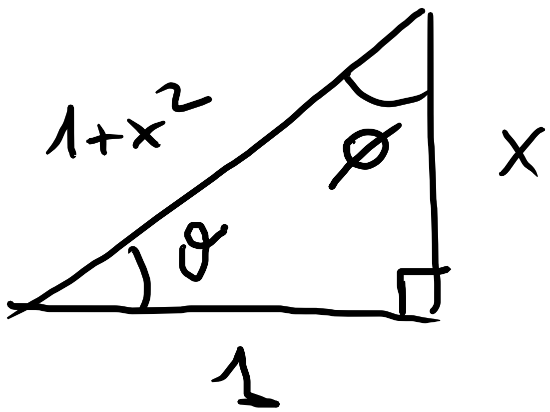
$$\lim_{x\to+\infty}\frac{\arctan x-\frac{\pi}{2}}{1/x}$$

F. INDET. $\frac{0}{0}$

Applico De Hop.

$$\lim_{x\to+\infty}\frac{\frac{d}{dx}\left(\arctan x-\frac{\pi}{2}\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}=$$

$$=\lim_{x\to+\infty}\frac{\frac{1}{1+x^2}}{\left(-\frac{1}{x^2}\right)}=\lim_{x\to+\infty}-\frac{x^2}{1+x^2}=-1\overset{\text{De Hop.}}{\Longrightarrow}\lim_{x\to+\infty}\frac{\arctan x-\frac{\pi}{2}}{1/x}=-1$$



$$\tan\vartheta = x$$

$$\vartheta = \arctan x$$

$$\tan\phi = \frac{1}{x}$$

$$\vartheta + \phi = \frac{\pi}{2}$$

$$\phi = \arctan\frac{1}{x}$$

$$\boxed{\arctan\left(\frac{1}{x}\right) + \arctan x = \frac{\pi}{2}}$$

$$\lim_{x\to 0} \frac{\arctan t}{t} \overset{!}{=} 1$$

sost $x = \frac{1}{t}$

$$\arctan x = \frac{\pi}{2} - \arctan\left(\frac{1}{x}\right) \longrightarrow \lim_{x\to+\infty} -x \cdot \arctan\left(\frac{1}{x}\right)$$

Es 1 $\quad C_n = n(n^2+8)$ è un multiplo di 3?

• Base dell'induzione:

Provo per $n=0$ $\quad C_0 = 0 \cdot (0^2+8) = 0$ è multiplo di 3

Vale per $n=0$

• Passo dell'induzione

Suppongo vero che C_n sia multiplo di 3 (ipotesi induttiva), provo che C_{n+1} sia multiplo di 3.

$\cancel{C_{n+1} = C_n + 1}$ $\qquad C_{n+1} = (n+1)\left[(n+1)^2+8\right]$

$$C_{n+1} = (n+1)\left[(n+1)^2 + 8\right] \qquad \boxed{C_n = n(n^2+8)}$$

$$C_{n+1} = (n+1)\left[n^2 + 2n + 1 + 8\right]$$

$$C_{n+1} = n\left[(n^2+8) + (2n+1)\right] + \left[n^2 + 2n + 9\right] \cdot 1$$

$$= \underbrace{n(n^2+8)}_{\downarrow} + n(2n+1) + n^2 + 2n + 9$$

$$= C_n + 3n^2 + 3n + 9$$

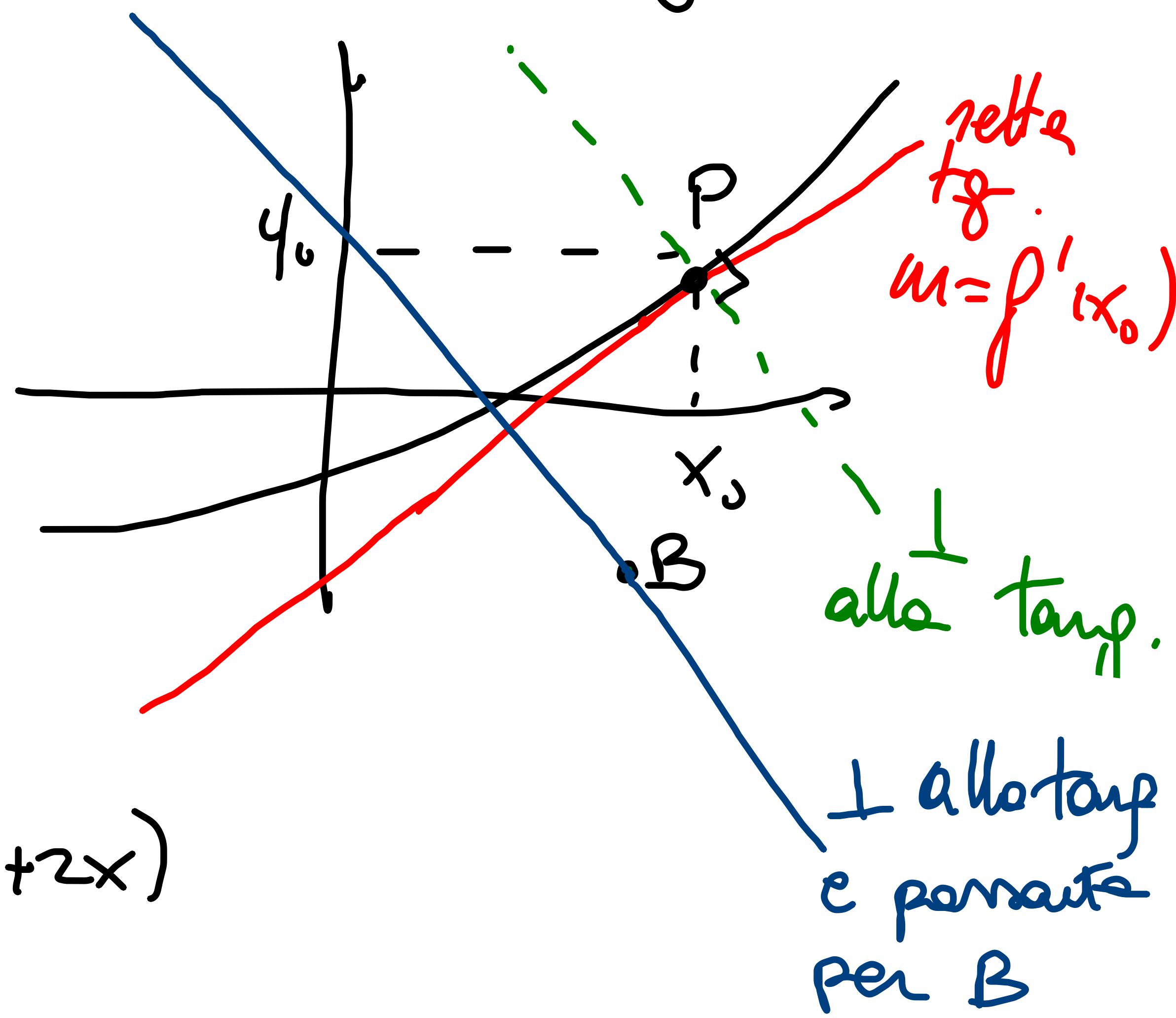
$$= \underbrace{C_n}_{\text{multiplo di 3 per ipotesi}} + \underbrace{3(n^2+n+3)}_{\text{è multiplo di 3}} \Longrightarrow C_{n+1} \text{ è multiplo di 3}$$

$$\lim_{n\to+\infty} \frac{C_{n+1}}{3C_n} = \lim_{n\to+\infty} \frac{(n+1)(n^2+2n+9)}{3n(n^2+8)} =$$

$$= \lim_{n\to+\infty} \frac{n^3+\cdots}{3n^3+\cdots} = \frac{1}{3}$$

Es 3

$f(x) = (1+2x)^x$ retta $\perp$ alla retta tg in $P = (1,3)$



retta passante per un punto

$y - y_P = m(x - x_P)$

Retta tangente a $f(x)$ in x_P

$y - y_P = f'(x_P)(x - x_P)$

$$f(x) = e^{\ln(1+2x)^x} = e^{x\ln(1+2x)}$$

$$f'(x) = \underbrace{e^{x\ln(1+2x)}}_{=} \cdot \overbrace{\left(\ln(1+2x) + \frac{2x}{1+2x}\right)}^{\text{derivata dell'esponente}}$$

$$= (1+2x)^x \cdot \left(\ln(1+2x) + \frac{2x}{1+2x}\right)$$

$$m_t = f'(x_P) = f'(1) = 3 \cdot \left(\ln 3 + \frac{2}{3}\right) = 3\ln 3 + 2$$

$$m_\perp \cdot m_t = -1 \qquad m_\perp = -\frac{1}{m_t} = -\frac{1}{3\ln 3 + 2}$$

$$y - y_B = m_\perp (x - x_B) \qquad B = (0,3)$$

$$y - 3 = -\frac{1}{3\ln 3 + 2} \cdot x \implies \boxed{y = -\frac{x}{3\ln 3 + 2} + 3}$$

Es 4

$$f(x) = \frac{x^2}{1-\ln|x|}$$

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

Dominio:

$$\begin{cases} |x| > 0 \rightarrow x \neq 0 \\ 1-\ln|x| \neq 0 \rightarrow \ln|x| \neq 1 \quad |x| \neq e \end{cases}$$

$e \qquad e$

$\rightarrow x \neq \pm e$

Positività

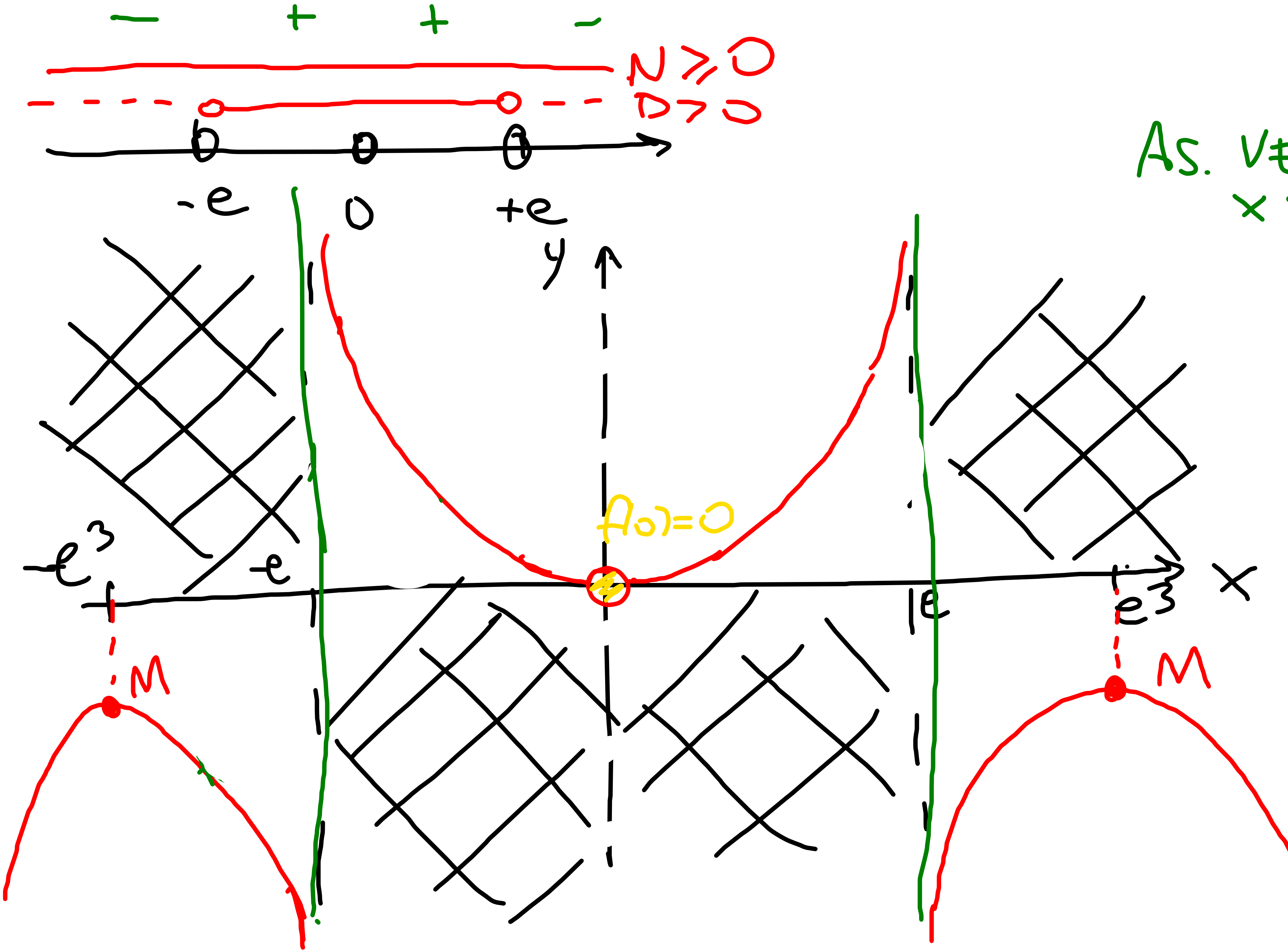
$$\frac{x^2}{1-\ln|x|} \geq 0$$

$N \geq 0$ $x^2 \geq 0$ $\forall x \in \mathbb{R}$

$D > 0$ $1-\ln|x| > 0$

$\ln|x| < 1$

$|x| < e$



AS. VERTICALI

$x = \pm e$

Intersezione asse y

$$\begin{cases} y = f(x) \\ x = 0 \end{cases} \longrightarrow \text{fuori dominio}$$

Intersezione asse x

$$\begin{cases} y = f(x) \\ y = 0 \end{cases} \longrightarrow 0 = \frac{x^2}{1 - \ln|x|}$$

$x^2 = 0 \quad x = 0$

Soluzione da escludere per dominio

NO INTERSEZIONI

Limiti per punti particolari del dominio

$$\lim_{x \to \pm\infty} \frac{x^2}{1-\ln|x|} = -\infty$$

De Hop.

De Hop. $$\lim_{x \to \pm\infty} \frac{2x}{-\frac{1}{x}} = \lim_{x \to \pm\infty} -2x^2 = -\infty$$

$$\lim_{x \to \pm e} \frac{x^2}{1-\ln|x|} = \infty \Longrightarrow$$ ASINTOTO VERTICALE in $x = \pm e$

$$\boxed{\lim_{x \to 0} \frac{x^2}{1-\ln|x|} = 0}$$

Intorno allo 0 $\quad f(x) \simeq \frac{x^2}{1} = x^2$

$$f'(x) = \frac{2x\,(1-\ln|x|) - x^2\left(-\frac{1}{\cancel{x}}\right)}{(1-\ln|x|)^2}$$

$$= \frac{x\,(3-\ln|x|)}{(1-\ln|x|)^2}$$

$f'(x) = 0$ (cerco punti stazionari)

$x(3-\ln|x|) = 0$

- $x = 0 \longrightarrow$ fuori dominio
- $3 - \ln|x| = 0 \qquad x = \pm e^3$

$f'(x) \geq 0$

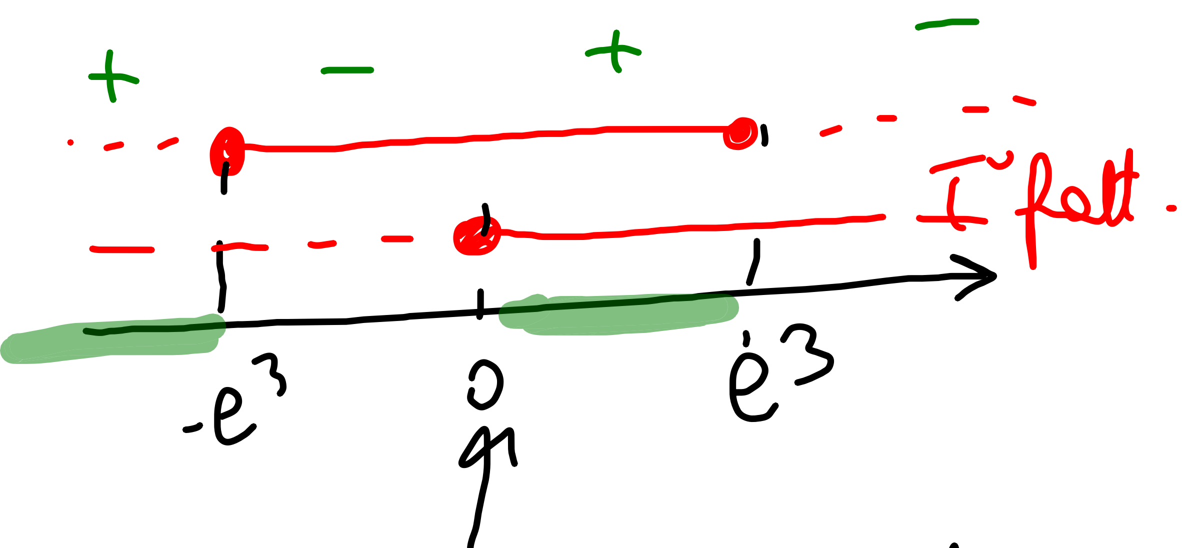
$$\frac{x(3-\ln|x|)}{(1-\ln|x|)^2} \geq 0$$

$N \geq 0$ $\quad x(3-\ln|x|) \geq 0$

$x \geq 0$ I° fatt.

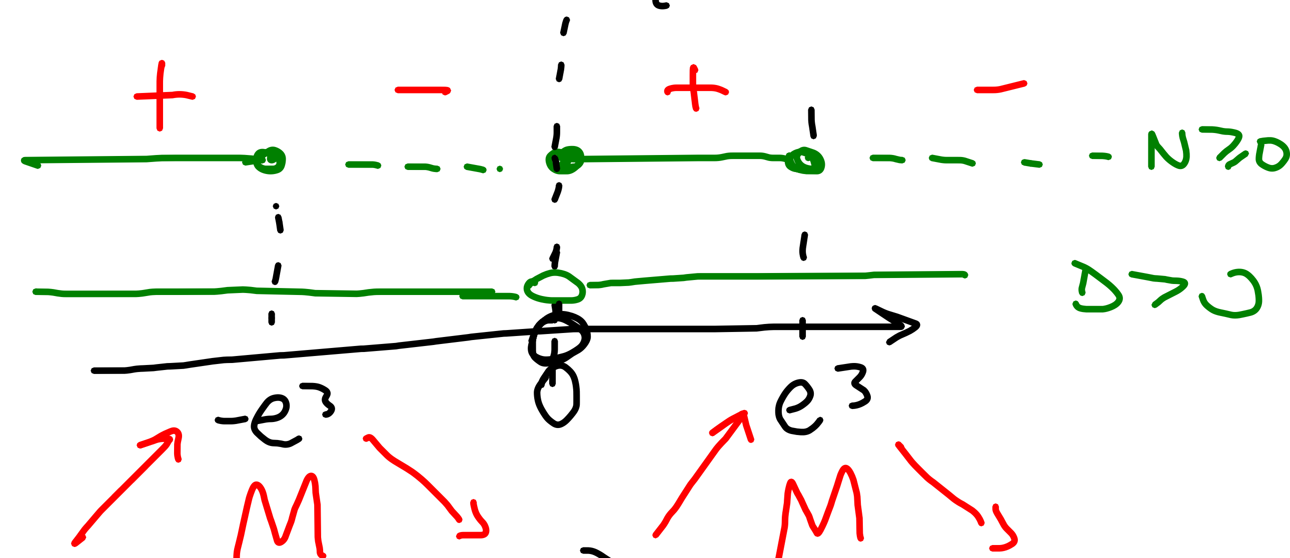
$|x| \leq e^3$ II° fatt.

$D > 0 \rightarrow x \neq 0$



Solo per il numeratore

MAX in $x = \pm e^3$



Per $f'(x)$

fuori dominio

Se pongo $\boxed{f(0)=0}$ la funzione è continua nell'origine?

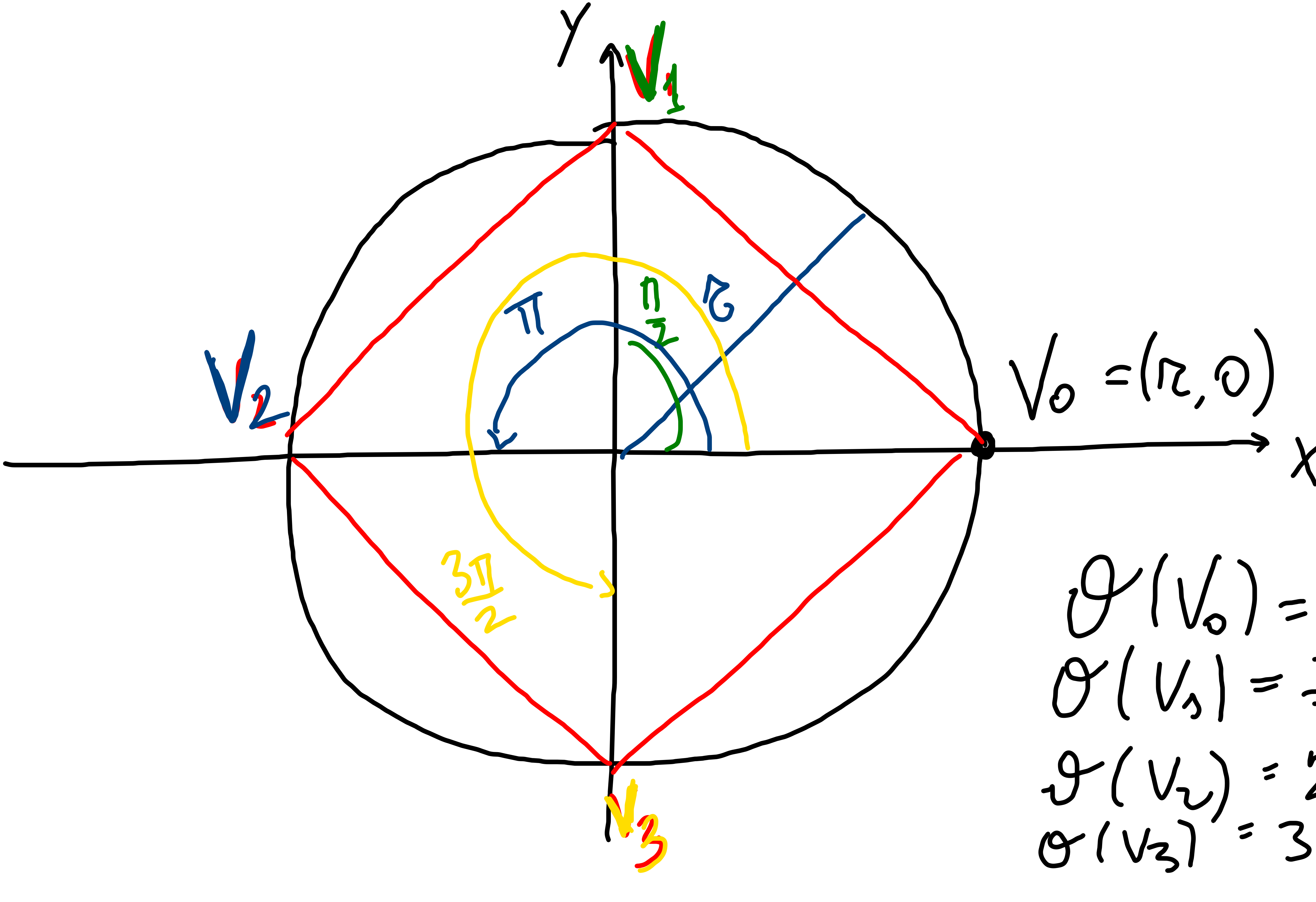
Sì, perché $\lim_{x\to 0} f(x)=0 = f(0)$

$$f''(x) = \Big\{ (1-\ln|x|)^2 \cdot \left(1(3-\ln|x|) + \cancel{x}\left(-\frac{1}{\cancel{x}}\right)\right) -$$

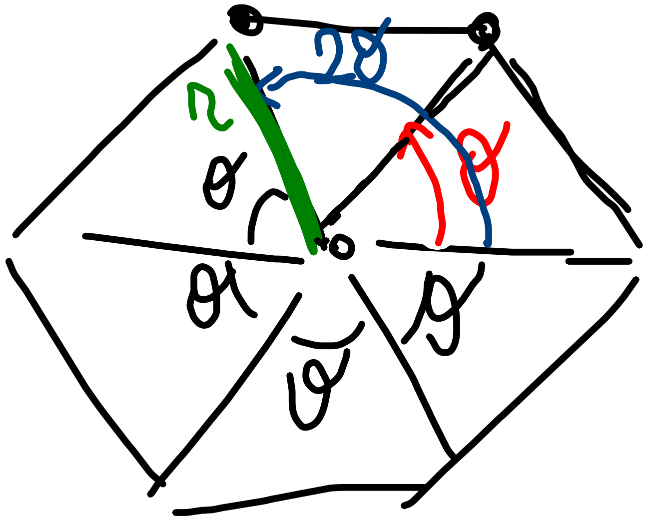
$$- 2(1-\ln|x|) \cdot \left(-\frac{1}{\cancel{x}}\right) \cdot \left(\cancel{x}(3-\ln|x|)\right) \Big\} \cdot \frac{1}{(1-\ln|x|)^4}$$

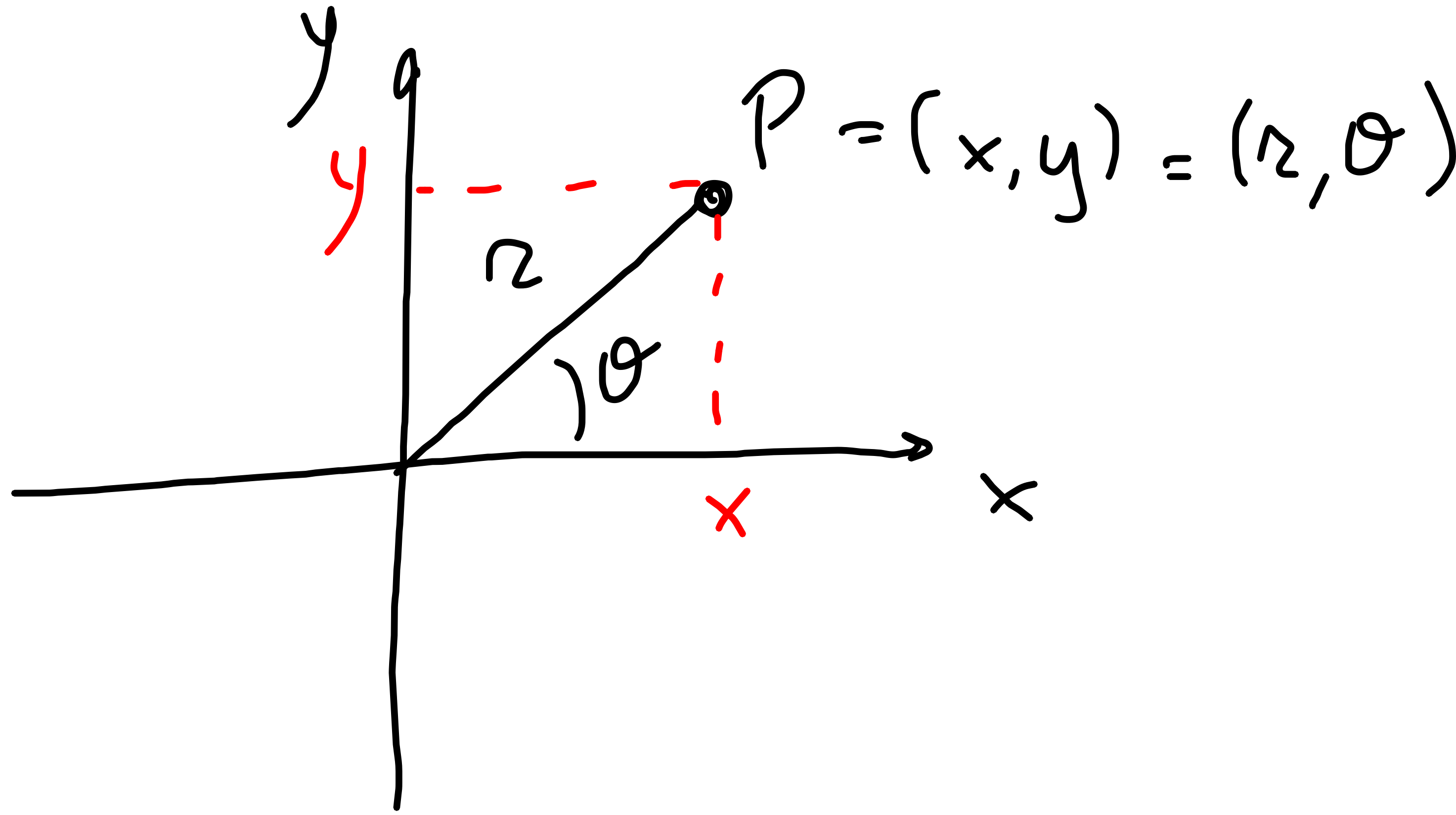
$$= \frac{(1-\ln|x|)^{\cancel{2}}(2-\ln|x|) + 2\cancel{(1-\ln|x|)}(3-\ln|x|)}{(1-\ln|x|)^{\cancel{4}\,3}}$$

$$= \frac{(1-\ln|x|)(2-\ln|x|) + 2(3-\ln|x|)}{(1-\ln|x|)^3}$$



$$\theta(V_0) = 0$$
$$\theta(V_1) = \frac{\pi}{2}$$
$$\theta(V_2) = 2 \cdot \frac{\pi}{2}$$
$$\theta(V_3) = 3 \cdot \frac{\pi}{2}$$





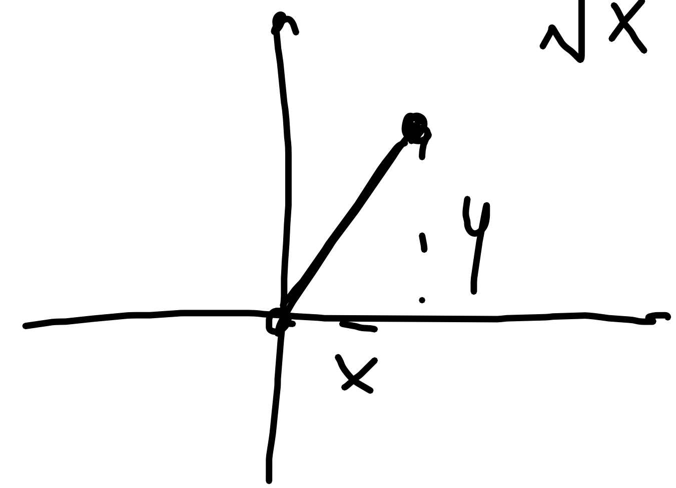
$$x = r\cos\theta$$
$$y = r\sin\theta$$

Nel problema r non cambia

$$\theta_n = n \cdot \frac{2\pi}{N}$$

$$\begin{cases} x = r\cos\left(n\frac{2\pi}{N}\right) \\ y = r\sin\left(n\frac{2\pi}{N}\right) \end{cases}$$

$$\theta = \frac{2\pi}{N}$$



$$\sqrt{x^2+y^2}$$

Sono punti

Sono punti

$$\ell_n = \| V_n - V_{n-1} \| =$$

$$\| \left(r\cos(n\theta) - r\cos((n-1)\theta),\ r\sin(n\theta) - r\sin((n-1)\theta) \right) \|$$

$$= r \left\{ \cos^2(n\theta) + \cos^2((n-1)\theta) - 2\cos(n\theta)\cos((n-1)\theta) + \sin^2(n\theta) + \sin^2((n-1)\theta) - 2\sin(n\theta)\sin((n-1)\theta) \right\}^{\frac{1}{2}}$$

1

1

$$l_n = r\left(2 - 2\cos(n\theta)\cos((n-1)\theta) - 2\sin(n\theta)\sin((n-1)\theta)\right)^{\frac{1}{2}}$$

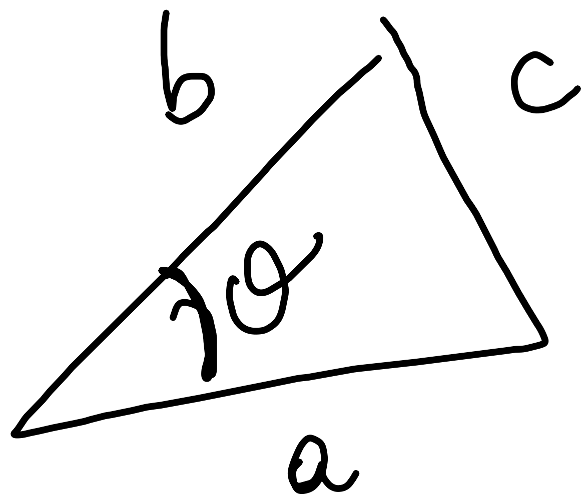
$$= r\left(2 - 2\cos\left(n\theta + (n-1)\theta\right)\right)^{\frac{1}{2}}$$

$$= r\left(2 - 2\cos\left(\theta(2n-1)\right)\right)^{\frac{1}{2}}$$

$$=$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\ell = \sqrt{\ell^2} = 2r\sin\left(\frac{\pi}{N}\right)$$



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$a = b = r \qquad c = \ell \qquad \theta = \frac{2\pi}{N}$$

$$\ell^2 = r^2 + r^2 - 2r^2\cos\theta$$

$$\overset{!}{=} 2r^2(1 - \cos\theta) = 2r^2\left(\overbrace{1 - \cos^2\frac{\theta}{2}}^{\sin^2\frac{\theta}{2}} + \sin^2\frac{\theta}{2}\right)$$

$$= 2r^2 \cdot 2\sin^2\frac{\theta}{2} = 4r^2\sin^2\frac{\cancel{2}\pi}{\cancel{2}N}$$