# Ex3

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$$W=\{(x_1,x_2,x_3)\in\mathbb{R}^3 \mid 2x_1+x_2=0\}\subseteq\mathbb{R}^3$$

1) BASE O.N. DI $W$?

2) COMPLETARE A BASE $\mathcal{B}$ O.N. DI $\mathbb{R}^3$.

3) $M_{\mathcal{B}}^{\mathcal{C}}$? $M_{\mathcal{C}}^{\mathcal{B}}$? (con $\mathcal{C}$ CANONICA)

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1) $W=\{(x_1,x_2,x_3)\in\mathbb{R}^3 \mid x_2=-2x_1,\ x_1,x_3\in\mathbb{R}\}$

$=\{(x_1,-2x_1,x_3)\in\mathbb{R}^3 \mid x_1,x_3\in\mathbb{R}\}$

$=\langle(1,-2,0),(0,0,1)\rangle$ ← BASE DI $W$

ORTONORMALIZZIAMO

$\langle(1,-2,0),(0,0,1)\rangle = 1\cdot 0+(-2)\cdot 0+0\cdot 1=0$

→ SONO GIÀ ORTOGONALI →

DOBBIAMO SOLO NORMALIZZARE.

$$\Rightarrow \mathcal{B}_W=\left\{\frac{(1,2,0)}{\|(1,2,0)\|},\frac{(0,0,1)}{\|(0,0,1)\|}\right\}=$$

$$=\left\{\left(\frac{1}{\sqrt{5}},\frac{-2}{\sqrt{5}},0\right),(0,0,1)\right\}.$$

2) CALCOLIAMO $W^\perp$ E TROVIAMO BASE O.N. $\mathcal{B}_{W^\perp}$ DI $W^\perp$

$\Rightarrow \mathcal{B}=\mathcal{B}_W\cup\mathcal{B}_{W^\perp}$ BASE O.N. DI $\mathbb{R}^3$.

$$W^\perp=\{x\in\mathbb{R}^3 \mid \langle x,w\rangle=0\ \forall w\in W\}=$$

$$=\{x\in\mathbb{R}^3 \mid \langle x,w\rangle=0\ \forall w\in\mathcal{B}_W\}$$

$=$ ( SOLUZIONI DI $\begin{cases}\langle x,\frac{1}{\sqrt{5}}(1,-2,0)\rangle=0\\ \langle x,(0,0,1)\rangle=0\end{cases}$ )

$$\begin{cases}\langle x,\frac{1}{\sqrt{5}}(1,-2,0)\rangle=0\\ \langle x,(0,0,1)\rangle=0\end{cases}\rightarrow\begin{cases}\frac{1}{\sqrt{5}}x_1-\frac{2}{\sqrt{5}}x_2+0x_3=0\\ x_3=0\end{cases}$$

$$\rightarrow\begin{cases}x_1-2x_2=0\\ x_3=0\end{cases}\rightarrow\begin{cases}x_1=2x_2\\ x_3=0\end{cases}$$

$$\Rightarrow W^\perp=\{(2x_2,x_2,0)\mid x_2\in\mathbb{R}\}$$

$=\langle(2,1,0)\rangle=$

$=\langle\frac{(2,1,0)}{\|(2,1,0)\|}\rangle=\langle\left(\frac{2}{\sqrt{5}},\frac{1}{\sqrt{5}},0\right)\rangle$

↑ NORMALIZZIAMO ↑ BASE O.N. DI $W^\perp$

$$\Rightarrow \mathcal{B}=\left\{\underbrace{\left(\frac{1}{\sqrt{5}},\frac{-2}{\sqrt{5}},0\right),(0,0,1)}_{\mathcal{B}_W},\underbrace{\left(\frac{2}{\sqrt{5}},\frac{1}{\sqrt{5}},0\right)}_{\mathcal{B}_{W^\perp}}\right\}$$ ✓✓

3) $$M_{\mathcal{B}}^{\mathcal{C}}=M_{\mathcal{B}}^{\mathcal{C}}(id_{\mathbb{R}^3})=\begin{pmatrix}\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}}\\ -\frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}}\\ 0 & 1 & 0\end{pmatrix}.$$

$\mathcal{B}$ BASE O.N. $\Rightarrow M_{\mathcal{B}}^{\mathcal{C}}$ ORTOGONALE

$$\Rightarrow M_{\mathcal{C}}^{\mathcal{B}}=\left(M_{\mathcal{B}}^{\mathcal{C}}\right)^{-1}={}^tM_{\mathcal{B}}^{\mathcal{C}}=\begin{pmatrix}\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} & 0\\ 0 & 0 & 1\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0\end{pmatrix}$$

✓✓.