Ex6

martedì 18 gennaio 2022 17:46

$U \subseteq \mathbb{R}^3$ SPAZIO DELLE SOL. DI

$$\begin{cases} x_1+x_2+2x_3=0 \\ x_1-x_2+x_3=0 \end{cases}$$

(i) DIM. E BASE O.N. DI U.

RISOLVO IL SISTEMA

$$\begin{cases} x_1+x_2+2x_3=0 \\ x_1-x_2+x_3=0 \end{cases} \xrightarrow{R_2+R_1} \begin{cases} x_1+x_2+2x_3=0 \\ 2x_1 \qquad +3x_3=0 \end{cases}$$

$$\rightarrow \begin{cases} x_1=-\frac{3}{2}x_3 \\ x_2=-x_1-2x_3=\left(\frac{3}{2}-2\right)x_3=-\frac{1}{2}x_3 \end{cases}$$

$$\Rightarrow U=\left\{ x=(x_1,x_2,x_3)\in\mathbb{R}^3 \;\middle|\; \begin{matrix} x_1=-\frac{3}{2}x_3 \\ x_2=-\frac{1}{2}x_3 \end{matrix} \right\}$$

$$=\left\{ x=\left(-\frac{3}{2}c,-\frac{1}{2}c,c\right) \mid c\in\mathbb{R} \right\}$$

$$=\left\langle \left(-\frac{3}{2},-\frac{1}{2},1\right)\right\rangle=\langle(3,1,-2)\rangle.$$

$\rightarrow \dim U = 1.$ ✓

BASE O.N.?

$\{v=(3,1,-2)\}$ BASE DI U

$$\rightarrow \quad u=\frac{v}{\|v\|}=\frac{1}{\sqrt{3^2+1^2+(-2)^2}}(3,1,-2)=\frac{1}{\sqrt{14}}(3,1,-2)$$

$\rightarrow \{u\}$ BASE O.N. DI U ✓✓

(ii) EQUAZIONE LINEARE IN x_1,x_2,x_3 CHE ABBIA $U^\perp$ COME SP. DELLE SOL.?

$$U^\perp=\{x\in\mathbb{R}^3 \mid \langle x,u\rangle=0\}.$$

$$\langle x,u\rangle=0 \iff \underbrace{\frac{3}{\sqrt{14}}x_1+\frac{1}{\sqrt{14}}x_2-\frac{2}{\sqrt{14}}x_3=0}$$

$\rightarrow U^\perp$ SP. DELLE SOL. DI QUESTA EQ. ✓✓
(O MOLTIPLICATA PER $\sqrt{14}$)

(iii) ESTENDERE $\{u\}$ A UNA BASE O.N. DI $\mathbb{R}^3$.

EQUIVALE A TROVARE $\{u_1,u_2\}$ BASE O.N. DI $U^\perp$

($\Rightarrow \{u, u_1, u_2\}$ BASE O.N. DI $\mathbb{R}^3$)

$U^\perp=\{$SOLUZIONI DI $3x_1+x_2-2x_3=0\}$ $\rightarrow$ RICAVO $x_2=-3x_1+2x_3$

$$\rightarrow U^\perp=\{(x_1,-3x_1+2x_3,x_3) \mid x_1,x_3\in\mathbb{R}\}$$

$$\rightarrow U^\perp=\langle\underbrace{(1,-3,0)}_{v_1},\underbrace{(0,2,1)}_{v_2}\rangle.$$

ORTONORMALIZZO v_1,v_2:

$$u_1=\frac{v_1}{\|v_1\|}=\frac{1}{\sqrt{1^2+(-3)^2+0^2}}(1,-3,0)=\frac{1}{\sqrt{10}}(1,-3,0).$$

$$v_2'=v_2-\langle v_2,u_1\rangle u_1=$$

$$=(0,2,1)-\left(0\cdot\frac{1}{\sqrt{10}}+2\cdot\left(\frac{-3}{\sqrt{10}}\right)+1\cdot\frac{0}{\sqrt{10}}\right)\cdot\frac{1}{\sqrt{10}}(1,-3,0)$$

$$=(0,2,1)+\frac{6}{10}(1,-3,0)=$$

$$=\left(0+\frac{6}{10},\frac{20}{10}-\frac{18}{10},\frac{10}{10}+0\right)=\left(\frac{6}{10},\frac{2}{10},\frac{10}{10}\right)$$

$$=\frac{1}{5}(3,2,5).$$

$$\rightarrow u_2=\frac{v_2'}{\|v_2'\|}=\frac{5}{\sqrt{3^2+2^2+5^2}}\;\frac{1}{5}(3,2,5)=\frac{1}{\sqrt{38}}(3,2,5)$$

$\rightarrow$ BASE O.N. DI $\mathbb{R}^3$: $\{u, u_1, u_2\}$ ✓✓✓