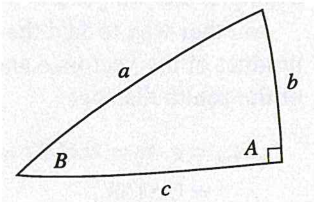
ESERCIZI

TRIGONOMETRIA SFERICA OK

For the spherical triangle we have to use the equations in (2.7), which are now simply:

$$\sin B \sin a = \sin b\,,$$
$$\cos B \sin a = \cos b \sin c\,,$$
$$\cos a = \cos b \cos c\,.$$



The first equation gives the sine of B:

$$\sin B = \sin b/\sin a\,.$$

Dividing the second equation by the third one, we get the cosine of B:

$$\cos B = \tan c/\tan a\,.$$

And the tangent is obtained by dividing the first equation by the second one:

$$\tan B = \tan b/\sin c\,.$$

The third equation is the equivalent of the Pythagorean theorem for rectangular triangles.

Example 2.2 *The Coordinates of New York City*

The geographic coordinates are 41° north and 74° west of Greenwich, or $\phi = +41°$, $\lambda = -74°$. In time units, the longitude would be $74/15\,\mathrm{h} = 4\,\mathrm{h}\ 56\,\mathrm{min}$ west of Greenwich. The geocentric latitude is obtained from

$$\tan\phi' = \frac{b^2}{a^2}\tan\phi = \left(\frac{6{,}356{,}752}{6{,}378{,}137}\right)^2 \tan 41°$$
$$= 0.86347 \quad\Rightarrow\quad \phi' = 40°\ 48'\ 34''\,.$$

The geocentric latitude is $11'\ 26''$ less than the geographic latitude.

THE HOUR CIRCLES CONVERGENCE OF TO CELESTIAL POLES

Example 2.3 The angular separation of two objects in the sky is quite different from their coordinate difference.

Suppose the coordinates of a star A are $\alpha_1 = 10\,\mathrm{h}$, $\delta_1 = 70°$ and those of another star B, $\alpha_2 = 11\,\mathrm{h}$, $\delta_2 = 80°$.

Using the Pythagorean theorem for plane triangles, we would get

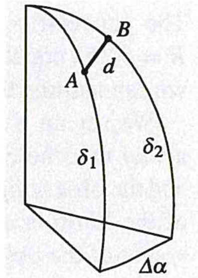
$$d = \sqrt{(15°)^2 + (10°)^2} = 18°\,.$$

NO

TEORIA

But if we use the third equation in (2.7), we get

$$\begin{aligned}\cos d &= \cos(\alpha_1 - \alpha_2)\\ &\quad\times \sin(90° - \delta_1)\sin(90° - \delta_2)\\ &\quad+ \cos(90° - \delta_1)\cos(90° - \delta_2)\\ &= \cos(\alpha_1 - \alpha_2)\cos\delta_1\cos\delta_2\\ &\quad+ \sin\delta_1\sin\delta_2\\ &= \cos 15° \cos 70° \cos 80°\\ &\quad+ \sin 70° \sin 80°\\ &= 0.983\,,\end{aligned}$$



which yields $d = 10.6°$. The figure shows why the result obtained from the Pythagorean theorem is so far from being correct: hour circles (circles with $\alpha = \text{constant}$) approach each other towards the poles and their angular separation becomes smaller, though the coordinate difference remains the same.

Example 2.4 Find the altitude and azimuth of the Moon in Helsinki at midnight at the beginning of 1996.

The right ascension is $\alpha = 2\,\mathrm{h}\ 55\,\mathrm{min}\ 7\,\mathrm{s} = 2.9186\,\mathrm{h}$ and declination $\delta = 14°\ 42' = 14.70°$, the sidereal time is $\Theta = 6\,\mathrm{h}\ 19\,\mathrm{min}\ 26\,\mathrm{s} = 6.3239\,\mathrm{h}$ and latitude $\phi = 60.16°$.

The hour angle is $h = \Theta - \alpha = 3.4053\,\mathrm{h} = 51.08°$. Next we apply the equations in (2.16):

$$\sin A \cos a = \sin 51.08° \cos 14.70° = 0.7526\,,$$
$$\begin{aligned}\cos A \cos a &= \cos 51.08° \cos 14.70° \sin 60.16°\\ &\quad - \sin 14.70° \cos 60.16°\\ &= 0.4008\,,\end{aligned}$$
$$\begin{aligned}\sin a &= \cos 51.08° \cos 14.70° \cos 60.16°\\ &\quad + \sin 14.70° \sin 60.16°\\ &= 0.5225\,.\end{aligned}$$

Thus the altitude is $a = 31.5°$. To find the azimuth we have to compute its sine and cosine:

$$\sin A = 0.8827\,,\qquad \cos A = 0.4701\,.$$

Hence the azimuth is $A = 62.0°$. The Moon is in the southwest, 31.5 degrees above the horizon. Actually, this would be the direction if the Moon were infinitely distant.

GOOD APPROXIMATION

$$d = \sqrt{15^2 \cdot \cos^2 75° + (10°)^2} \approx 10{,}7°$$

$$\downarrow \frac{70+80}{2}$$

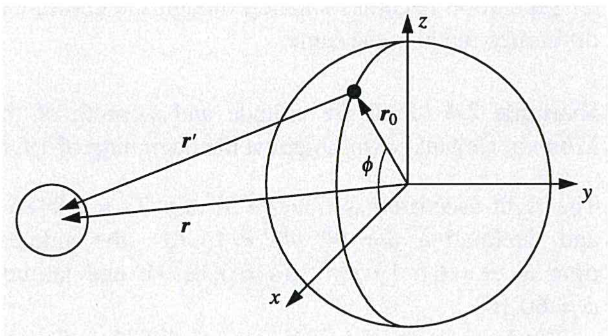
TEORIA RISING AND SETTING TIMES
2.6

Example 2.5 Find the topocentric place of the Moon in the case of the previous example.

The geocentric distance of the Moon at that time is $R = 62.58$ equatorial radii of the Earth. For simplicity, we can assume that the Earth is spherical.

We set up a rectangular coordinate frame in such a way that the z axis points towards the celestial pole and the observing site is in the xz plane. When the radius of the Earth is used as the unit of distance, the radius vector of the observing site is

$$\boldsymbol{r}_0 = \begin{pmatrix} \cos\phi \\ 0 \\ \sin\phi \end{pmatrix} = \begin{pmatrix} 0.4976 \\ 0 \\ 0.8674 \end{pmatrix} .$$



The radius vector of the Moon is

$$\boldsymbol{r} = R \begin{pmatrix} \cos\delta\cos h \\ -\cos\delta\sin h \\ \sin\delta \end{pmatrix} = 62.58 \begin{pmatrix} 0.6077 \\ -0.7526 \\ 0.2538 \end{pmatrix} .$$

The topocentric place of the Moon is

$$\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{r}_0 = \begin{pmatrix} 37.53 \\ -47.10 \\ 15.02 \end{pmatrix} .$$

We divide this vector by its length 62.07 to get the unit vector $\boldsymbol{e}$ pointing to the direction of the Moon. This can be expressed in terms of the topocentric coordinates δ' and h':

$$\boldsymbol{e} = \begin{pmatrix} 0.6047 \\ -0.7588 \\ 0.2420 \end{pmatrix} = \begin{pmatrix} \cos\delta'\cos h' \\ -\cos\delta'\sin h' \\ \sin\delta' \end{pmatrix} ,$$

which gives $\delta' = 14.00°$ and $h' = 51.45°$. Next we can calculate the altitude and azimuth as in the previous example, and we get $a = 30.7°$, $A = 61.9°$.

Another way to find the altitude is to take the scalar product of the vectors $\boldsymbol{e}$ and $\boldsymbol{r}_0$, which gives the cosine of the zenith distance:

$$\cos z = \boldsymbol{e} \cdot \boldsymbol{r}_0 = 0.6047 \times 0.4976 + 0.2420 \times 0.8674 = 0.5108 ,$$

whence $z = 59.3°$ and $a = 90° - z = 30.7°$. We see that this is 0.8° less than the geocentric altitude; i.e. the difference is more than the apparent diameter of the Moon.

11'

Example 2.6 The coordinates of Arcturus are $\alpha = 14\,\mathrm{h}\ 15.7\,\mathrm{min}$, $\delta = 19° 1'1'$. Find the sidereal time at the moment Arcturus rises or sets in Boston ($\phi = 42°\ 19'$).

Neglecting refraction, we get

angolo orario

VEDI TEORIA 2.6

$$\cos h = -\tan 19°\ 11' \tan 42°\ 19' = -0.348 \times 0.910 = -0.317 .$$

Hence, $h = \pm 108.47° = 7\,\mathrm{h}\ 14\,\mathrm{min}$. The more accurate result is

$$\cos h = -\tan 19°\ 11' \tan 42°\ 19' - \frac{\sin 35'}{\cos 19°\ 11' \cos 42° 19'} = -0.331 ,$$

whence $h = \pm 109.35° = 7\,\mathrm{h}\ 17\,\mathrm{min}$. The plus and minus signs correspond to setting ant rising, respectively. When Arcturus rises, the sidereal time is

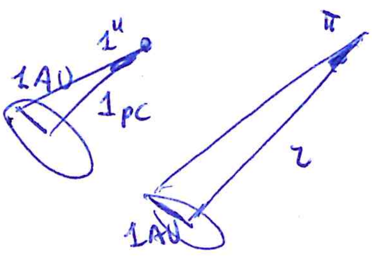
$$\Theta = \alpha + h = 14\,\mathrm{h}\ 16\,\mathrm{min} - 7\,\mathrm{h}\ 17\,\mathrm{min} = 6\,\mathrm{h}\ 59\,\mathrm{min}$$

and when it sets, the sidereal time is

$$\Theta = 14\,\mathrm{h}\ 16\,\mathrm{min} + 7\,\mathrm{h}\ 17\,\mathrm{min} = 21\,\mathrm{h}\ 33\,\mathrm{min} .$$

Note that the result is independent of the date: a star rises and sets at the same sidereal time every day.

Example 2.7 The proper motion of Aldebaran is $\mu = 0.20''/\mathrm{a}$ and parallax $\pi = 0.048''$. The spectral line of iron at $\lambda = 440.5$ nm is displaced 0.079 nm towards the



$$\mathrm{tg}\,\pi = \frac{1\,\mathrm{AU}}{r} = \frac{1\,\mathrm{pc}\ \mathrm{tg}\,1''}{r} \longrightarrow \mathrm{tg}\,\pi \sim \pi \qquad \pi = \frac{1}{r}$$

$$\mathrm{tg}\,1'' = \frac{1\,\mathrm{AU}}{1\,\mathrm{pc}} \qquad r = \frac{1}{\pi} \qquad r\,[\mathrm{pc}] \quad \pi\,['']$$

μ è misurato sulla lastra/immagine

$\mu = \sqrt{\mu_\alpha^2 \cos^2\delta + \mu_\delta^2}$ $v_T = \frac{\text{spazio angolare} \cdot \text{dist.}}{\text{tempo}} = \mu r$

4.74 × unità opportune

red. What are the radial and tangential velocities and the total velocity?

The radial velocity is found from

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} \qquad z = \frac{v_r}{c}$$

$$\Rightarrow v_r = \frac{0.079}{440.5} \cdot 3 \times 10^8 \text{ m/s} = 5.4 \times 10^4 \text{ m/s} = 54 \text{ km/s}.$$

The tangential velocity is now given by (2.40), since μ and π are in correct units:

$$v_t = 4.74\mu r = 4.74\mu/\pi = \frac{4.74 \times 0.20}{0.048} = 20 \text{ km/s}.$$

The total velocity is

$$v = \sqrt{v_r^2 + v_t^2} = \sqrt{54^2 + 20^2} \text{ km/s} = 58 \text{ km/s}.$$

Example 2.8 Find the local time in Paris (longitude $\lambda = 2°$) at 12:00.

Local time coincides with the zonal time along the meridian 15° east of Greenwich. Longitude difference $15° - 2° = 13°$ equals $(13°/15°) \times 60$ min = 52 minutes. The local time is 52 minutes less than the official time, or 11:08. This is mean solar time. To find the true solar time, we must add the equation of time. In early February, E.T. = −14 min and the true solar time is 11:08 − 14 min = 10:54. At the beginning of November, ET = +16 min and the solar time would be 11:24. Since −14 min and +16 min are the extreme values of E.T., the true solar time is in the range 10:54–11:24, the exact time depending on the day of the year. During daylight saving time, we must still subtract one hour from these times.

Example 2.9 *Estimating Sidereal Time*

Since the sidereal time is the hour angle of the vernal equinox ♈, it is 0 h when ♈ culminates or transits the south meridian. At the moment of the vernal equinox, the Sun is in the direction of ♈ and thus culminates at the same time as ♈. So the sidereal time at 12:00 local solar time is 0:00, and at the time of the vernal equinox, we have

$$\Theta = T + 12\,\text{h},$$

where T is the local solar time. This is accurate within a couple of minutes. Since the sidereal time runs about 4 minutes fast a day, the sidereal time, n days after the vernal equinox, is

$$\Theta \approx T + 12\,\text{h} + n \times 4\,\text{min}.$$

At autumnal equinox ♈ culminates at 0:00 local time, and sidereal and solar times are equal.

Let us try to find the sidereal time in Paris on April 15 at 22:00, Central European standard time (=23:00 daylight saving time). The vernal equinox occurs on the average on March 21; thus the time elapsed since the equinox is 10 + 15 = 25 days. Neglecting the equation of time, the local time T is 52 minutes less than the zonal time. Hence

$$\begin{aligned}\Theta &= T + 12\,\text{h} + n \times 4\,\text{min} \\ &= 21\,\text{h}\,8\,\text{min} + 12\,\text{h} + 25 \times 4\,\text{min} \\ &= 34\,\text{h}\,48\,\text{min} = 10\,\text{h}\,48\,\text{min}.\end{aligned}$$

The time of the vernal equinox can vary about one day in either direction from the average. Therefore the accuracy of the result is roughly 5 min.

Example 2.10 Find the rising time of Arcturus in Boston on January 10.

In Example 2.6 we found the sidereal time of this event, $\Theta = 6\,\text{h}\ 59\,\text{min}$. Since we do not know the year, we use the rough method of Example 2.9. The time between January 1 and vernal equinox (March 21) is about 70 days. Thus the sidereal time on January 1 is

$$\Theta \approx T + 12\,\text{h} - 70 \times 4\,\text{min} = T + 7\,\text{h}\,20\,\text{min},$$

from which

$$\begin{aligned}T &= \Theta - 7\,\text{h}\,20\,\text{min} = 6\,\text{h}\,59\,\text{min} - 7\,\text{h}\,20\,\text{min} \\ &= 30\,\text{h}\,59\,\text{min} - 7\,\text{h}\,20\,\text{min} = 23\,\text{h}\,39\,\text{min}.\end{aligned}$$

The longitude of Boston is 71° W, and the Eastern standard time is $(4°/15°) \times 60$ min = 16 minutes less, or 23:23.

Example 2.11 Find the sidereal time in Helsinki on April 15, 1982 at 20:00 UT.

The Julian date is $J = 2{,}445{,}074.5$ and

$$T = \frac{2{,}445{,}074.5 - 2{,}451{,}545.0}{36{,}525} = -0.1771526\,.$$

Next, we use (2.47) to find the sidereal time at 0 UT:

$$\Theta_0 = -1{,}506{,}521.0\,\mathrm{s} = -418\,\mathrm{h}\ 28\,\mathrm{min}\ 41\,\mathrm{s} = 13\,\mathrm{h}\ 31\,\mathrm{min}\ 19\,\mathrm{s}\,.$$

Since the sidereal time runs 3 min 57 s fast a day as compared to the solar time, the difference in 20 hours will be

$$\frac{20}{24} \times 3\,\mathrm{min}\ 57\,\mathrm{s} = 3\,\mathrm{min}\ 17\,\mathrm{s}\,,$$

and the sidereal time at 20 UT will be 13 h 31 min 19 s + 20 h 3 min 17 s = 33 h 34 min 36 s = 9 h 34 min 36 s.

At the same time (at 22:00 Finnish time, 23:00 daylight saving time) in Helsinki the sidereal time is ahead of this by the amount corresponding to the longitude of Helsinki, 25°, i.e. 1 h 40 min 00 s. Thus the sidereal time is 11 h 14 min 36 s.

2.17 Exercises

Exercise 2.1 Find the distance between Helsinki and Seattle along the shortest route. Where is the northernmost point of the route, and what is its distance from the North Pole? The longitude of Helsinki is 25°E and latitude 60°; the longitude of Seattle is 122°W and latitude 48°. Assume that the radius of the Earth is 6370 km.

VEDI TEORIA 2,5

Exercise 2.2 A star crosses the south meridian at an altitude of 85°, and the north meridian at 45°. Find the declination of the star and the latitude of the observer.

Exercise 2.3 Where are the following statements true?

a) Castor (α Gem, declination $\delta = 31°53'$) is circumpolar.
b) Betelgeuze (α Ori, $\delta = 7°\ 24'$) culminates at zenith.
c) α Cen ($\delta = -60°\ 50'$) rises to an altitude of 30°.

VEDI PAGINA DOPO

Exercise 2.4 In his *Old Man and the Sea* Hemingway wrote:

> *It was dark now as it becomes dark quickly after the Sun sets in September. He lay against the worn wood of the bow and rested all that he could. The first stars were out. He did not know the name of Rigel but he saw it and knew soon they would all be out and he would have all his distant friends.*

How was Hemingway's astronomy?

Exercise 2.5 The right ascension of the Sun on June 1, 1983, was 4 h 35 min and declination 22° 00′. Find the ecliptic longitude and latitude of the Sun and the Earth.

Exercise 2.6 Show that on the Arctic Circle the Sun

a) rises at the same sidereal time Θ_0 between December 22 and June 22,
b) sets at the same sidereal time Θ_0 between June 22 and December 22.

What is Θ_0?

Exercise 2.7 Derive the equations (2.24), which give the galactic coordinates as functions of the ecliptic coordinates.

Exercise 2.8 The coordinates of Sirius for the epoch 1900.0 were $\alpha = 6\,\mathrm{h}\ 40\,\mathrm{min}\ 45\,\mathrm{s}$, $\delta = -16°\ 35'$, and the components of its proper motion were $\mu_\alpha = -0.037\,\mathrm{s/a}$, $\mu_\delta = -1.12''\mathrm{a}^{-1}$. Find the coordinates of Sirius for 2000.0. The precession must also be taken into account.

Exercise 2.9 The parallax of Sirius is 0.375″ and radial velocity −8 km/s.

a) What are the tangential and total velocities of Sirius? (See also the previous exercise.)
b) When will Sirius be closest to the Sun?
c) What will its proper motion and parallax be then?

2.3

a) $\delta + \phi - 90° > 0$ CASTOR è CIRCUMPOLARE

$\phi > 90° - \delta$ $\delta = 31°53'$

$\phi > 58°07'$ (con rifrazione $\phi > 57°07'$)

b) BETELGEUZE $\delta = 7°24'$ culmine allo zenith

$\Downarrow$

$a_{max} = 90°$ ma $a_{max} = 90° - \phi + \delta$

$\Rightarrow 90° = 90° - \phi + \delta$ $\phi = \delta = 7°24'$

c) α Cen ($\delta = -60°50'$) rises to an altitude of 30°

$30° < a_{max} < 90°$

$a_{max} > 30°$ $90° - \phi + \delta > 30°$

$\phi < 90 - 30 + \delta$ $\boxed{\phi < -50'}$

$a_{max} < 90°$ $\cancel{90°} - \phi + \delta < \cancel{90°}$

$\phi > \delta$ $\boxed{\phi > -60°50'}$

$-60°50' < \phi < -50'$

TEORIA

the vernal equinox measured along the equator. This angle is the *right ascension* α (or R.A.) of the object, measured counterclockwise from ♈.

Since declination and right ascension are independent of the position of the observer and the motions of the Earth, they can be used in star maps and catalogues. As will be explained later, in many telescopes one of the axes (the hour axis) is parallel to the rotation axis of the Earth. The other axis (declination axis) is perpendicular to the hour axis. Declinations can be read immediately on the declination dial of the telescope. But the zero point of the right ascension seems to move in the sky, due to the diurnal rotation of the Earth. So we cannot use the right ascension to find an object unless we know the direction of the vernal equinox.

Since the south meridian is a well-defined line in the sky, we use it to establish a local coordinate corresponding to the right ascension. The *hour angle* is measured clockwise from the meridian. The hour angle of an object is not a constant, but grows at a steady rate, due to the Earth's rotation. The hour angle of the vernal equinox is called the *sidereal time* Θ. Figure 2.11 shows that for any object,

$$\Theta = h + \alpha \,, \tag{2.11}$$

where h is the object's hour angle and α its right ascension.

Since hour angle and sidereal time change with time at a constant rate, it is practical to express them in units of time. Also the closely related right ascension is customarily given in time units. Thus 24 hours equals 360 degrees, 1 hour = 15 degrees, 1 minute of time = 15 minutes of arc, and so on. All these quantities are in the range [0 h, 24 h).

In practice, the sidereal time can be readily determined by pointing the telescope to an easily recognisable star and reading its hour angle on the hour angle dial of the telescope. The right ascension found in a catalogue is then added to the hour angle, giving the sidereal time at the moment of observation. For any other time, the sidereal time can be evaluated by adding the time elapsed since the observation. If we want to be accurate, we have to use a sidereal clock to measure time intervals. A sidereal clock runs 3 min 56.56 s fast a day as compared with an ordinary solar time clock:

$$\begin{aligned} &24\,\text{h solar time} \\ &\quad = 24\,\text{h}\ 3\,\text{min}\ 56.56\,\text{s sidereal time}\,. \end{aligned} \tag{2.12}$$

The reason for this is the orbital motion of the Earth: stars seem to move faster than the Sun across the sky; hence, a sidereal clock must run faster. (This is further discussed in Sect. 2.13.)

Transformations between the horizontal and equatorial frames are easily obtained from spherical

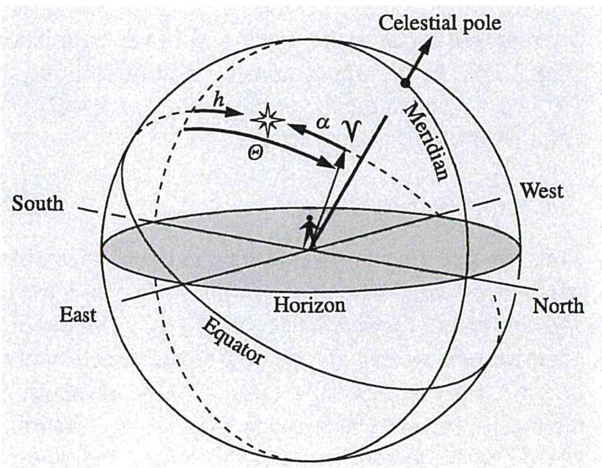


Fig. 2.11. The sidereal time Θ (the hour angle of the vernal equinox) equals the hour angle plus right ascension of any object

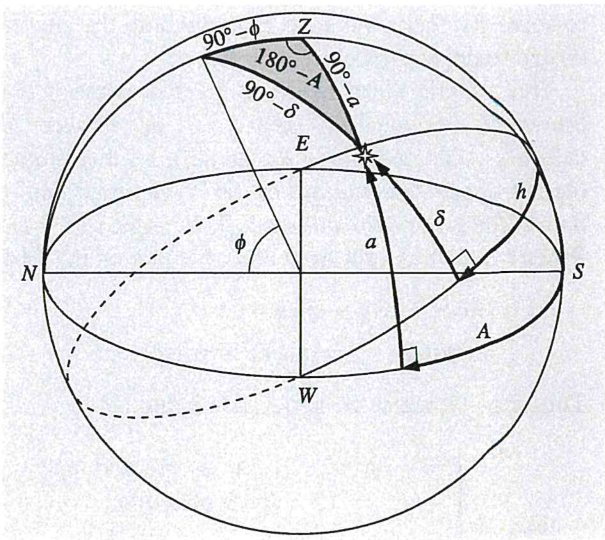


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

TEORÍA 2.5

trigonometry. Comparing Figs. 2.6 and 2.12, we find that we must make the following substitutions into (2.5):

$$\begin{aligned} \psi &= 90^\circ - A\,, \quad \theta = a\,, \\ \psi' &= 90^\circ - h\,, \quad \theta' = \delta\,, \quad \chi = 90^\circ - \phi\,. \end{aligned} \tag{2.13}$$

The angle ϕ in the last equation is the altitude of the celestial pole, or the latitude of the observer. Making the substitutions, we get

$$\begin{aligned} \sin h \cos\delta &= \sin A \cos a\,, \\ \cos h \cos\delta &= \cos A \cos a \sin\phi + \sin a \cos\phi\,, \\ \sin\delta &= -\cos A \cos a \cos\phi + \sin a \sin\phi\,. \end{aligned} \tag{2.14}$$

The inverse transformation is obtained by substituting

$$\begin{aligned} \psi &= 90^\circ - h\,, \quad \theta = \delta\,, \\ \psi' &= 90^\circ - A\,, \quad \theta' = a\,, \quad \chi = -(90^\circ - \phi)\,, \end{aligned} \tag{2.15}$$

whence

$$\begin{aligned} \sin A \cos a &= \sin h \cos\delta\,, \\ \cos A \cos a &= \cos h \cos\delta \sin\phi - \sin\delta \cos\phi\,, \\ \sin a &= \cos h \cos\delta \cos\phi + \sin\delta \sin\phi\,. \end{aligned} \tag{2.16}$$

Since the altitude and declination are in the range [−90°, +90°], it suffices to know the sine of one of these angles to determine the other angle unambiguously. Azimuth and right ascension, however, can have any value from 0° to 360° (or from 0 h to 24 h), and to solve for them, we have to know both the sine and cosine to choose the correct quadrant.

The altitude of an object is greatest when it is on the south meridian (the great circle arc between the celestial poles containing the zenith). At that moment (called *upper culmination*, or *transit*) its hour angle is 0 h. At the *lower culmination* the hour angle is $h = 12$ h. When $h = 0$ h, we get from the last equation in (2.16)

$$\begin{aligned} \sin a &= \cos\delta \cos\phi + \sin\delta \sin\phi \\ &= \cos(\phi - \delta) = \sin(90^\circ - \phi + \delta)\,. \end{aligned}$$

Thus the altitude at the upper culmination is

$$a_{max} = \begin{cases} 90^\circ - \phi + \delta\,, & \text{if the object culminates south of zenith}\,, \\ 90^\circ + \phi - \delta\,, & \text{if the object culminates north of zenith}\,. \end{cases} \tag{2.17}$$

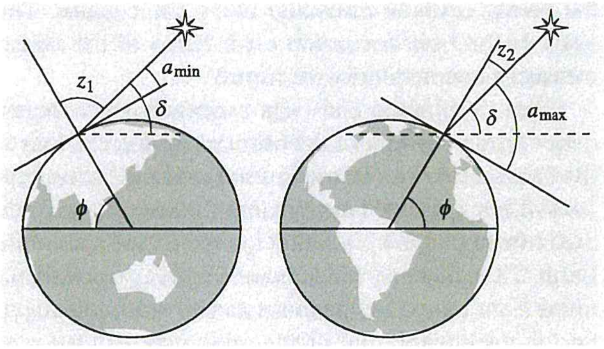


Fig. 2.13. The altitude of a circumpolar star at upper and lower culmination

The altitude is positive for objects with $\delta > \phi - 90^\circ$. Objects with declinations less than $\phi - 90^\circ$ can never be seen at the latitude ϕ. On the other hand, when $h = 12$ h we have

$$\begin{aligned} \sin a &= -\cos\delta \cos\phi + \sin\delta \sin\phi \\ &= -\cos(\delta + \phi) = \sin(\delta + \phi - 90^\circ)\,, \end{aligned}$$

and the altitude at the lower culmination is

$$a_{min} = \delta + \phi - 90^\circ\,. \tag{2.18}$$

Stars with $\delta > 90^\circ - \phi$ will never set. For example, in Helsinki ($\phi \approx 60^\circ$), all stars with a declination higher than 30° are such *circumpolar* stars. And stars with a declination less than −30° can never be observed there.

We shall now study briefly how the (α, δ) frame can be established by observations. Suppose we observe a circumpolar star at its upper and lower culmination (Fig. 2.13). At the upper transit, its altitude is $a_{max} = 90^\circ - \phi + \delta$ and at the lower transit, $a_{min} = \delta + \phi - 90^\circ$. Eliminating the latitude, we get

$$\delta = \frac{1}{2}(a_{min} + a_{max})\,. \tag{2.19}$$

Thus we get the same value for the declination, independent of the observer's location. Therefore we can use it as one of the absolute coordinates. From the same observations, we can also determine the direction of the celestial pole as well as the latitude of the observer. After these preparations, we can find the declination of any object by measuring its distance from the pole.

The equator can be now defined as the great circle all of whose points are at a distance of 90° from the

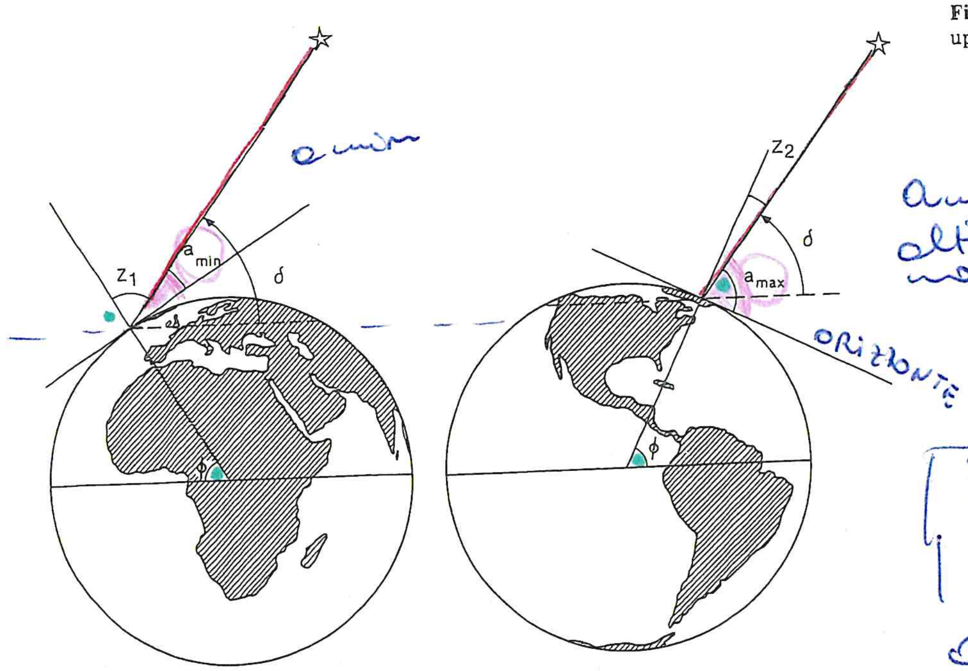


Fig. 2.14. The altitude of a circum... upper and lower culmination

a_{max} altezza max

parallele

z = distanza zenitale

δ = declinazione

stella rossa e Terra gialla

ϕ = latitudine dell'osservatore

$a_{min} = 90 - z$

$z = 180° - \phi - \delta$

$a_{min} = 90° - 180 + \phi + \delta$

$a_{min} = \delta + \phi - 90°$

* è CIRCUMPOLARE se $a_{min} > 0$

$\delta + \phi - 90° > 0$

$\boxed{\delta > 90° - \phi}$

es. 45° $\delta > 45°$ sono circumpolari

$a_{max} = 90° - z \qquad z = \phi - \delta$

$a_{max} = 90° - (\phi - \delta) = 90° - \phi + \delta$

stella * è VISIBILE se $a_{max} > 0$ cioè $90° - \phi + \delta > 0$

$\boxed{\delta > \phi - 90°}$

es. supp. $\phi = 45°$ come a TS

* con $\delta > -45°$ si vedono

* con $\delta < -45°$ NO

CASI LIMITE

$\phi = 0° \Rightarrow \delta > 90°$ - nessuna stella è circumpolare

$\phi = +90° \Rightarrow \delta > 0°$ tutte le stelle dell'emisfero Nord sono circumpolari.

CASI LIMITE

$\phi = 0°$ = oss. su equatore

$\Rightarrow \delta > -90°$ cioè TUTTE le stelle si vedono

$\phi = +90°$ = Polo Nord

$\Rightarrow \delta > 0°$ cioè vedo solo ½ delle stelle!

pole. The zero point of the second coordinate (right ascension) can then be defined as the point where the Sun seems to cross the equator from south to north.

In practice the situation is more complicated, since the direction of Earth's rotation axis changes due to perturbations. Therefore the equatorial coordinate frame is nowadays defined using certain standard objects the positions of which are known very accurately. The best accuracy is achieved by using the most distant objects, *quasars* (Sect. 18.7), which remain in the same direction over very long intervals of time.

2.6 Rising and Setting Times

From the last equation (2.16), we find the hour angle h of an object at the moment its altitude is a:

$$\cos h = -\tan\delta\tan\phi + \frac{\sin a}{\cos\delta\cos\phi} \,. \tag{2.20}$$

This equation can be used for computing rising and setting times. Then $a = 0$ and the hour angles corresponding to rising and setting times are obtained from

altezza = 0

$$\cos h = -\tan\delta\tan\phi \,. \tag{2.21}$$

latitudine

If the right ascension α is known, we can use (2.11) to compute the sidereal time Θ. (Later, in Sect. 2.14, we shall study how to transform the sidereal time to ordinary time.)

If higher accuracy is needed, we have to correct for the refraction of light caused by the atmosphere of the Earth (see Sect. 2.9). In that case, we must use a small negative value for a in (2.20). This value, the *horizontal refraction*, is about $-34'$.

The rising and setting times of the Sun given in almanacs refer to the time when the upper edge of the Solar disk just touches the horizon. To compute these times, we must set $a = -50'$ $(= -34' - 16')$.

Also for the Moon almanacs give rising and setting times of the upper edge of the disk. Since the distance of the Moon varies considerably, we cannot use any constant value for the radius of the Moon, but it has to be calculated separately each time. The Moon is also so close that its direction with respect to the background stars varies due to the rotation of the Earth. Thus the rising and setting times of the Moon are defined as the instants when the altitude of the Moon is $-34' - s + \pi$, where s is the apparent radius ($15.5'$ on the average) and π the parallax ($57'$ on the average). The latter quantity is explained in Sect. 2.9.

Finding the rising and setting times of the Sun, planets and especially the Moon is complicated by their motion with respect to the stars. We can use, for example, the coordinates for the noon to calculate estimates for the rising and setting times, which can then be used to interpolate more accurate coordinates for the rising and setting times. When these coordinates are used to compute new times a pretty good accuracy can be obtained. The iteration can be repeated if even higher precision is required.

2.7 The Ecliptic System

The orbital plane of the Earth, the *ecliptic*, is the reference plane of another important coordinate frame. The ecliptic can also be defined as the great circle on the celestial sphere described by the Sun in the course of one year. This frame is used mainly for planets and other bodies of the solar system. The orientation of the Earth's equatorial plane remains invariant, unaffected by annual motion. In spring, the Sun appears to move from the southern hemisphere to the northern one (Fig. 2.14). The time of this remarkable event as well as the direction to the Sun at that moment are called the *vernal equinox*. At the vernal equinox, the Sun's right ascension and declination are zero. The equatorial and ecliptic

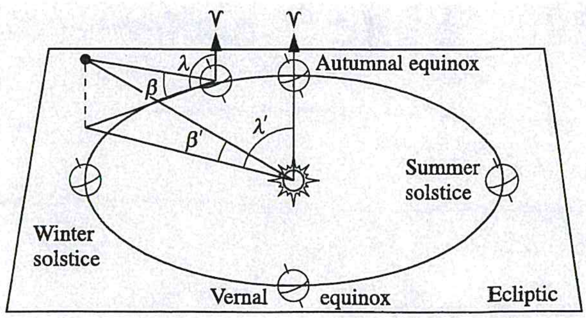


Fig. 2.14. The ecliptic geocentric (λ, β) and heliocentric (λ', β') coordinates are equal only if the object is very far away. The geocentric coordinates depend also on the Earth's position in its orbit

a, A altezza e azimut
δ, h declinazione e angolo orario α ≡ R.A.
Θ tempo siderale Θ = h + α
Θ = angolo orario di γ, punto d'Ariete