Ex4

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$v = (1,-1,0)$, $W = \langle v \rangle^\perp$.

$P_W : \mathbb{R}^3 \to W \subseteq \mathbb{R}^3$ PROIEZIONE ORTOGONALE SU W.

1) ESPRESSIONE ESPLICITA DI P_W?

2) $M_{\mathcal{C}}^{\mathcal{C}}(P_W)$? ($\mathcal{C}$ BASE CANONICA)

3) BASE O.N. CHE DIAGONALIZZA P_W?

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1) $W = \langle v \rangle^\perp = \{ x \in \mathbb{R}^3 \mid \langle x, v \rangle = 0 \}$

$= \{ (x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 - x_2 = 0 \}$ $\quad x_1, x_3 \in \mathbb{R}$

$= \{ (x_1, x_1, x_3) \in \mathbb{R}^3 \mid x_1, x_3 \in \mathbb{R} \}$

$= \langle \underbrace{(1,1,0), (0,0,1)}_{\text{BASE DI } W} \rangle$

ORTONORMALIZZIAMO

$$u_1 = \frac{(1,1,0)}{\|(1,1,0)\|} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$v_2' = (0,0,1) - \left\langle (0,0,1), \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right) \right\rangle \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)$$

$$= (0,0,1) - 0 \cdot \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right) = (0,0,1)$$

$$u_2 = \frac{(0,0,1)}{\|(0,0,1)\|} = (0,0,1)$$

$\Rightarrow \mathcal{B}_W = \{u_1, u_2\}$ BASE O.N. DI W

$\Rightarrow \mathcal{B} = \{ \underbrace{u_1, u_2}_{W}, \underbrace{\tfrac{v}{\|v\|}}_{\langle v \rangle = W^\perp} \}$ BASE O.N. DI $\mathbb{R}^3$.

$$\forall x \in \mathbb{R}^3 \quad x = \langle x, u_1 \rangle u_1 + \langle x, u_2 \rangle u_2 + \left\langle x, \tfrac{v}{\|v\|} \right\rangle \tfrac{v}{\|v\|}$$

$$\Rightarrow P_W(x) = \langle x, u_1 \rangle u_1 + \langle x, u_2 \rangle u_2 .$$

$\left(\text{O } P_W(x) = x - \left\langle x, \tfrac{v}{\|v\|} \right\rangle \tfrac{v}{\|v\|} \right)$.

SCRIVIAMO ESPLICITAMENTE:

$$\langle x, u_1 \rangle = \left\langle (x_1,x_2,x_3), \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right) \right\rangle = \tfrac{1}{\sqrt{2}} x_1 + \tfrac{1}{\sqrt{2}} x_2$$

$$\langle x, u_2 \rangle = \langle (x_1,x_2,x_3), (0,0,1) \rangle = x_3$$

$$\Rightarrow P_W(x) = \left(\tfrac{1}{\sqrt{2}} x_1 + \tfrac{1}{\sqrt{2}} x_2\right)\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right) + x_3 (0,0,1) =$$

$$= \left(\tfrac{1}{2} x_1 + \tfrac{1}{2} x_2, \tfrac{1}{2} x_1 + \tfrac{1}{2} x_2, x_3\right).$$

2)
$$M_{\mathcal{C}}^{\mathcal{C}}(P_W) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

$$\begin{cases} 1^a \text{ COLONNA} = P_W(e_1) \\ 2^a \text{ COLONNA} = P_W(e_2) \\ 3^a \text{ COLONNA} = P_W(e_3) \end{cases}$$

3) RIPRENDIAMO $\mathcal{B} = \{ \underbrace{u_1, u_2}_{\text{BASE O.N. DI } W}, \underbrace{\tfrac{v}{\|v\|}}_{\text{B O.N. DI } W^\perp} \}$.

ALLORA: $P_W(u_1) = u_1$ E $P_W(u_2) = u_2$

PERCHÉ $P_W|_W = id_W$;

E $P_W\left(\frac{v}{\|v\|}\right) = 0$ PERCHÉ $v \perp W$

$$\Rightarrow M_{\mathcal{B}}^{\mathcal{B}}(P_W) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \checkmark\checkmark$$

(PERCHÉ LE COLONNE SONO $P_W(u_1)$, $P_W(u_2)$ E $P_W\left(\frac{v}{\|v\|}\right)$ SCRITTE RISPETTO A $\mathcal{B} = \{u_1, u_2, \frac{v}{\|v\|}\}$)

! PER COMPLETEZZA:

$v = (1,-1,0) \Rightarrow \|v\| = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$

$\Rightarrow \frac{v}{\|v\|} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$. $\checkmark\checkmark\checkmark$