Image Processing for Physicists

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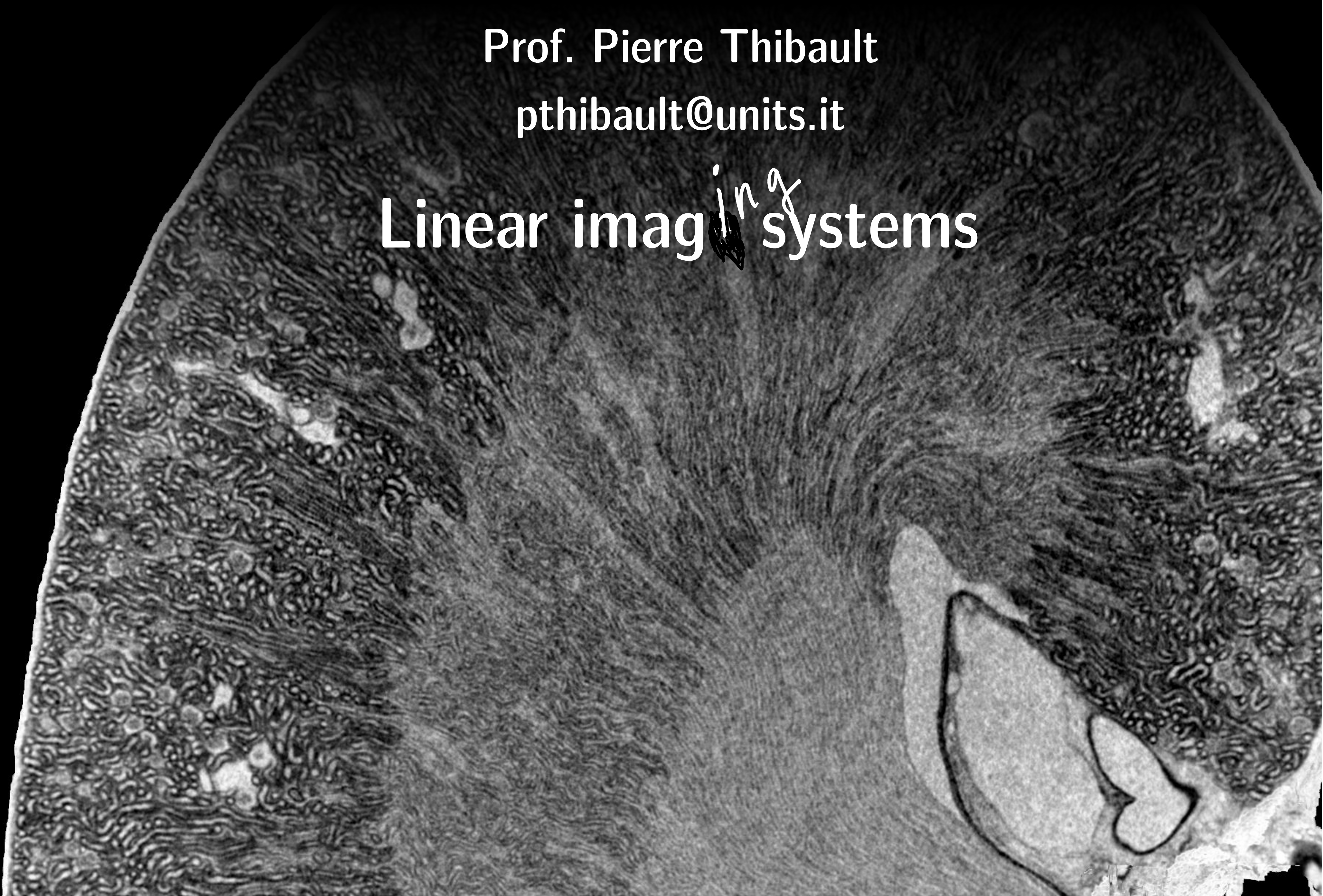
Linear imaging systems



Overview

- Definition of resolution
- Imaging systems:
  - Linear transfer model
  - Noise

Resolution

"the smallest detail that can be distinguished"

- No unique definition
  - Numerical aperture ← microscopy, photography
  - Pixel size
  - Other criteria (PSF, MTF)
- What is "detail"?
- What is "distinguish"?

Resolution

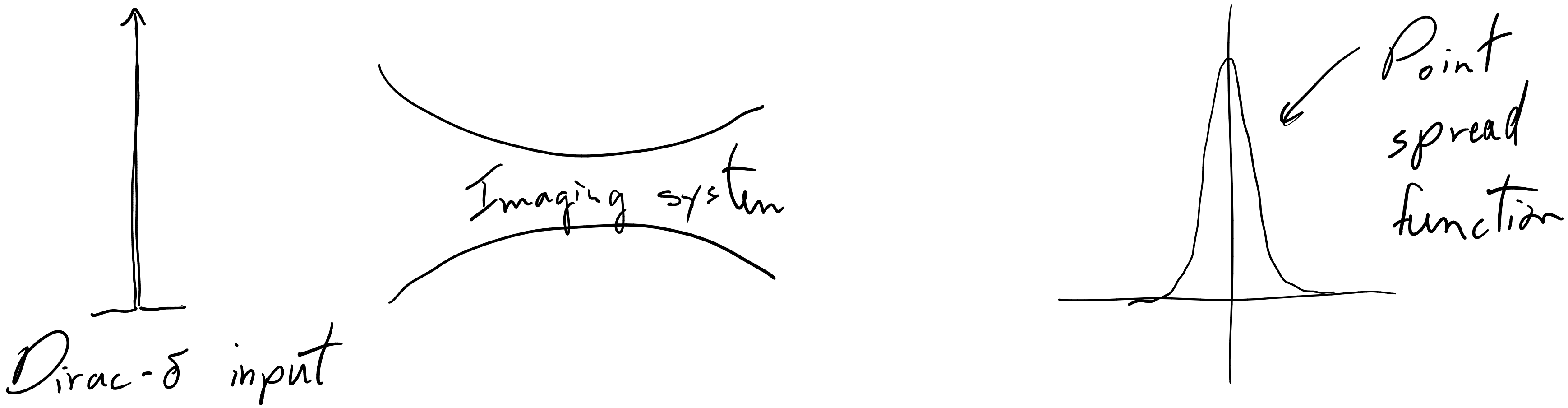
1280 x 1280

640 x 640

- **not** simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear translation-invariant systems

- Point spread function ("impulse response")



- LTI system: convolution with PSF

$$f(x,y) = \int dx'dy' \; f(x',y') \, \delta(x-x') \, \delta(y-y') \quad \leftarrow \text{input}$$

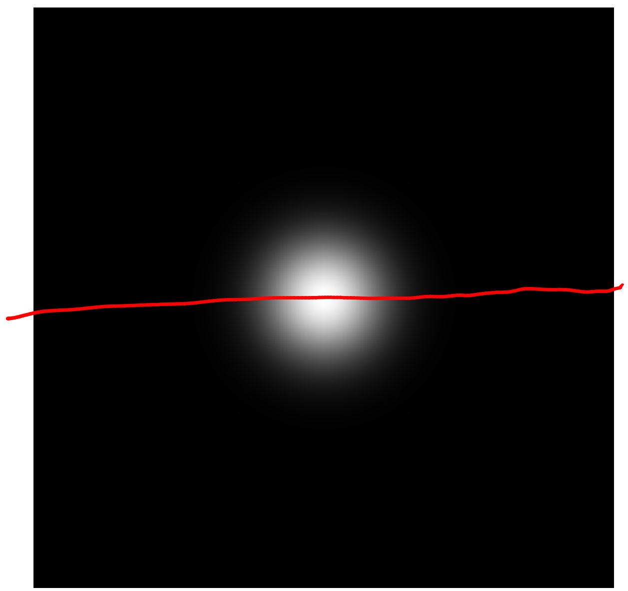
} system — PSF ↓

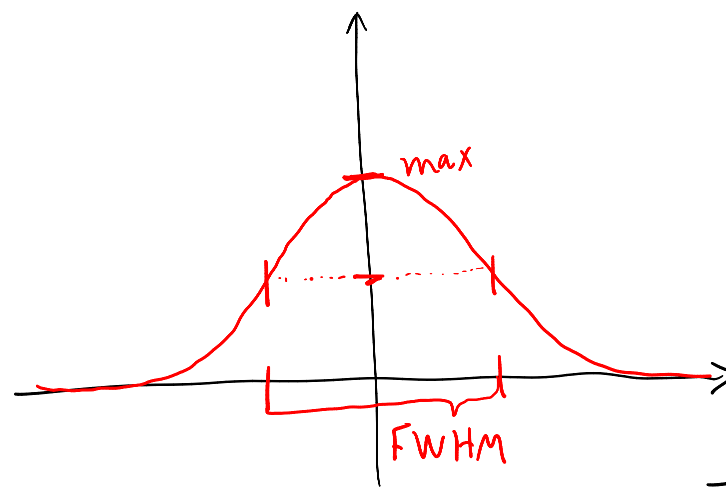
$$\text{output} \rightarrow \int dx'dy' \; f(x',y') \, h(x-x', y-y') = f * h$$

Point spread function

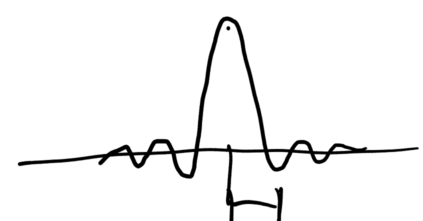
PSF and resolution

Commonly, resolution from PSF given by
"full width at half maximum"
FWHM





[Rayleigh criterion:
applies to imaging systems with a circular aperture → PSF = airy disc

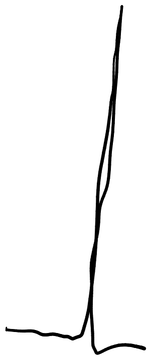


(to be continued)]

Measurement of the PSF

- Direct measurement from impulse

Generate sharp point as input



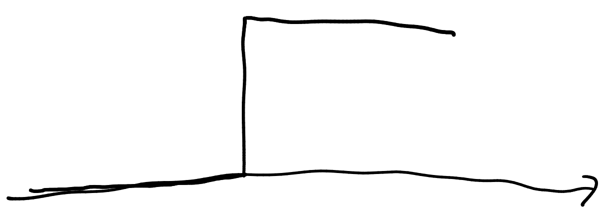
imaging

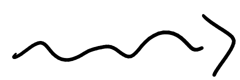
output = PSF!

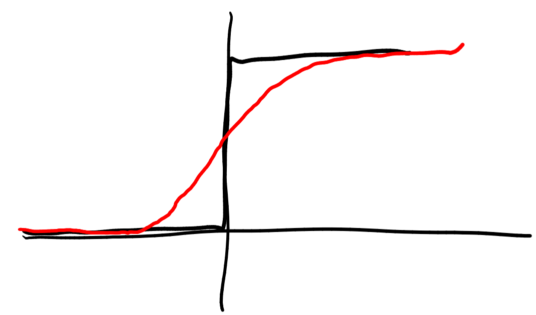
astronomy: easy: pick a bright star,

- Line-spread function

knife-edge







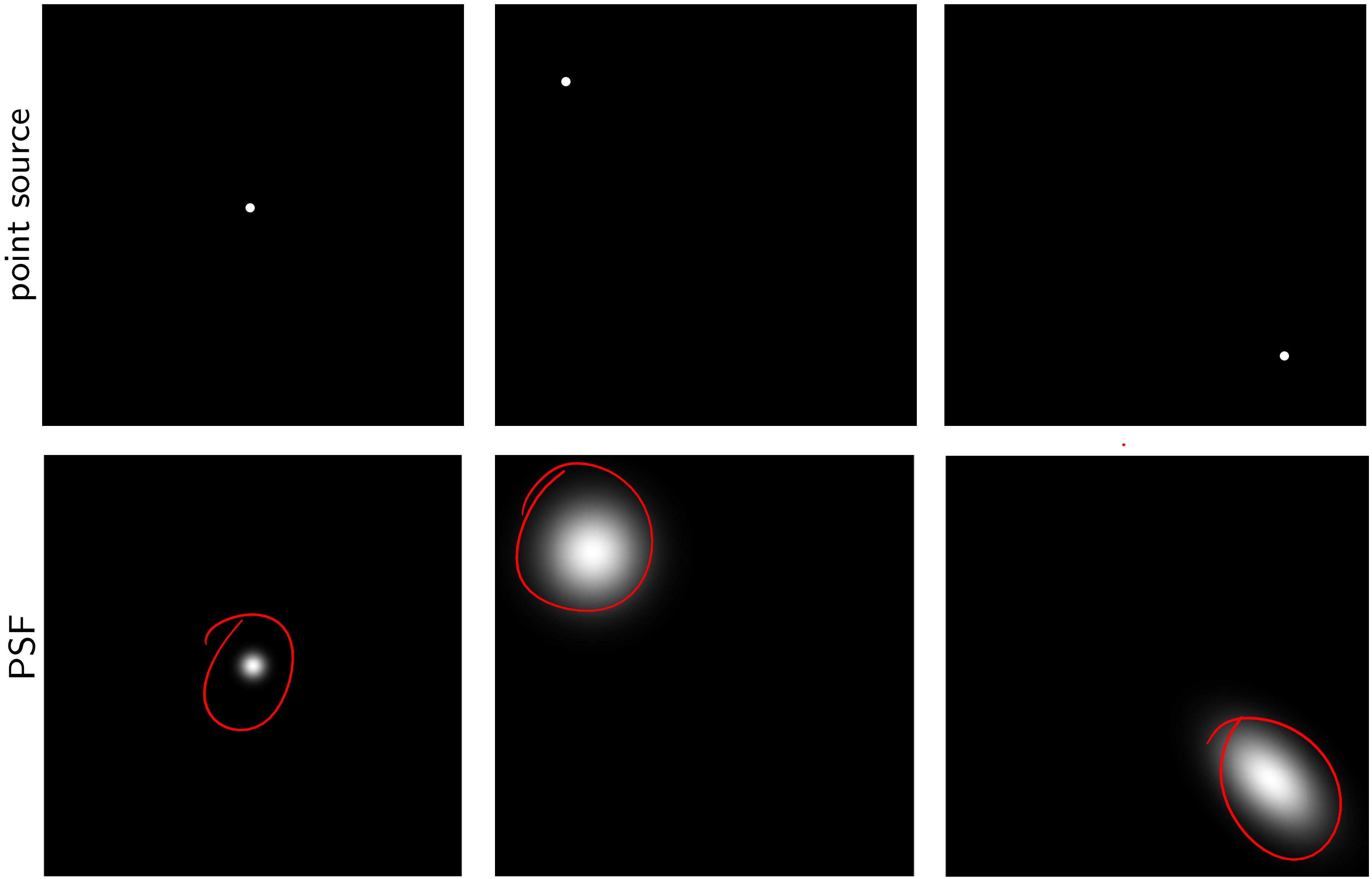
$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

"Heaviside step function"

$$\frac{\partial H}{\partial x} = \delta(x)$$

$\Rightarrow$ PSF = derivative of knife-edge measurement

PSF and translation invariance



- Not translation invariant $\rightarrow$ PSF depends on position $\rightarrow$ not a convolution
- Useful to model system imperfections, lens aberrations, ...

The Fourier picture

$$\mathcal{F}\{f * h\} = F(u) \cdot H(u)$$

↳ describes how an oscillating signal changes through the imaging system

H: F.T. of PSF = Optical Transfer Function (OTF)

$$\text{Imaging system}\left\{e^{2\pi i x u}\right\} = \lambda \cdot e^{2\pi i x u}$$

↳ reduced amplitude = $H(u)$

in other words, $e^{2\pi i x u}$ is an eigenvector of the imaging system with eigenvalue $H(u)$

Optical transfer function

Response of a system to an oscillating signal with well-defined frequency

$$OTF(u) = \mathcal{F}\{PSF\}$$

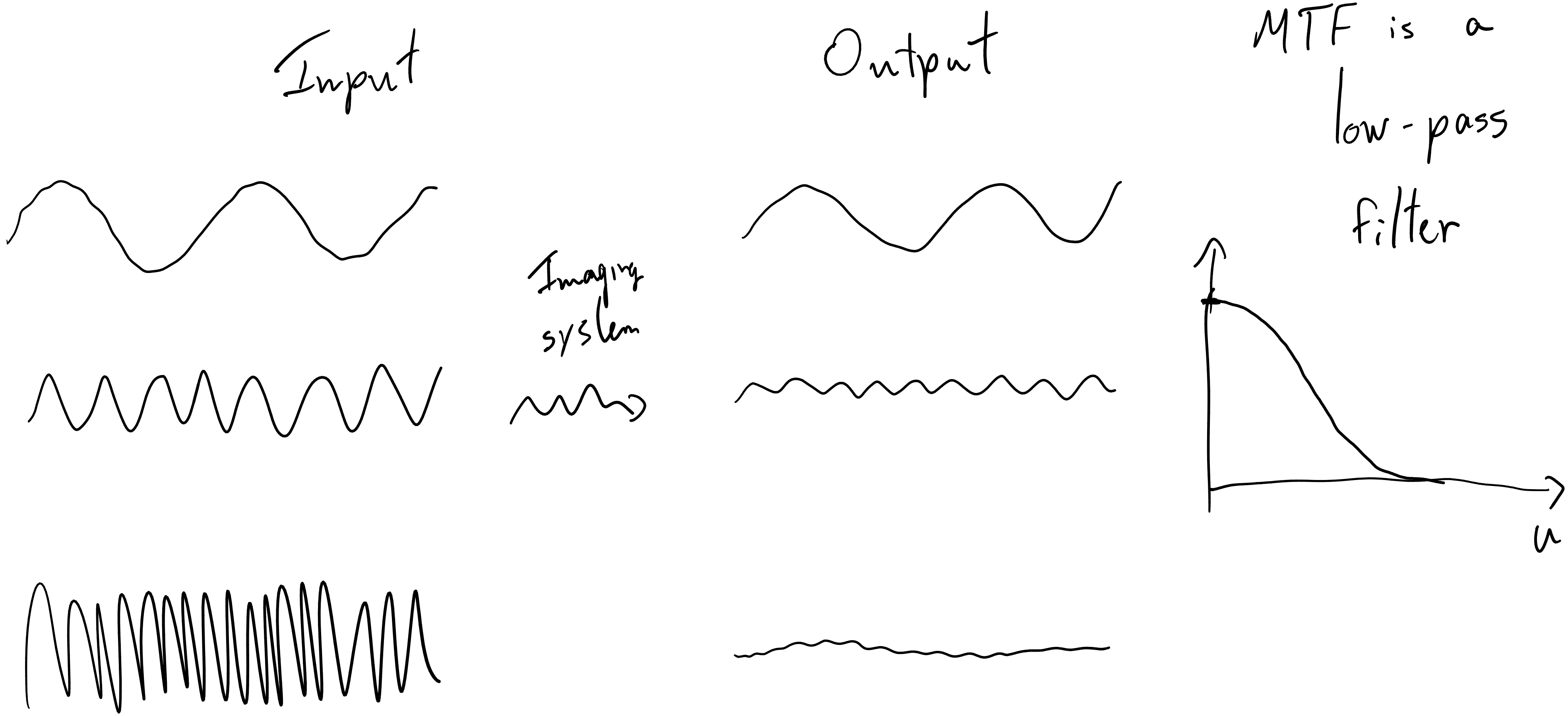
Amplitude: $|OTF| = MTF$ "modulation transfer function"

Phase: $\arg\{OTF\} = PTF$ "phase transfer function"

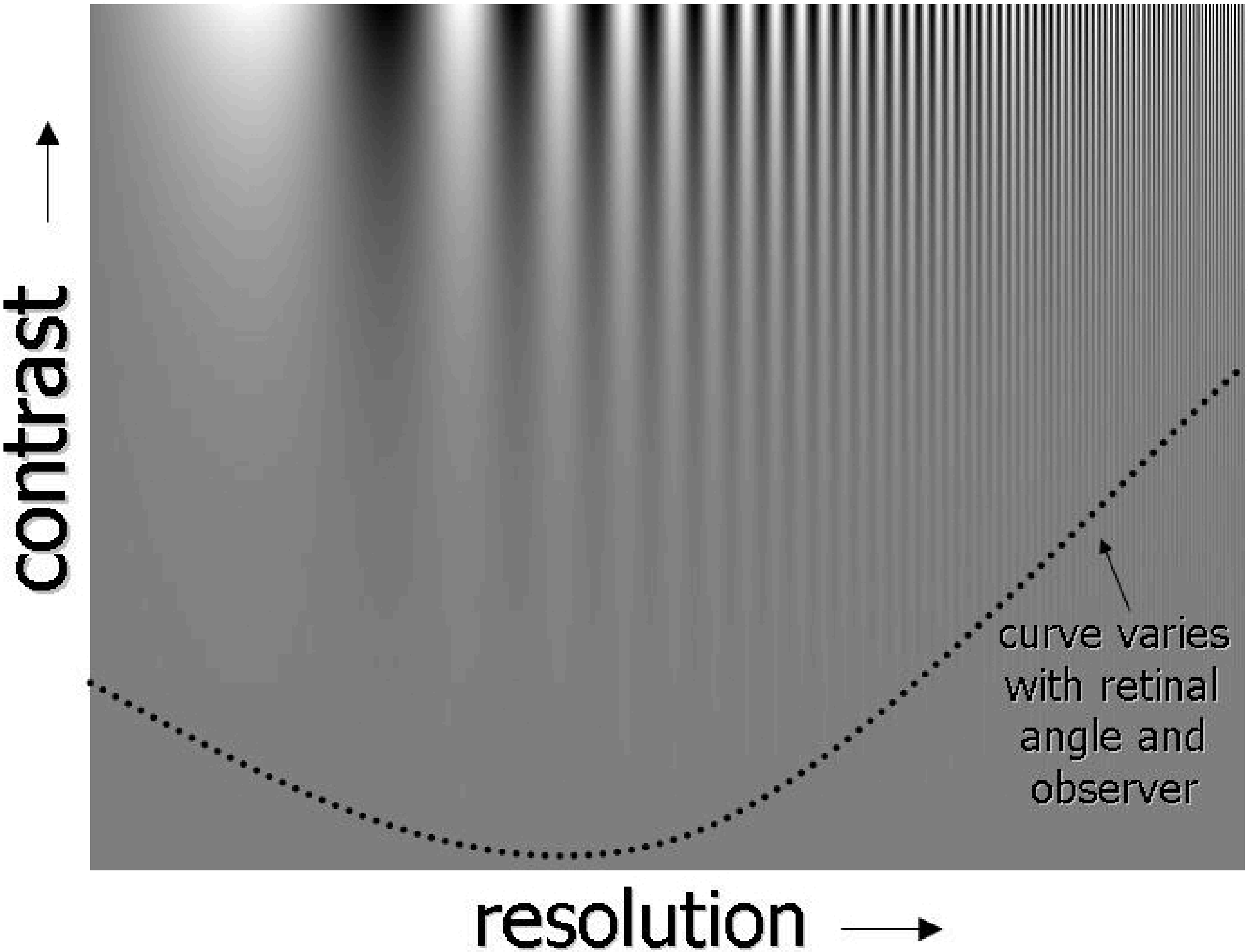
$$OTF = MTF \cdot e^{iPTF}$$

Modulation transfer function

Amplitude change of an oscillating signal for a given frequency



Eye MTF


contrast
curve varies
with retinal
angle and
observer
resolution

Campbell-Robson curve

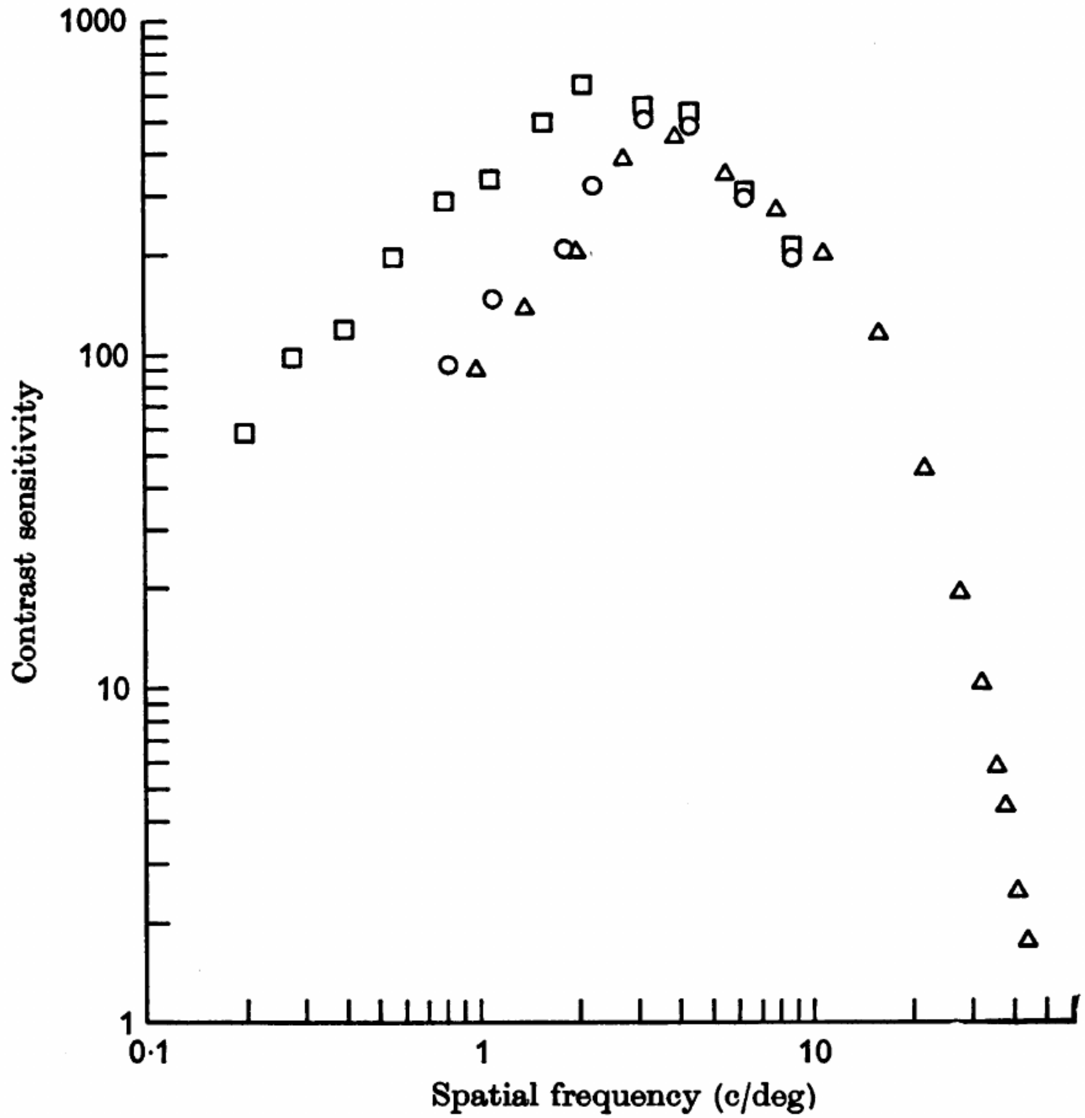
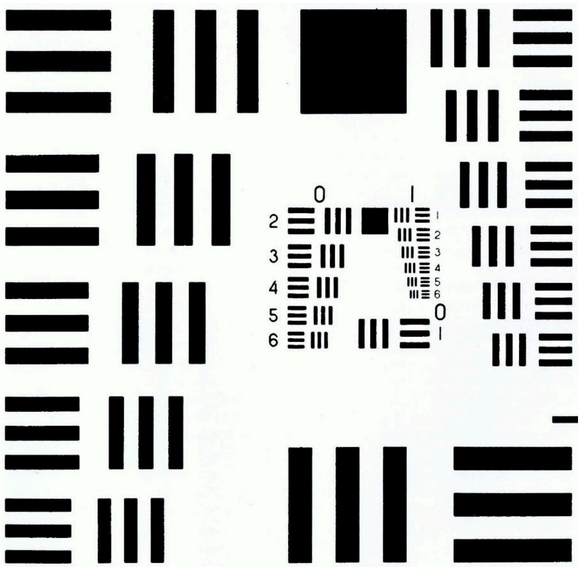


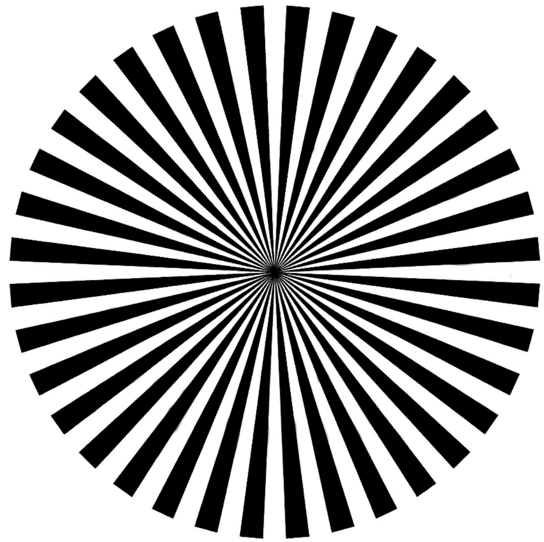
Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture 2° × 2°, △; viewing distance 57 cm, aperture 10° × 10°, □; viewing distance 57 cm, aperture 2° × 2°, ○.

1968

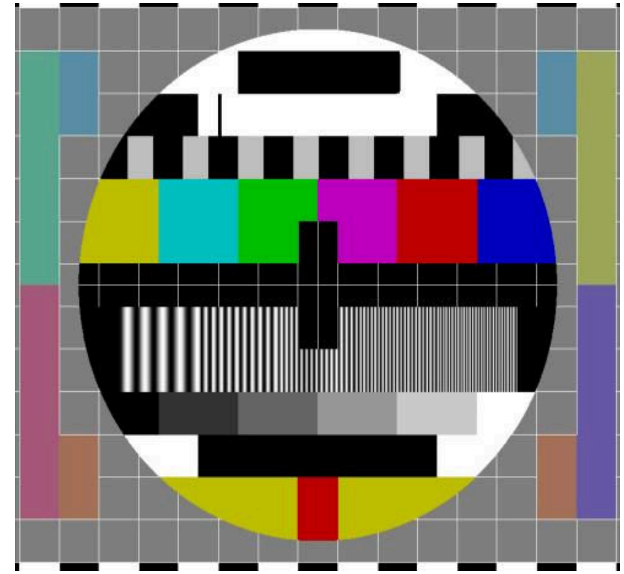
Measurement of MTF



US AF 1951



Siemens star

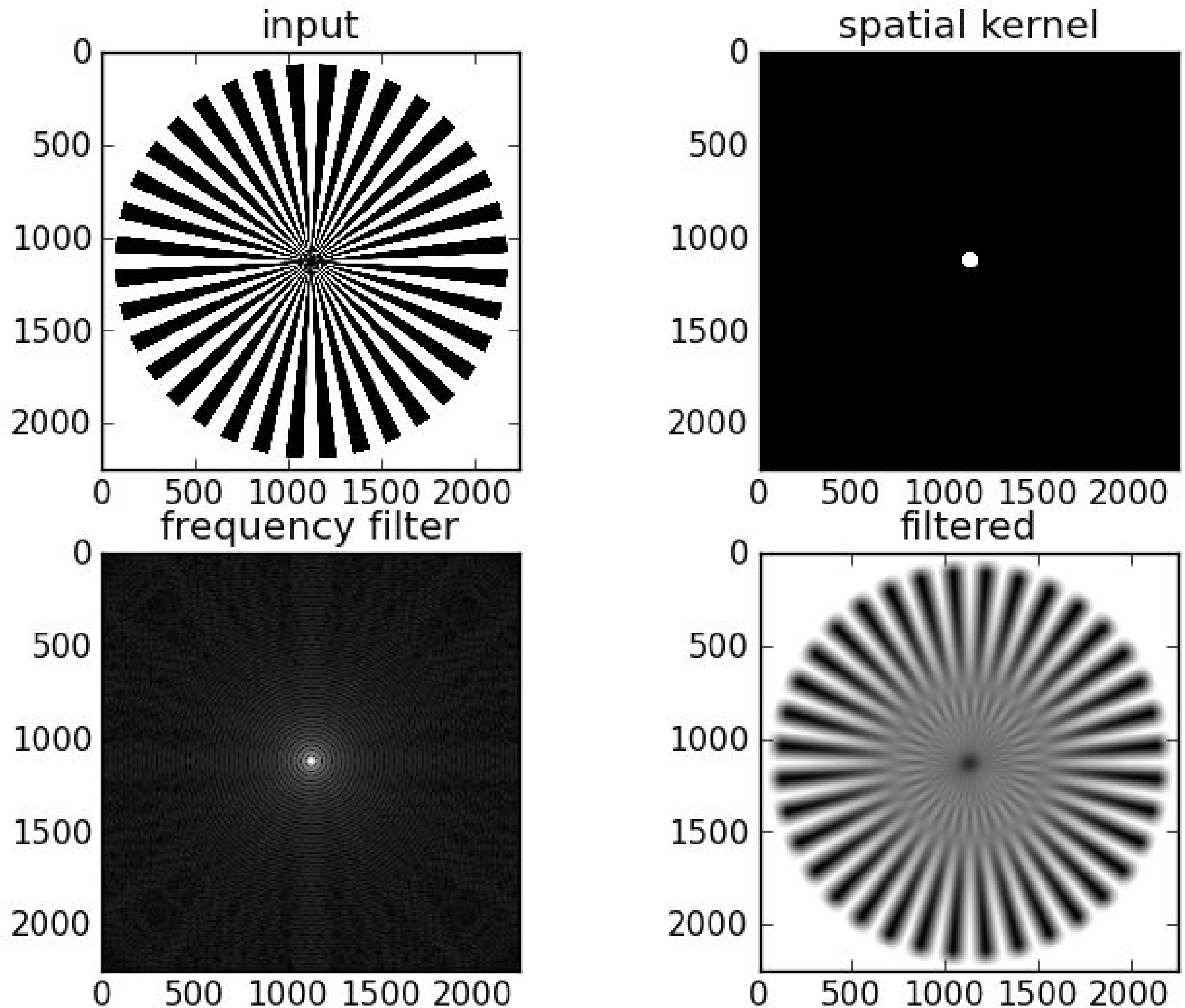


old TV

source: http://fotomagazin.de

Phase transfer function

describes how an oscillating signal changes in phase due to system



Phase transfer function

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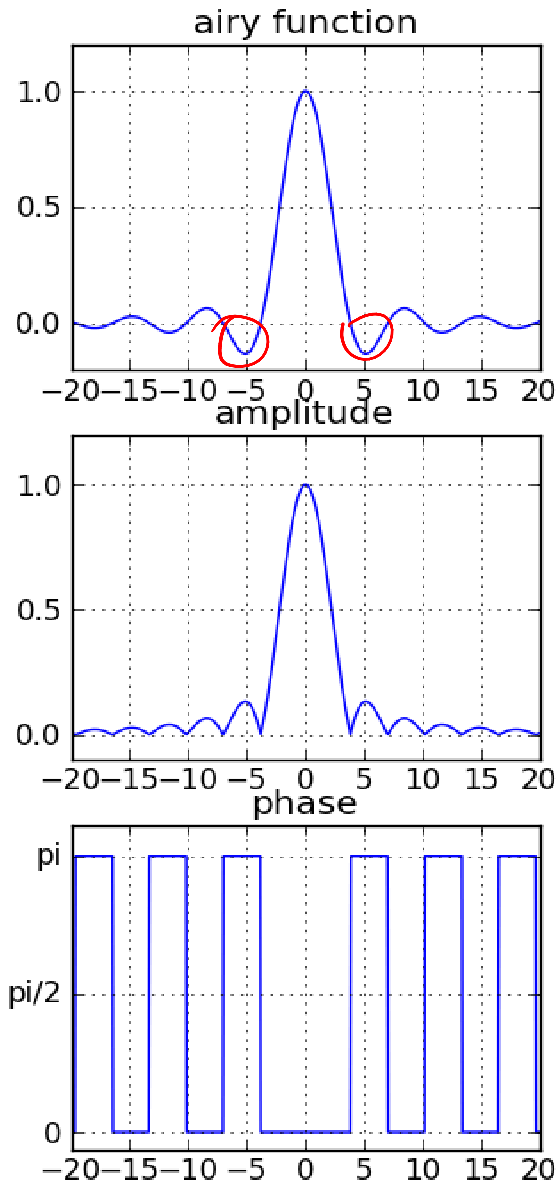
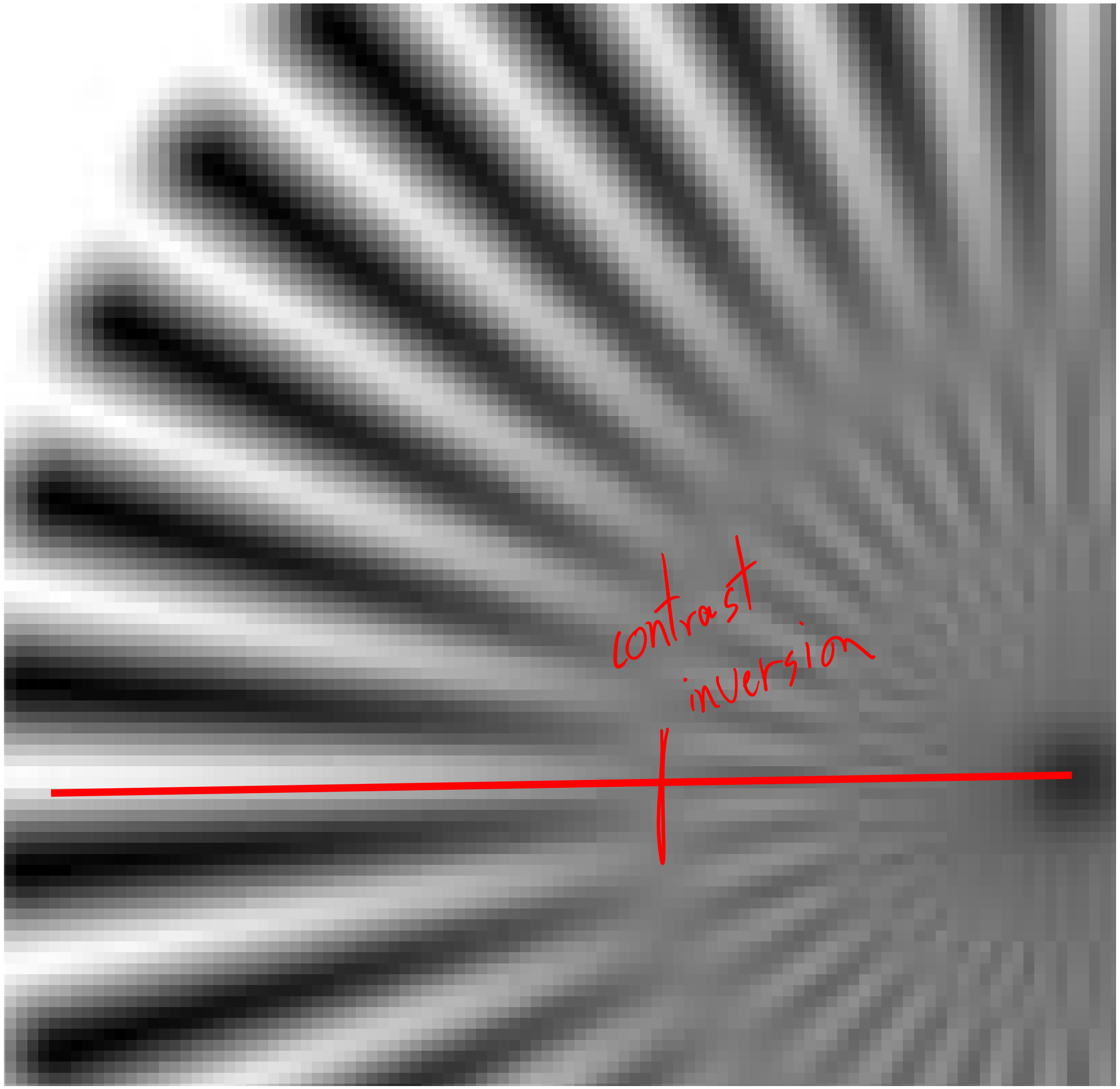
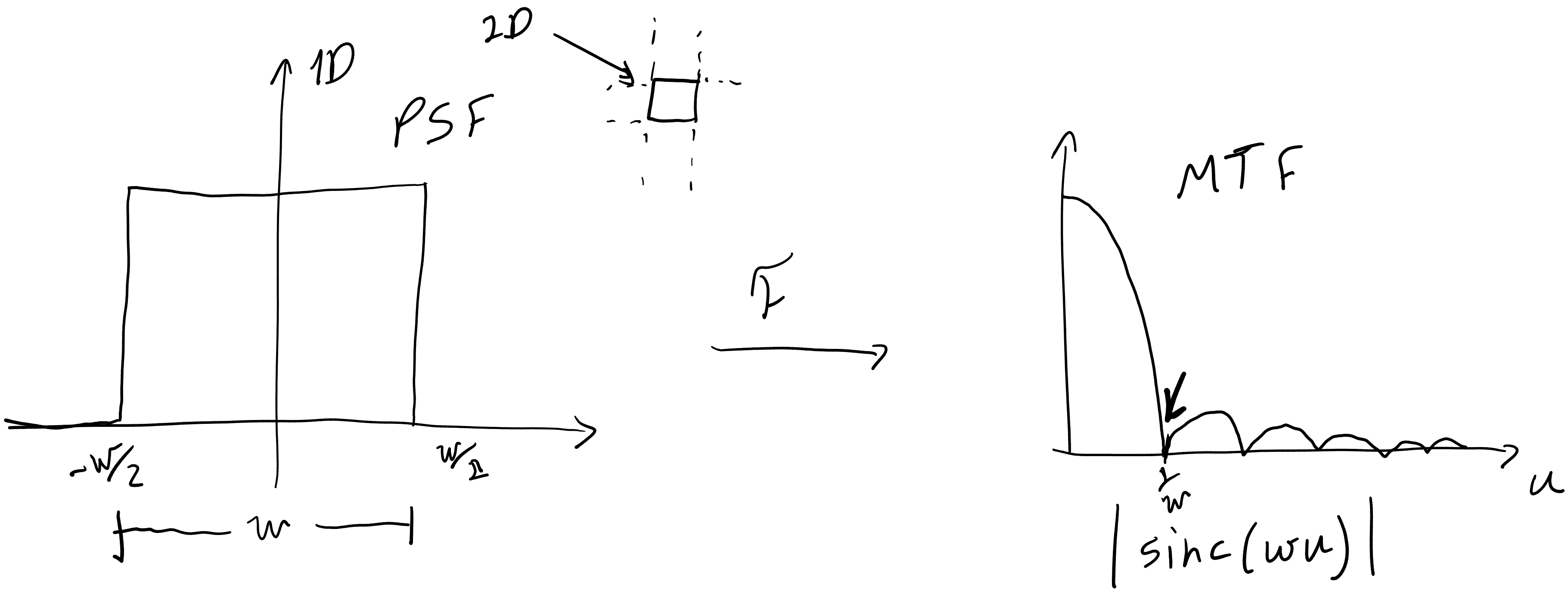


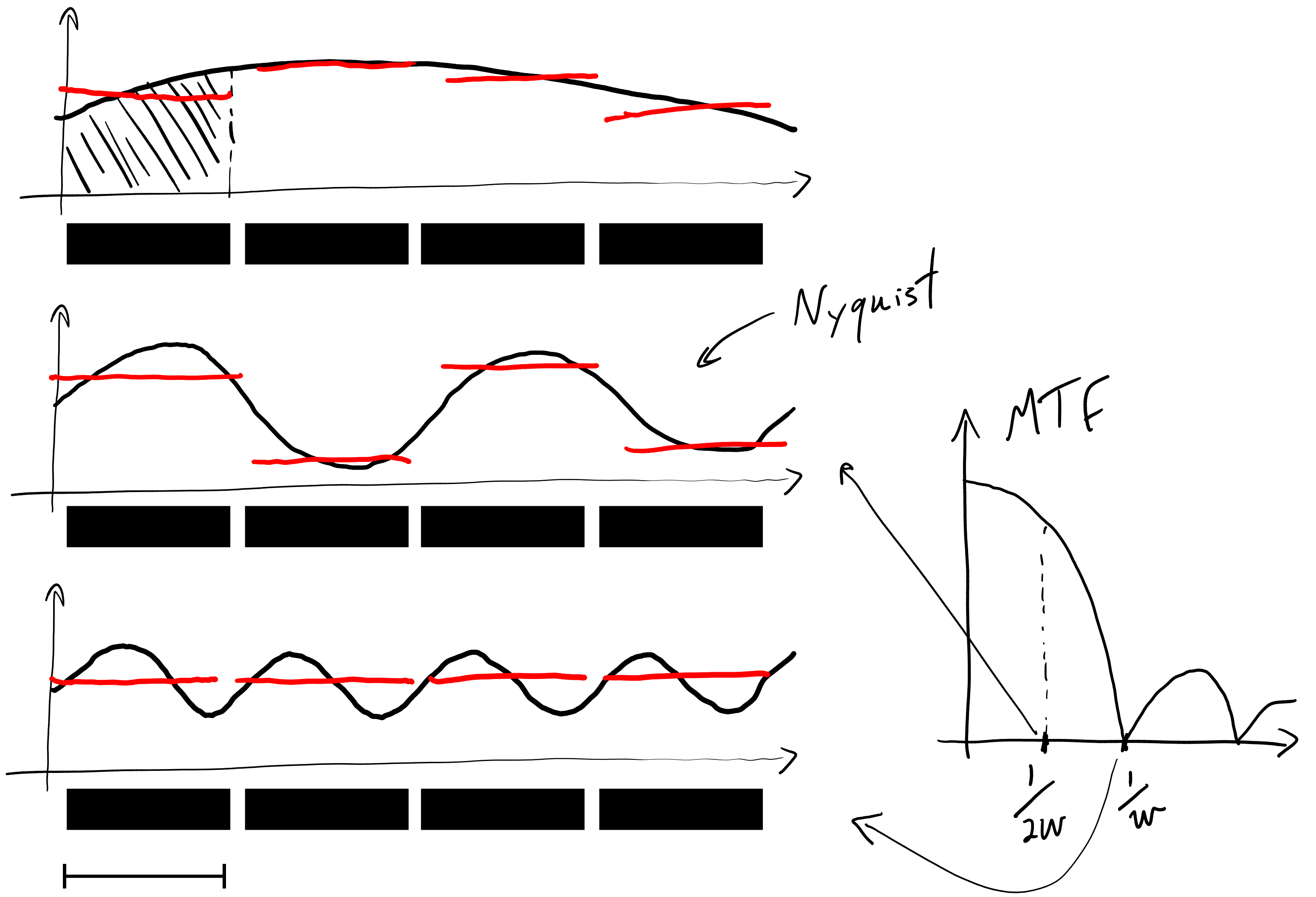


MTF of an ideal pixel



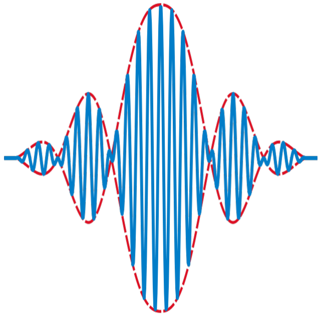
Pixel MTF

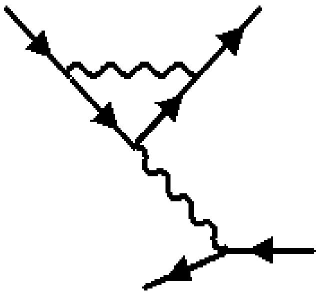
Modulation transfer function of a single detector pixel



Imaging as a linear filter

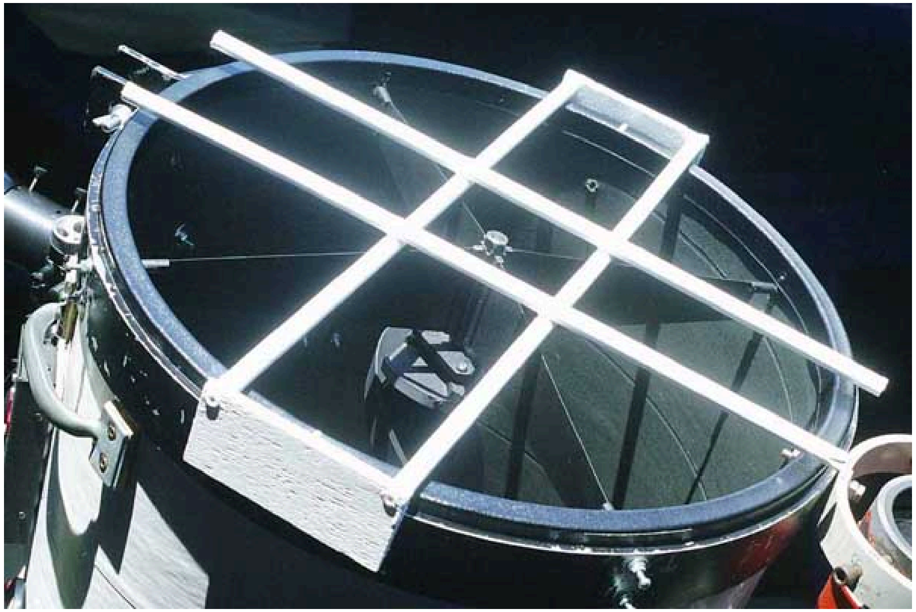
$$\text{output}(u) = \text{input}(u) \cdot MTF_{\text{optics}} \cdot MTF_{\text{detector}} \cdot MTF_{\text{algorithm}} \cdot \ldots$$





PSF examples

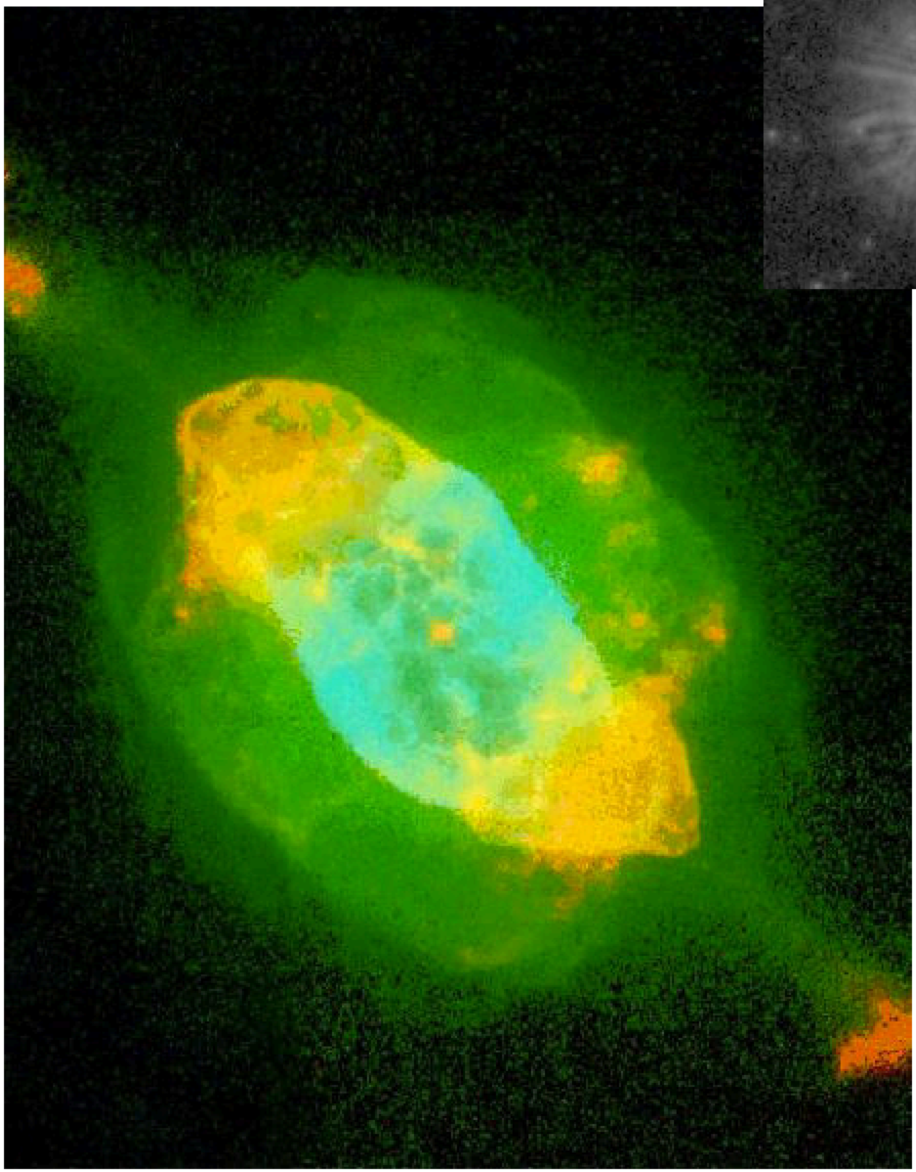
- isolated stars are essentially PSFs

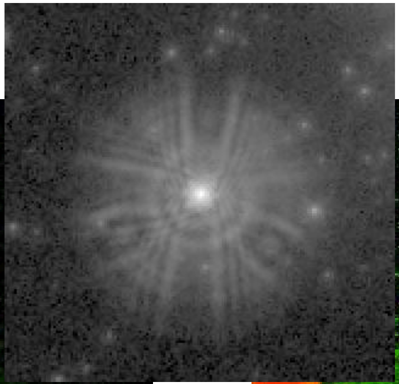


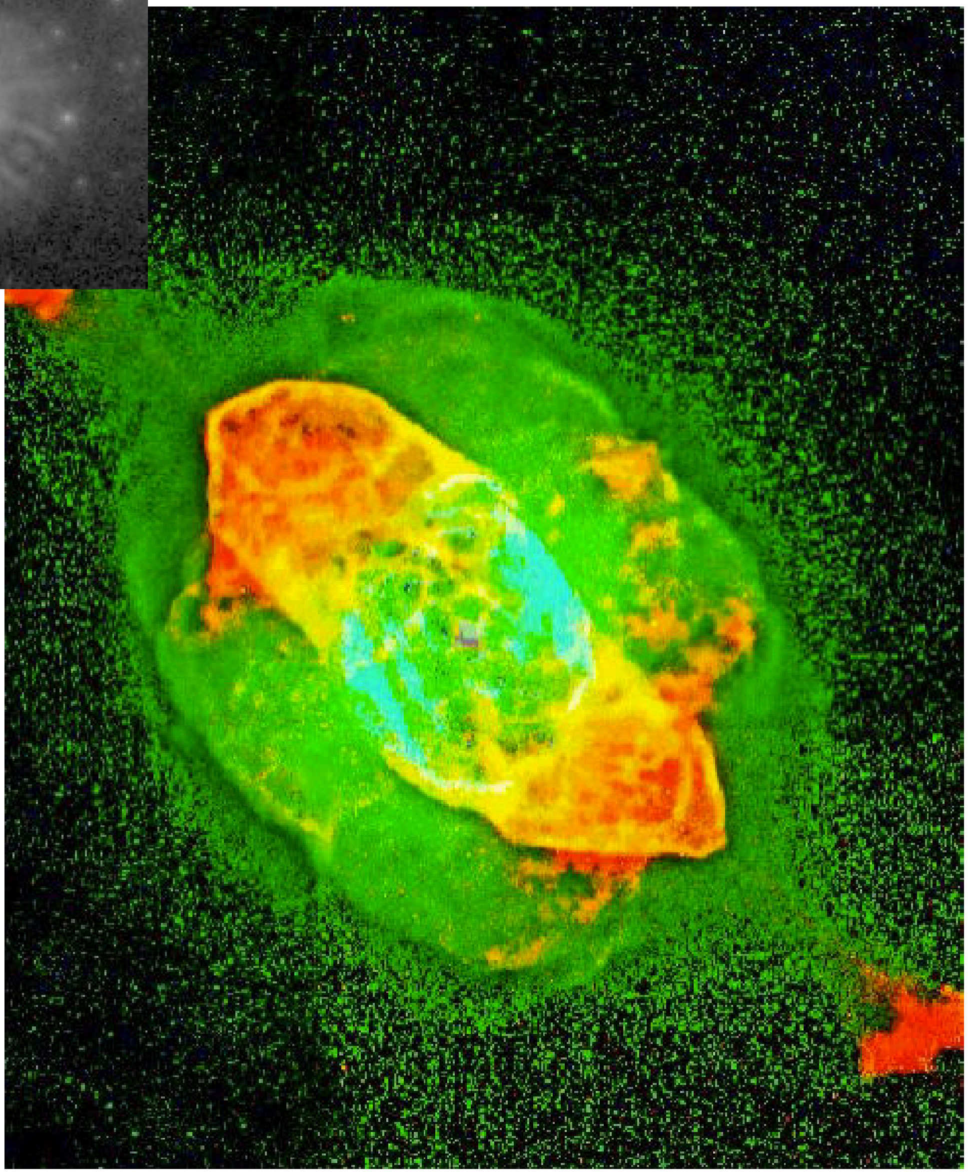
source: www.apod.nasa.gov

PSF examples

Hubble flawed mirror deconvolution (correction for spherical aberration)



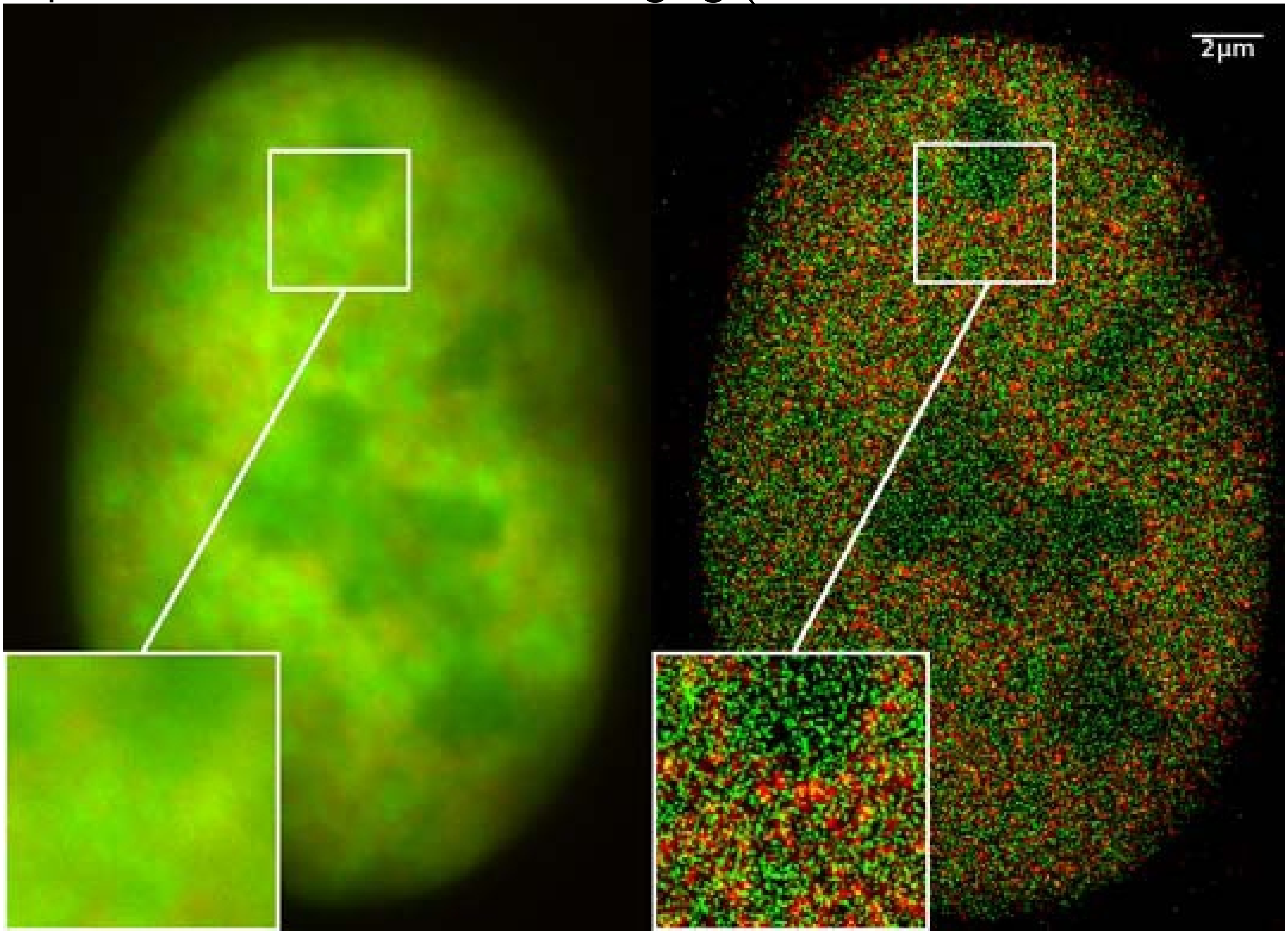




source: www.wikipedia.org

PSF examples

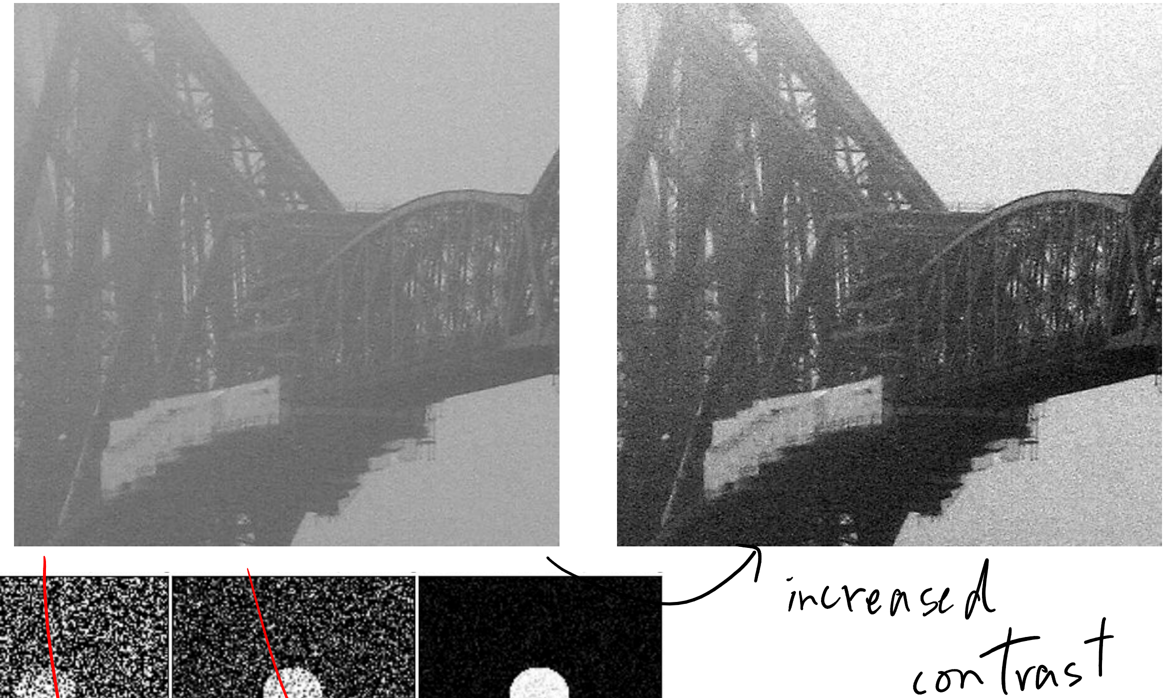
Super-resolution fluorescence imaging (STORM, STED, PALM, ...)

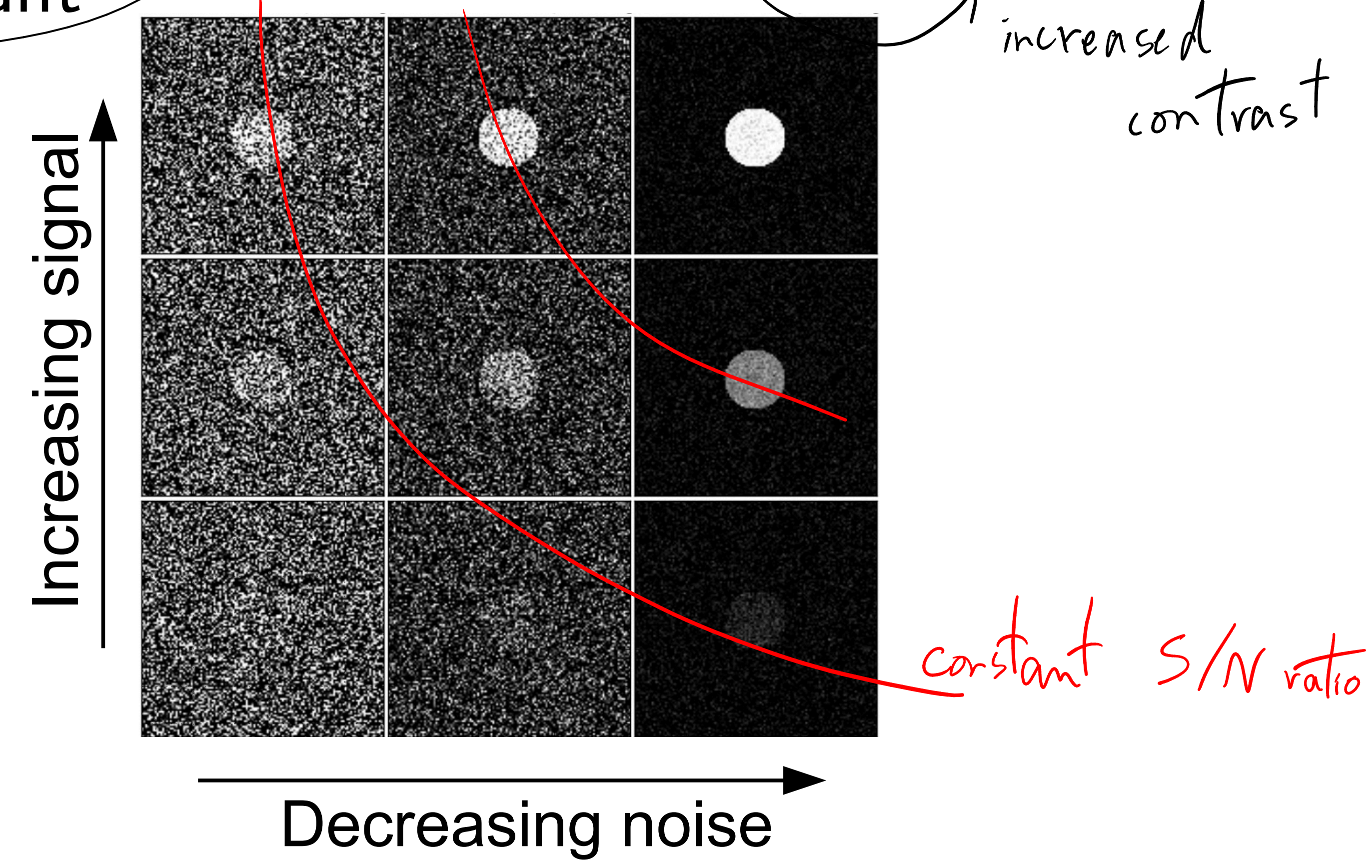


source: www.wikipedia.org

Contrast and noise

- Intensity operation: higher contrast, higher noise
- Contrast-to-noise remains constant





Random variables

- random variable, sample space

$X \qquad \Omega \qquad x \in \Omega$

$p(x) \leq 1$

$p(\Omega) = 1$

- probability density function → "PDF"

$$P(a < x < b) = \int_a^b \rho(x)\,dx \qquad \int_\Omega \rho(x)\,dx = 1$$

ρ ← probability density

- expectation value

$$E[f(x)] = \langle f \rangle = \int_\Omega f(x)\rho(x)\,dx$$

special case:

$$E[x] = \langle x \rangle = \mu = \int_\Omega x\,\rho(x)\,dx$$

μ: "mean"

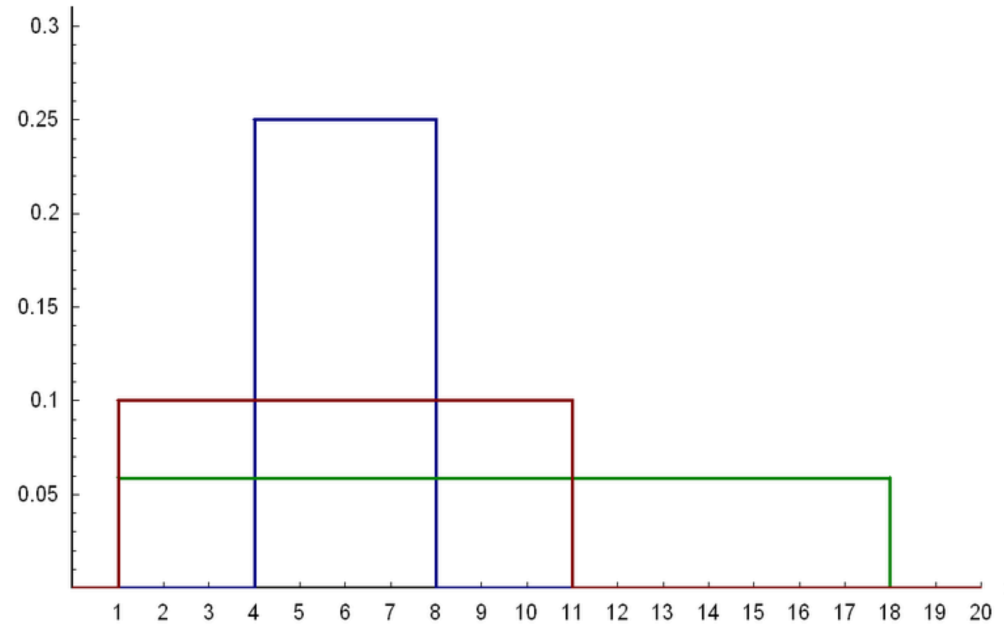
- variance

$$var(x) = V[x] = E[(x - E[x])^2] = \langle (x - \langle x \rangle)^2 \rangle$$

Uniform distribution

- probability density function

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



- expectation value

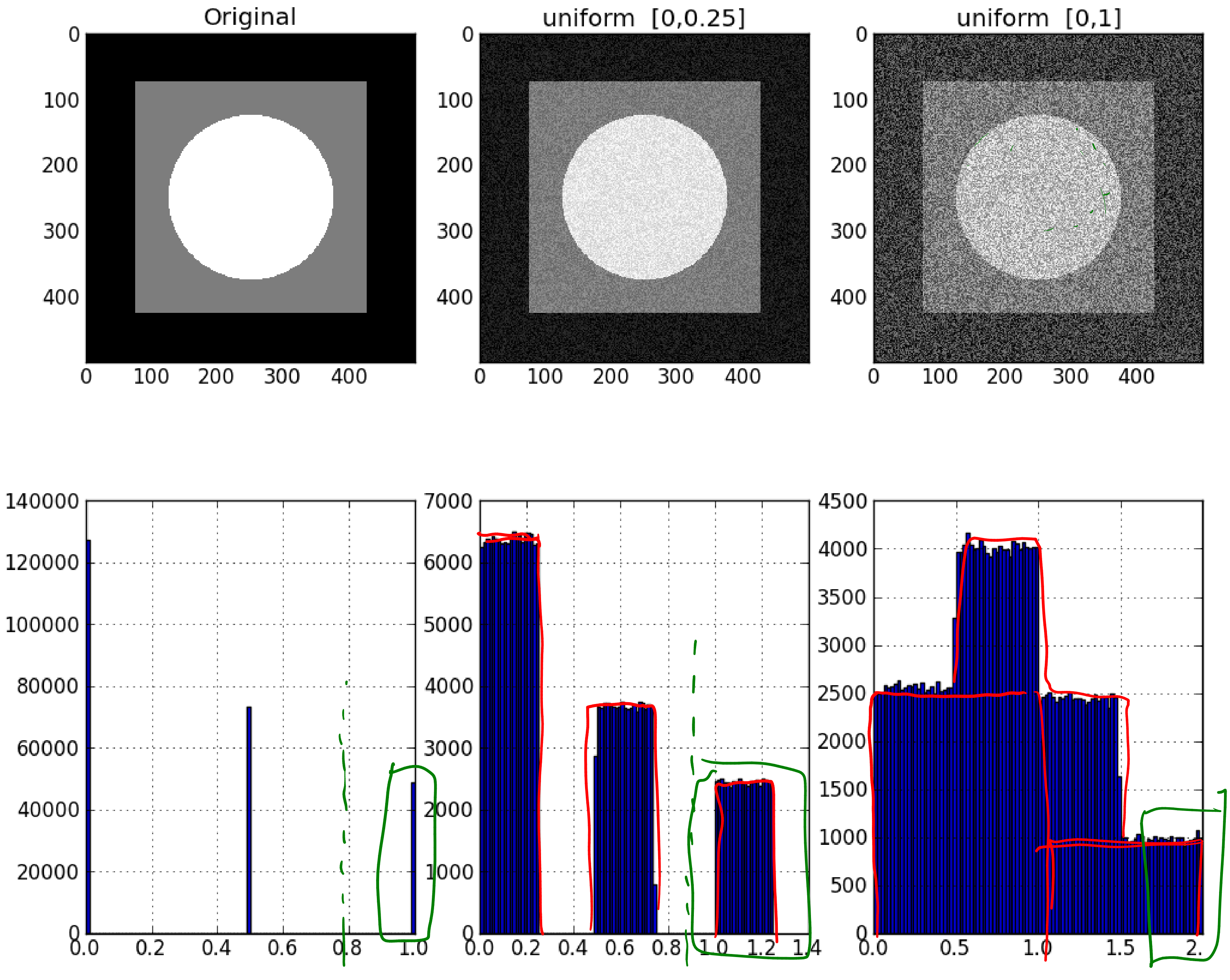
mean: $\frac{1}{2}(a+b)$

- variance

$$\frac{(b-a)^2}{12}$$

- occurrence not very common in physics, but useful to construct other probability distributions

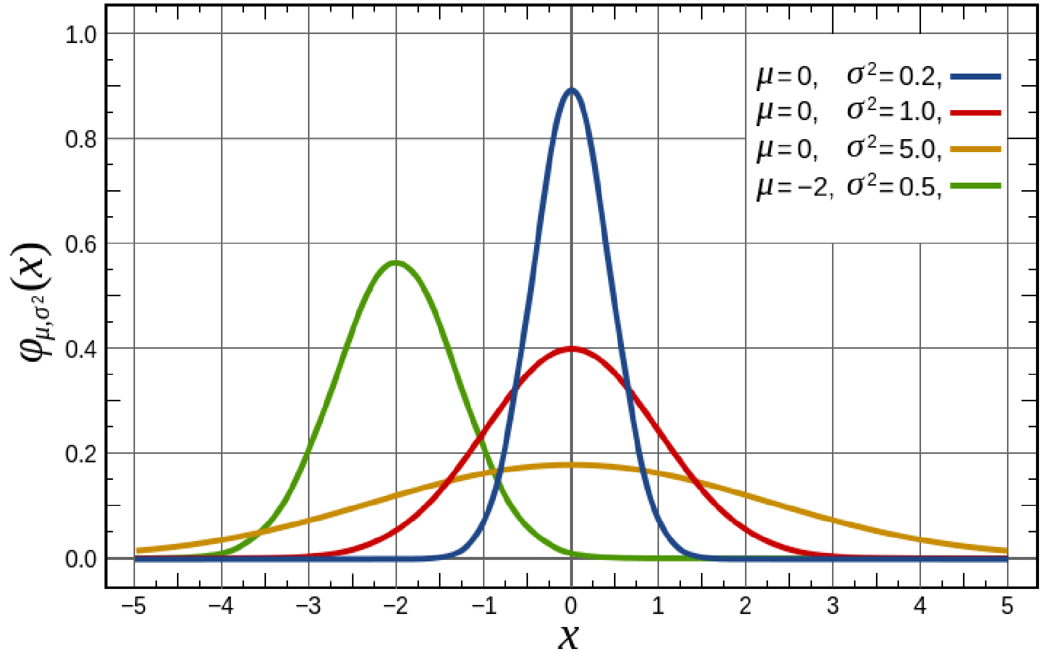
Uniform distribution



Gaussian distribution

- probability density function

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



- expectation value

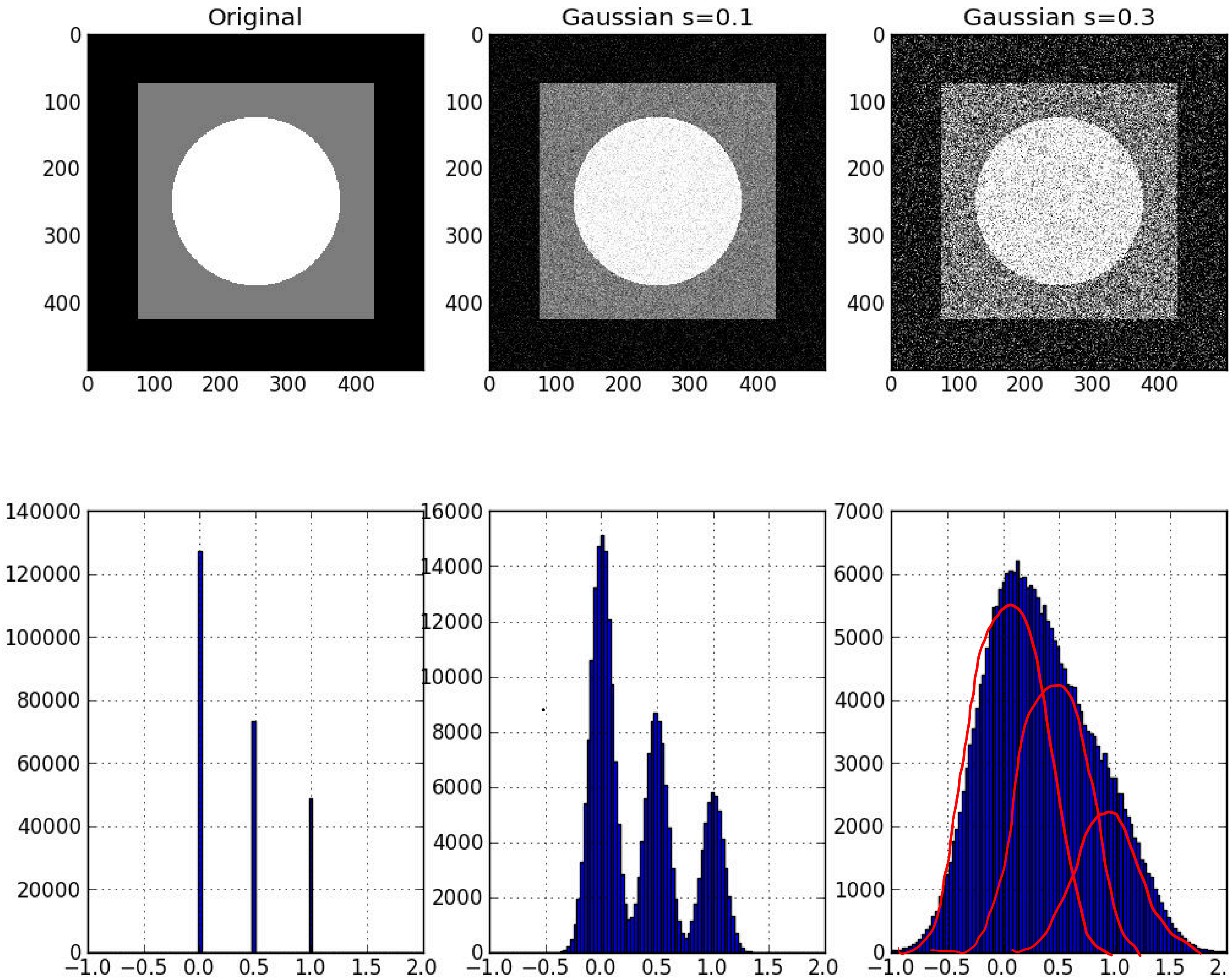
$$E[x] = \mu$$

- variance

$$V[x] = \sigma^2$$

- occurrence → very common (central limit theorem)

Gaussian distribution

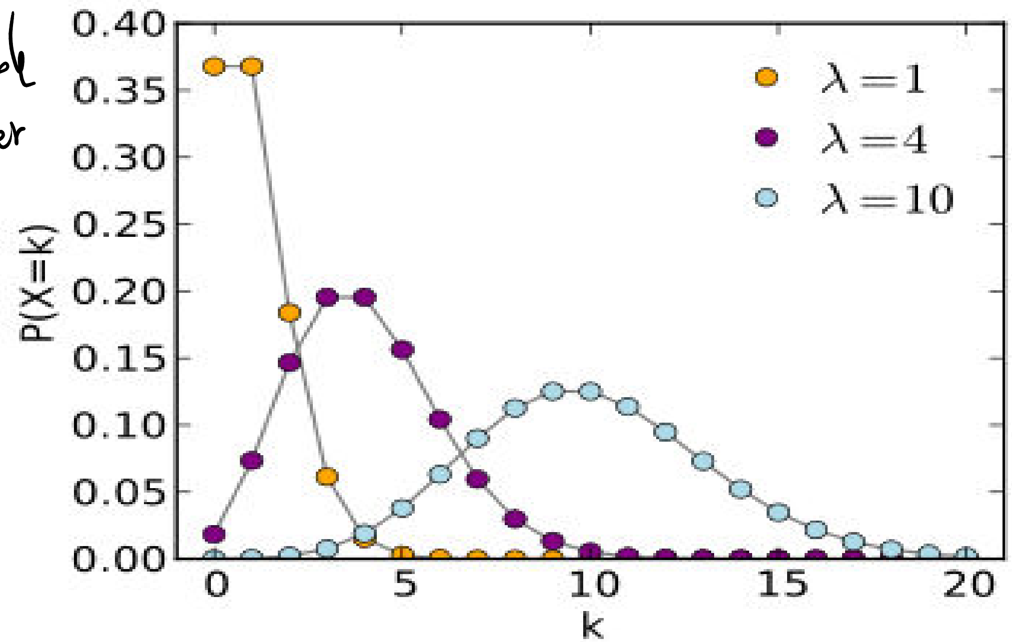


Poisson distribution

- probability mass function

random variable is integer

$$p(n) = \frac{1}{n!} \lambda^{n} e^{-\lambda}$$



- expectation value

$$E[n] = \lambda$$

S/N ratio : $\frac{E[n]}{\sqrt{V[n]}} = \sqrt{\lambda}$

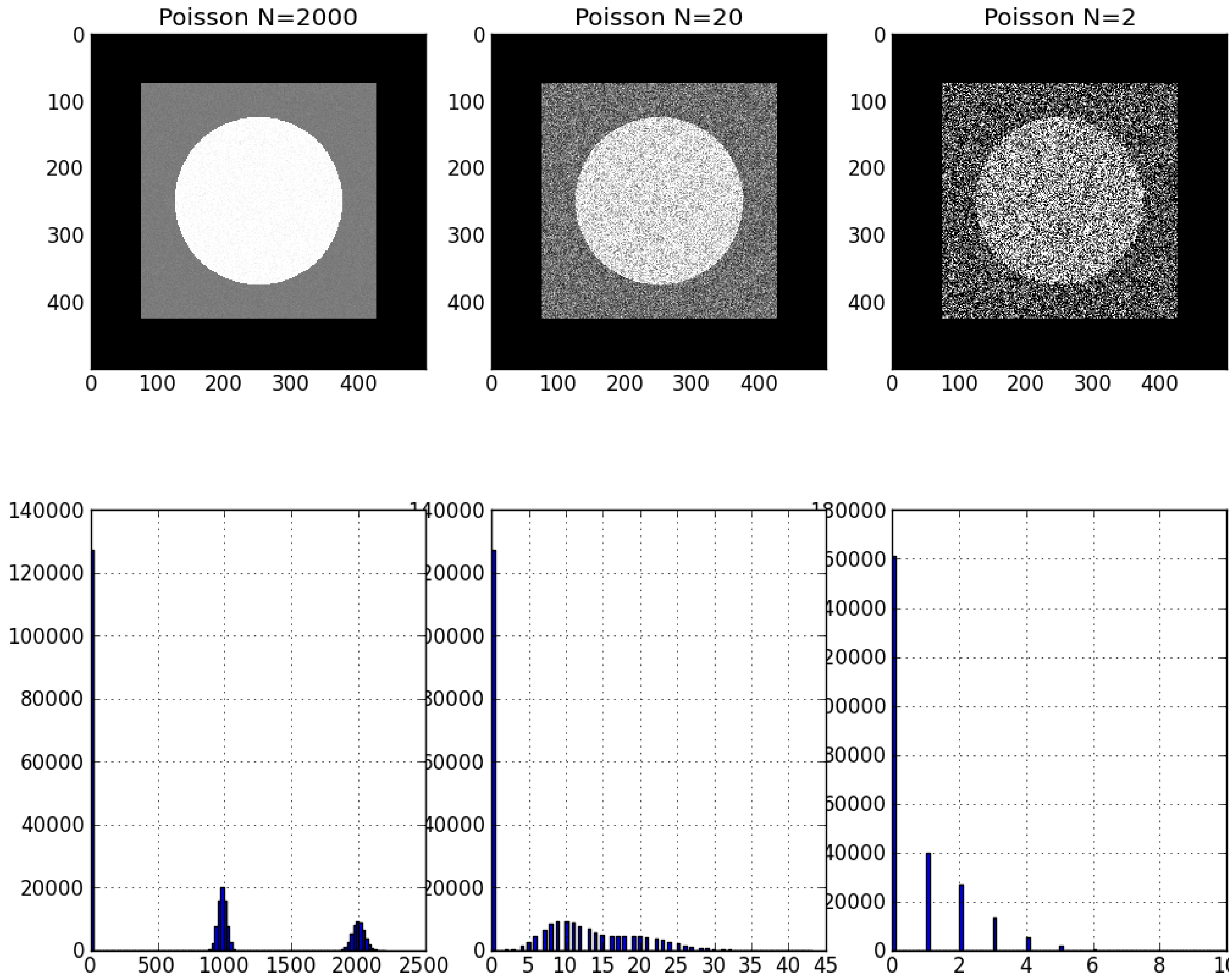
- variance

$$V[n] = \lambda$$

- occurrence counting process (photons, electrons)

"shot noise"

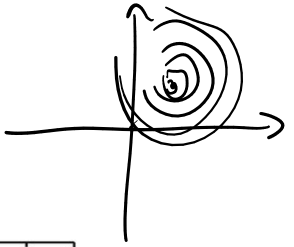
Poisson distribution



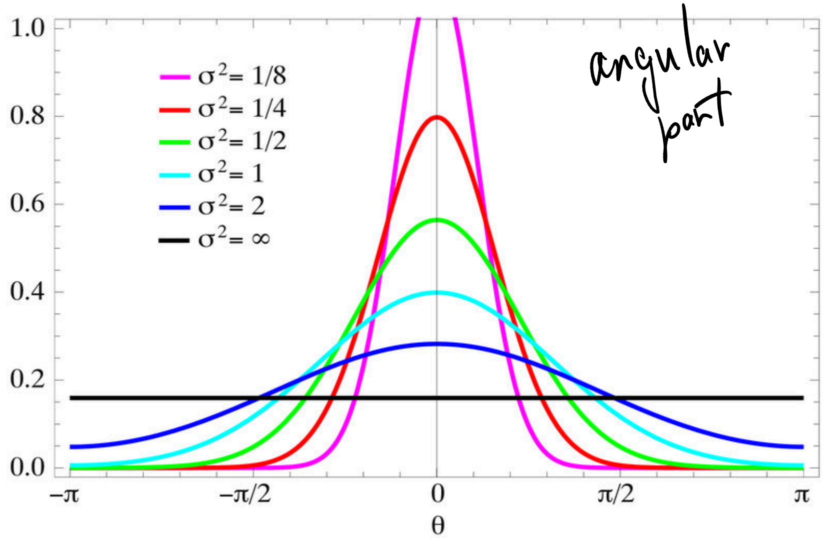
Poisson distribution

$S/N \propto \sqrt{\lambda}$

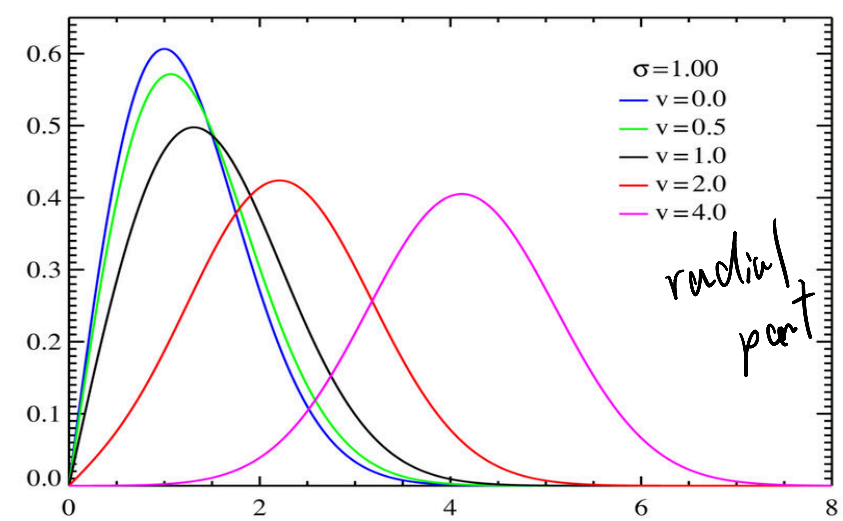
Many other distributions



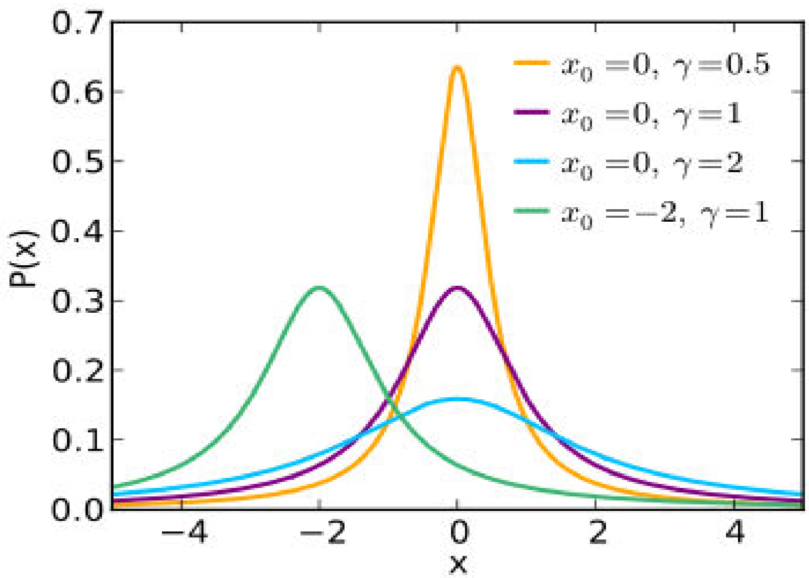
Wrapped normal distribution



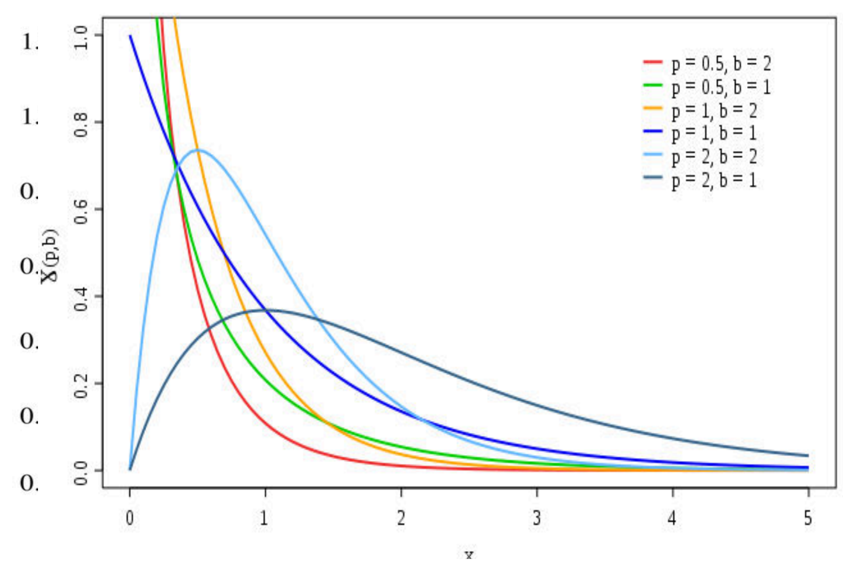
Rice distribution



Lorentz distribution



Gamma distribution



Detector noise (CCD)

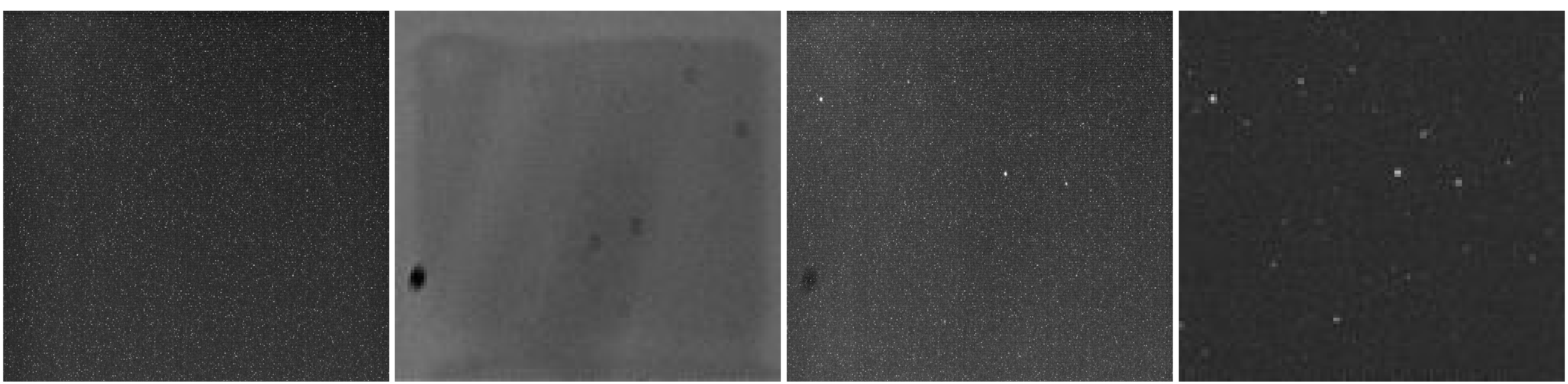
- Various sources:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gauss)
  - many other imperfections ...

- dark frame measures detector noise, hot pixels, dead pixels

- bright frame measures gain differences and imperfections (dust, etc)

uniform illumination "flat"

dark frame *bright frame* *raw image* *calibrated image*



source: H. Raab, Johannes-Kepler-Observatory, Linz

Correlation & Convolution

Convolution $f * g = \int_{-\infty}^{\infty} f(x')\, g(x - x')\, dx'$

Convolution theorem:

$$\mathcal{F}\{f * g\} = F \cdot G$$

Correlation:

$$f \circledast g = \int_{-\infty}^{\infty} f(-x')\, g(x - x')\, dx'$$

Fourier:

complex conjugate

$$\mathcal{F}\{f \circledast g\} = F^{*} \cdot G$$

Noise power spectrum

- power spectrum of pure noise image

noise "function" in image

$$n(x,y) \xrightarrow{\mathcal{F}} N(u,v)$$

$$NPS = E\left[|N(u,v)|^2 \right]$$

noise power spectrum

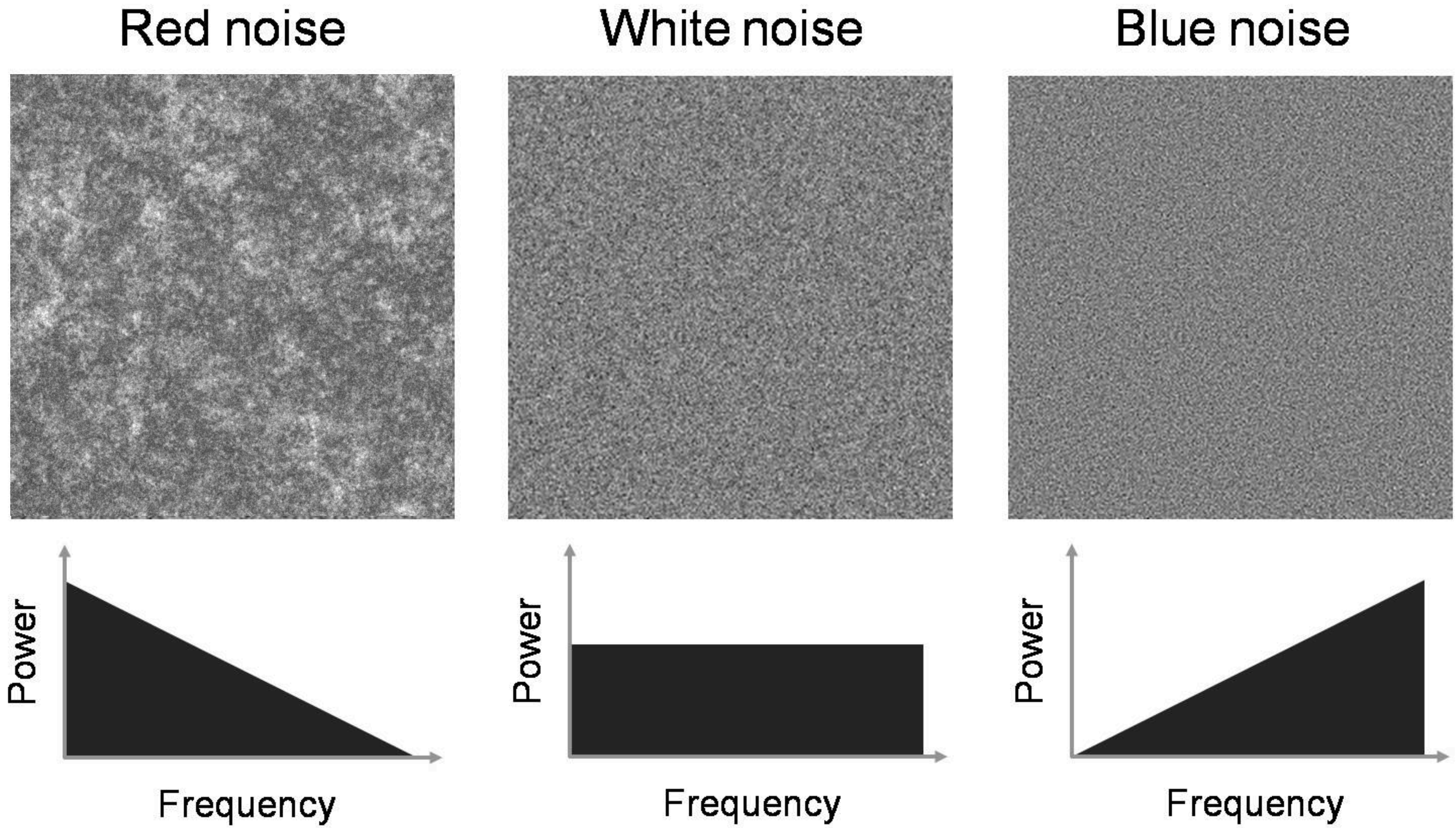
- connection to auto-correlation

$$|N(u,v)|^2 = N(u,v) \cdot N^*(u,v)$$

$$\mathcal{F}^{-1}\{NPS\} = \langle n(x,y) \circledast n(x,y) \rangle$$

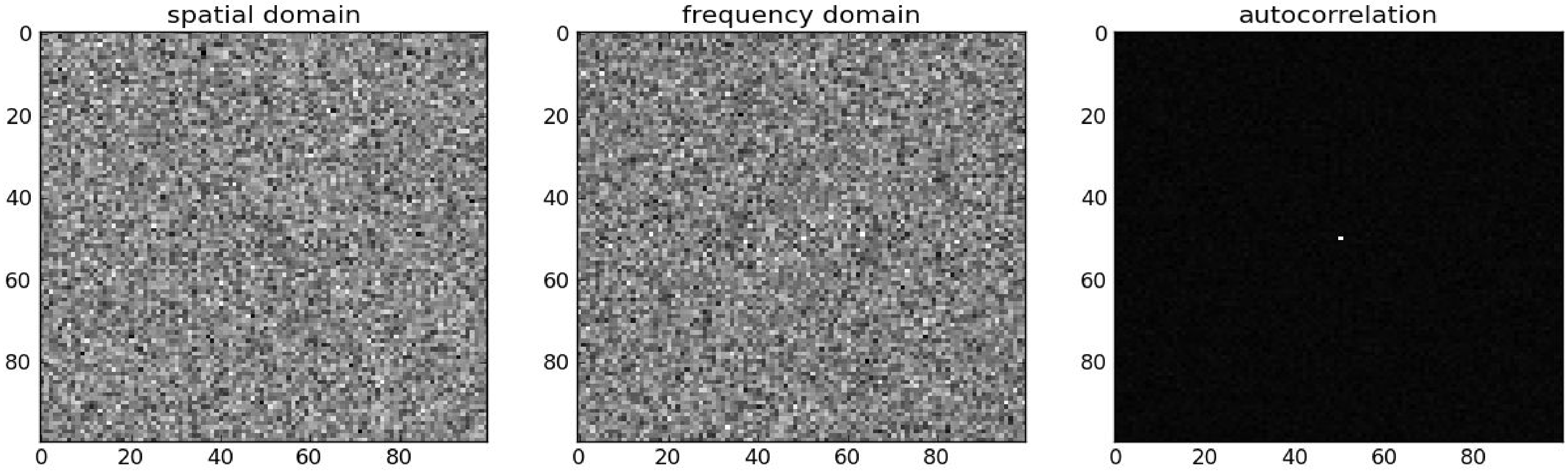
noise power spectrum $\overset{\mathcal{F}}{\longleftrightarrow}$ noise autocorrelation

Noise power spectrum



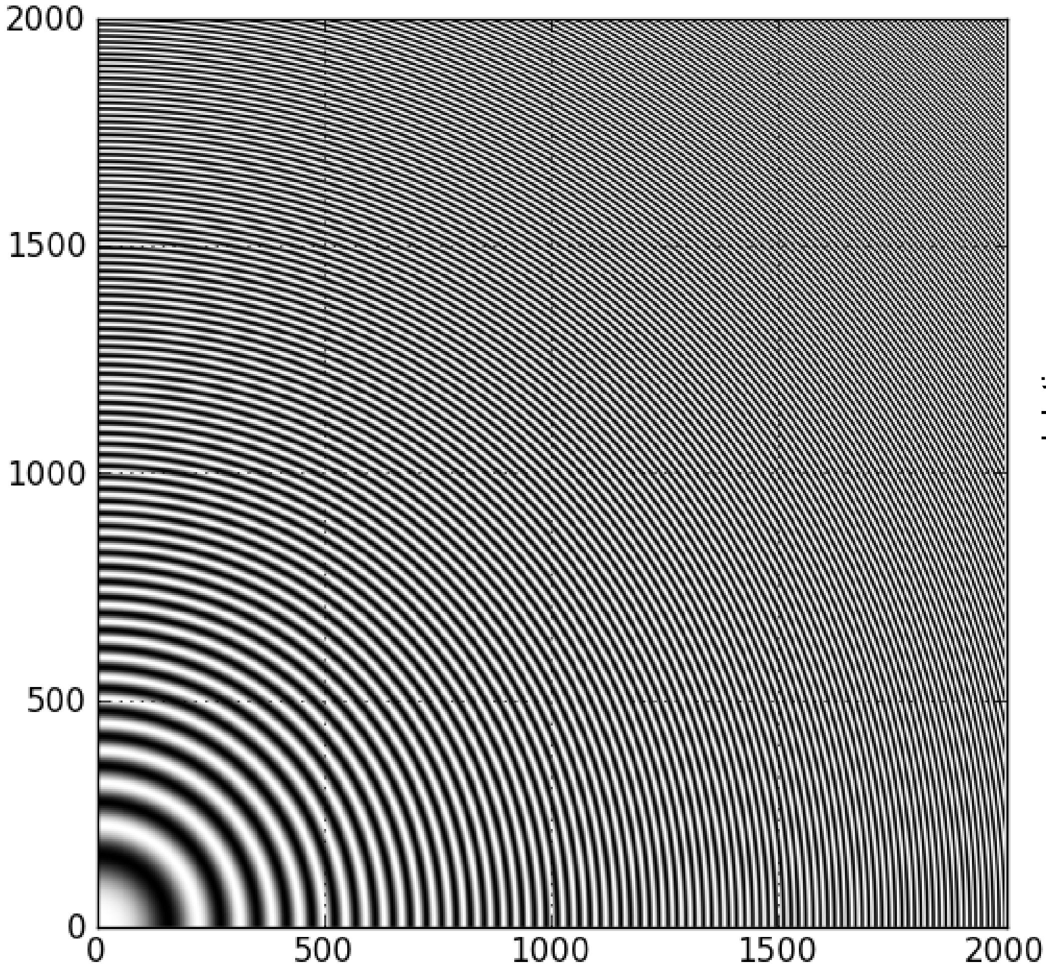
source: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

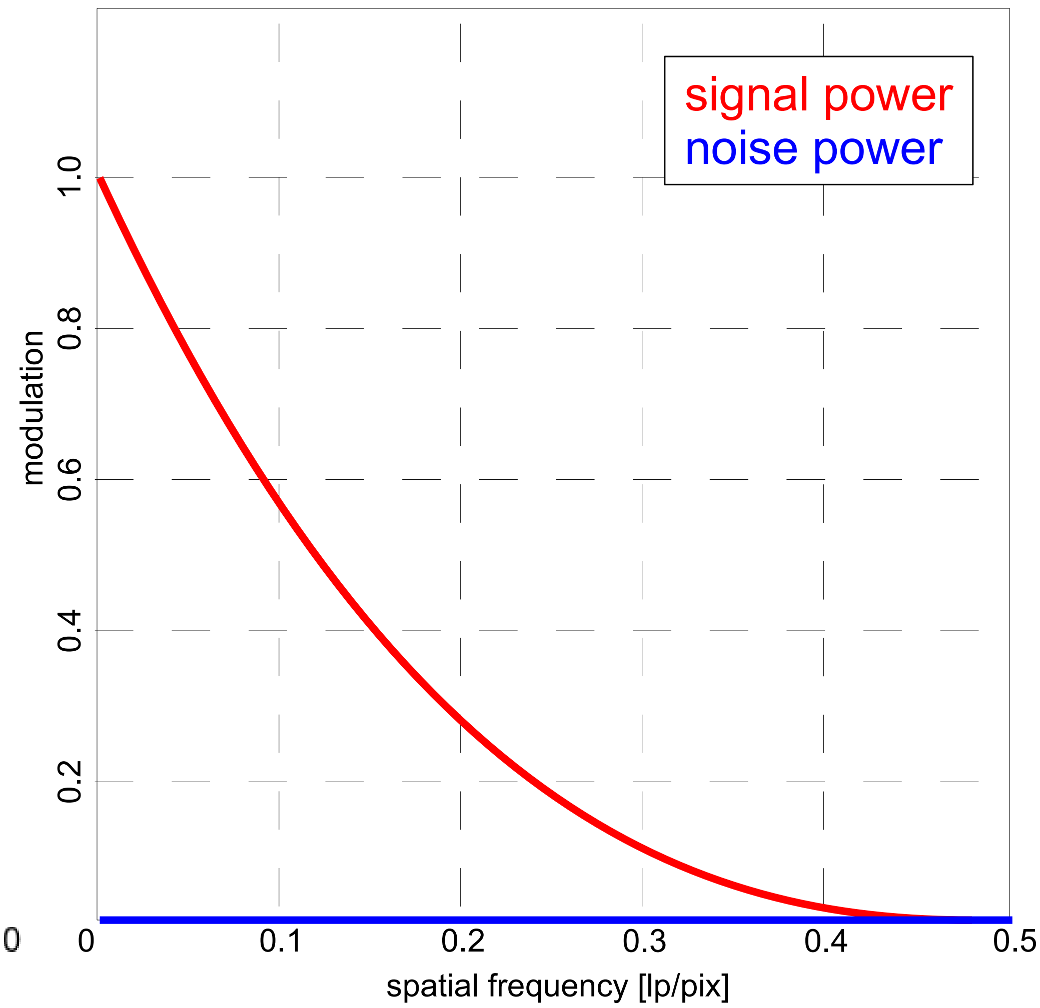
White noise



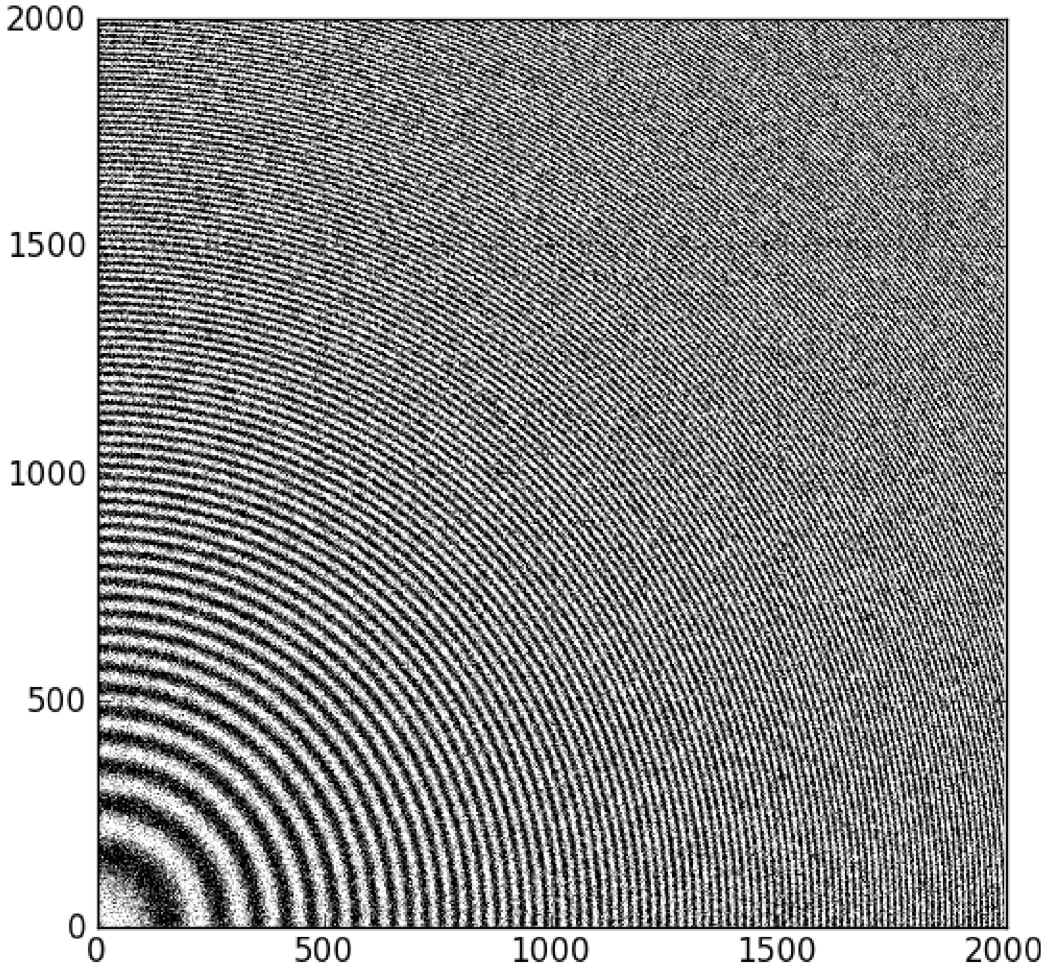
- white noise in spatial domain equals white noise in frequency domain
- white noise is perfectly uncorrelated
- all other types of noise are correlated to some degree
- white noise is an idealization

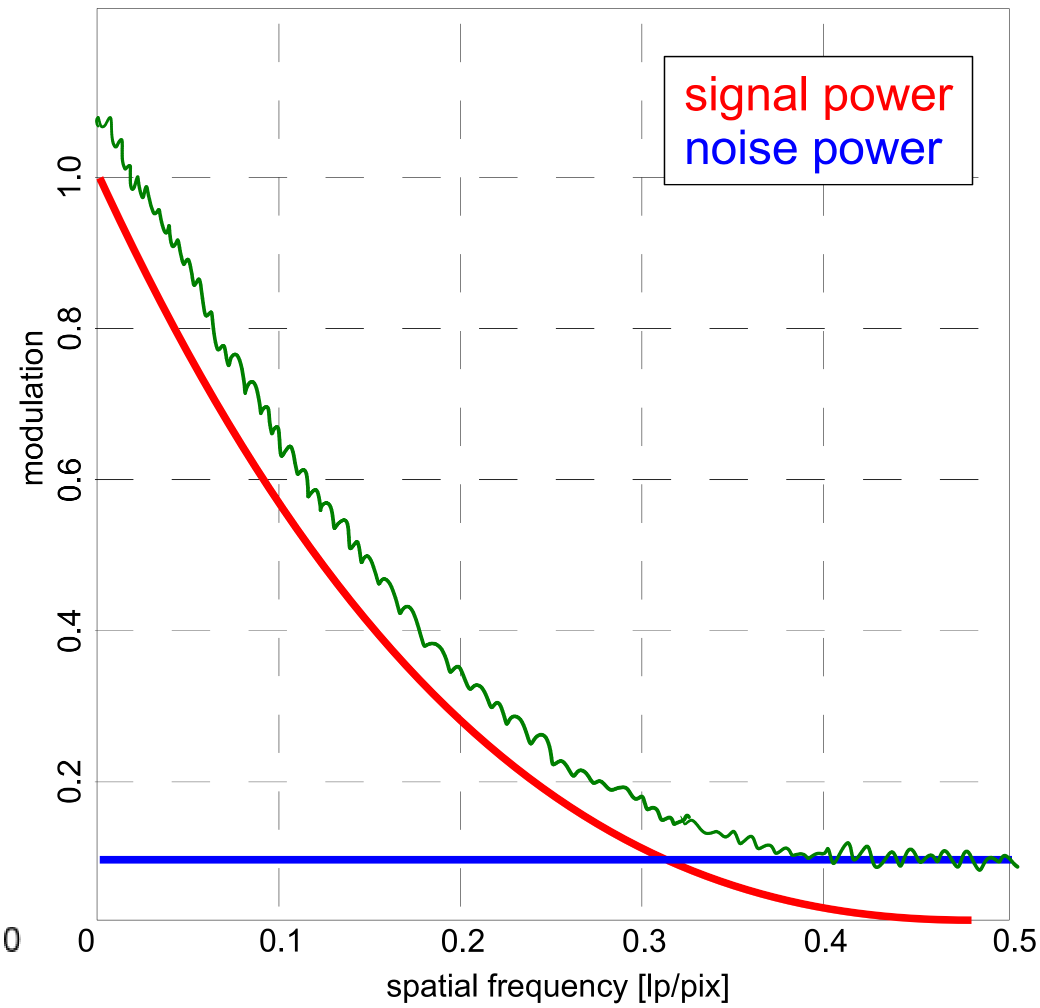
Signal power vs. noise power



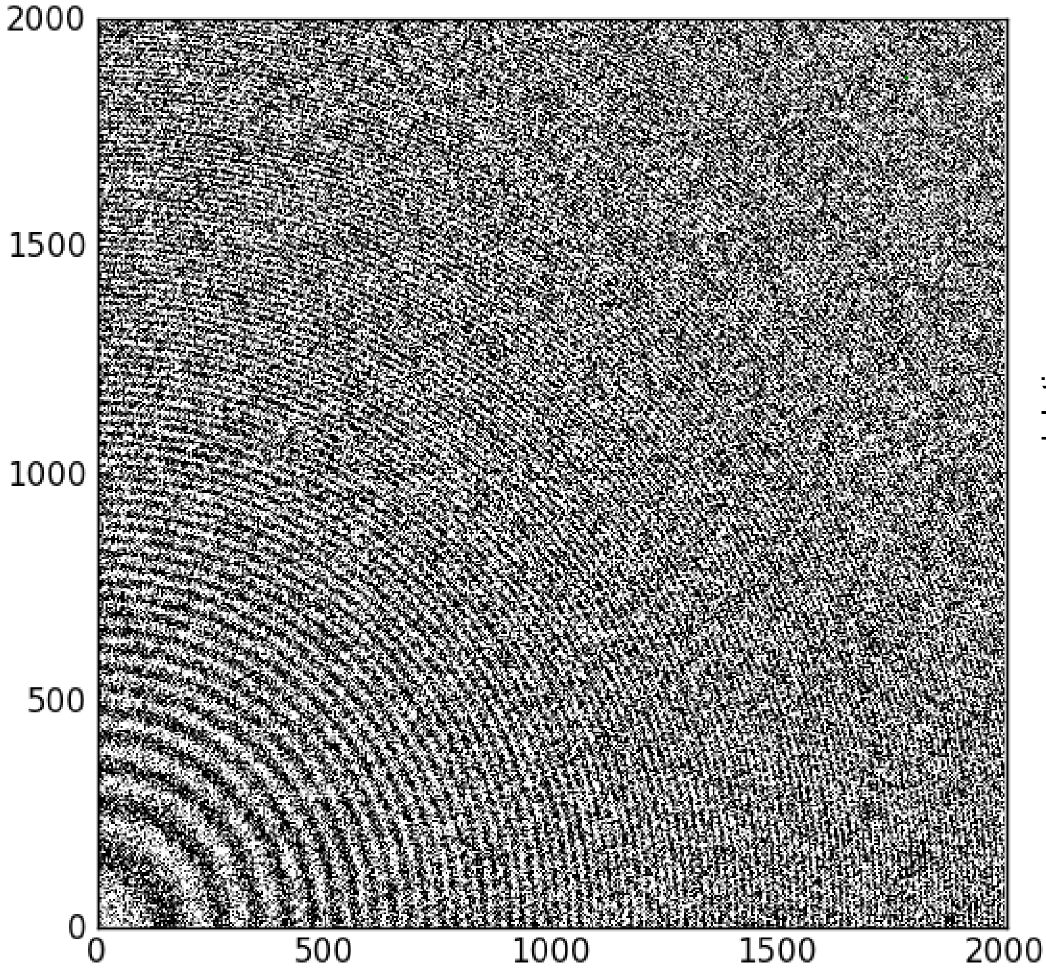


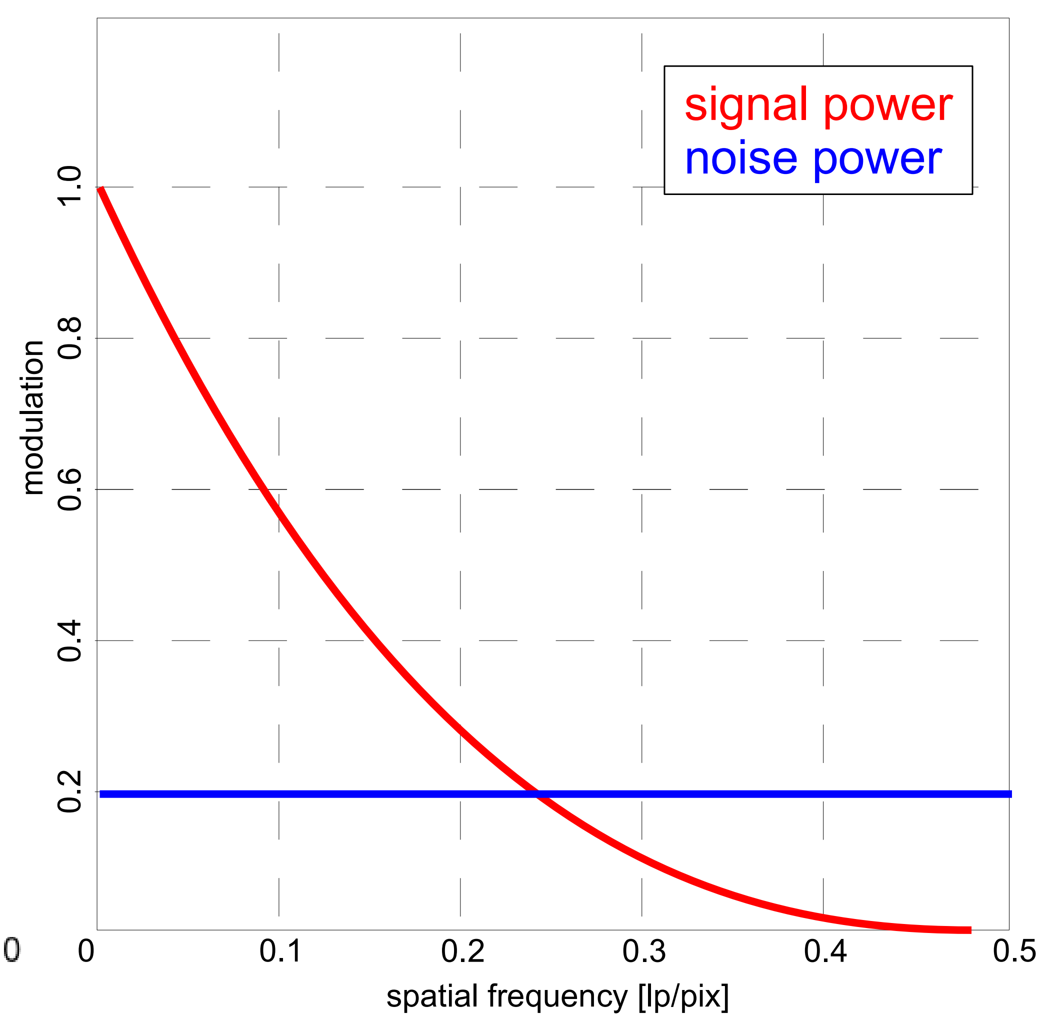
Signal power vs. noise power



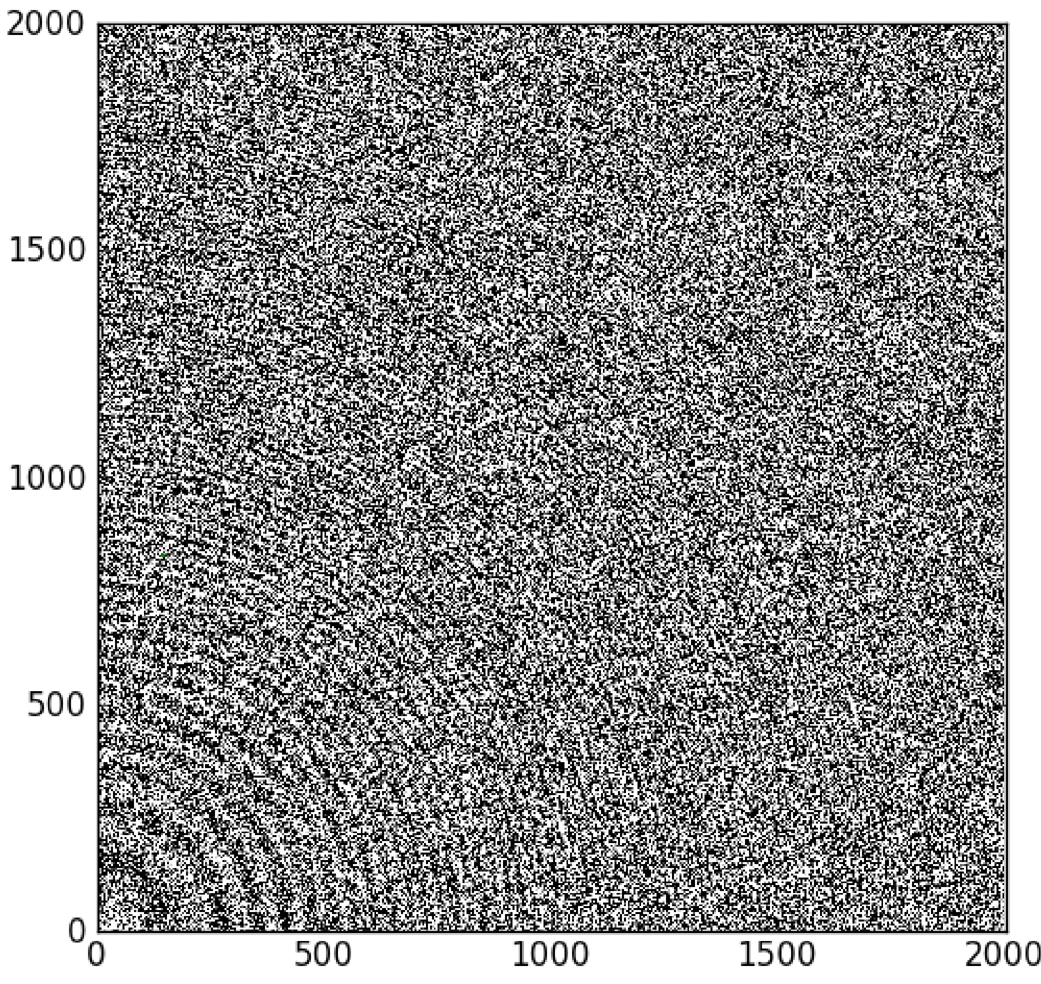


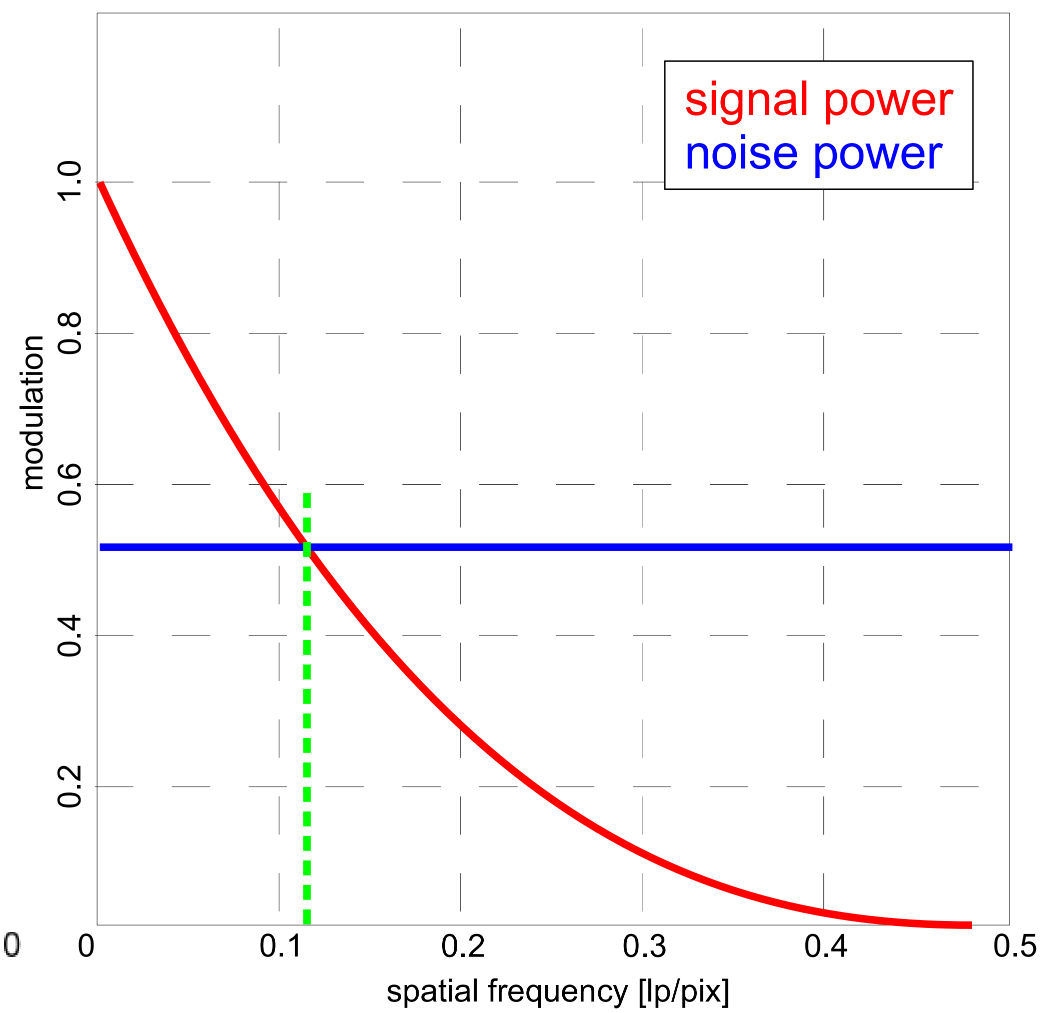
Signal power vs. noise power





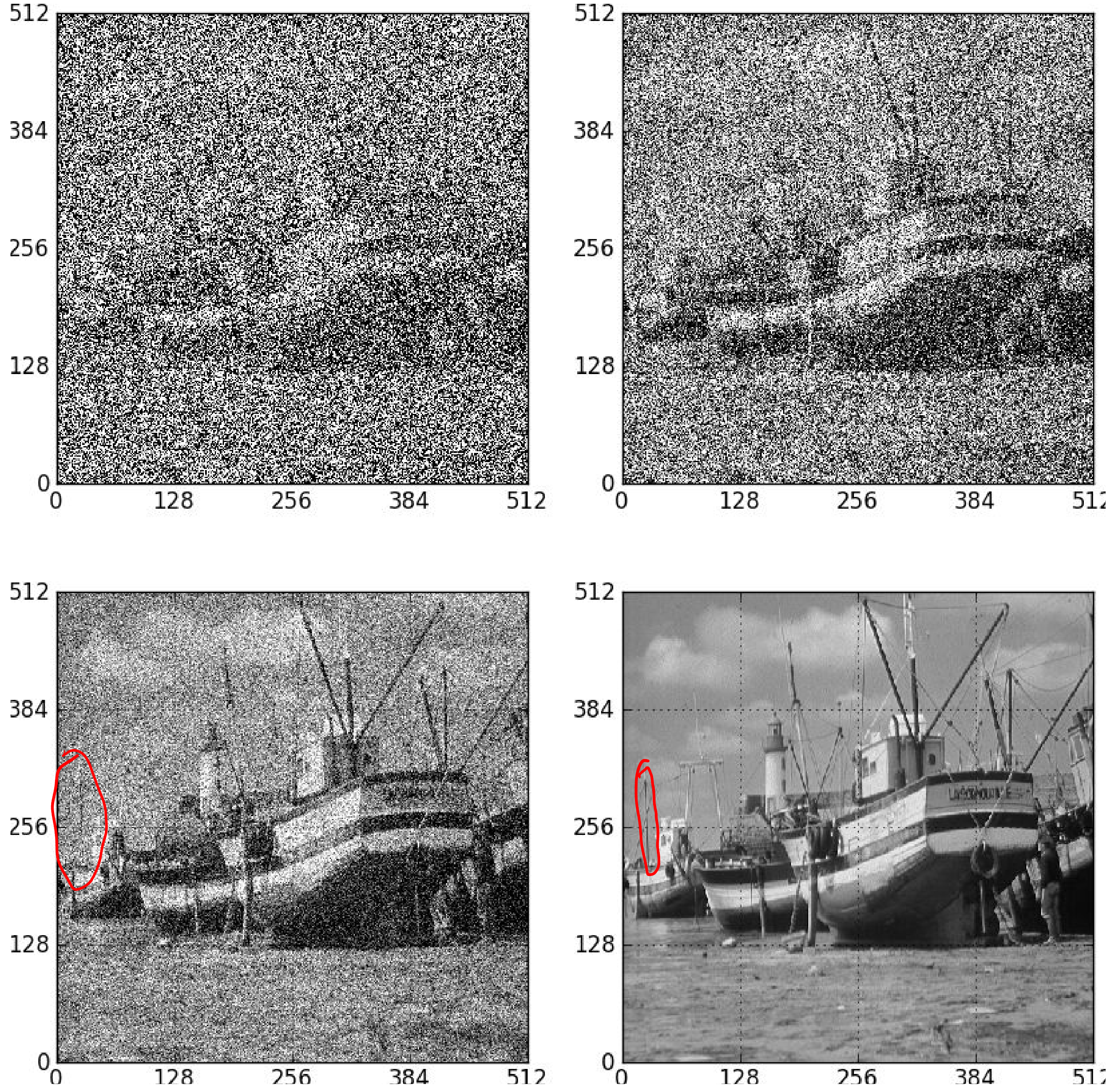
Signal power vs. noise power





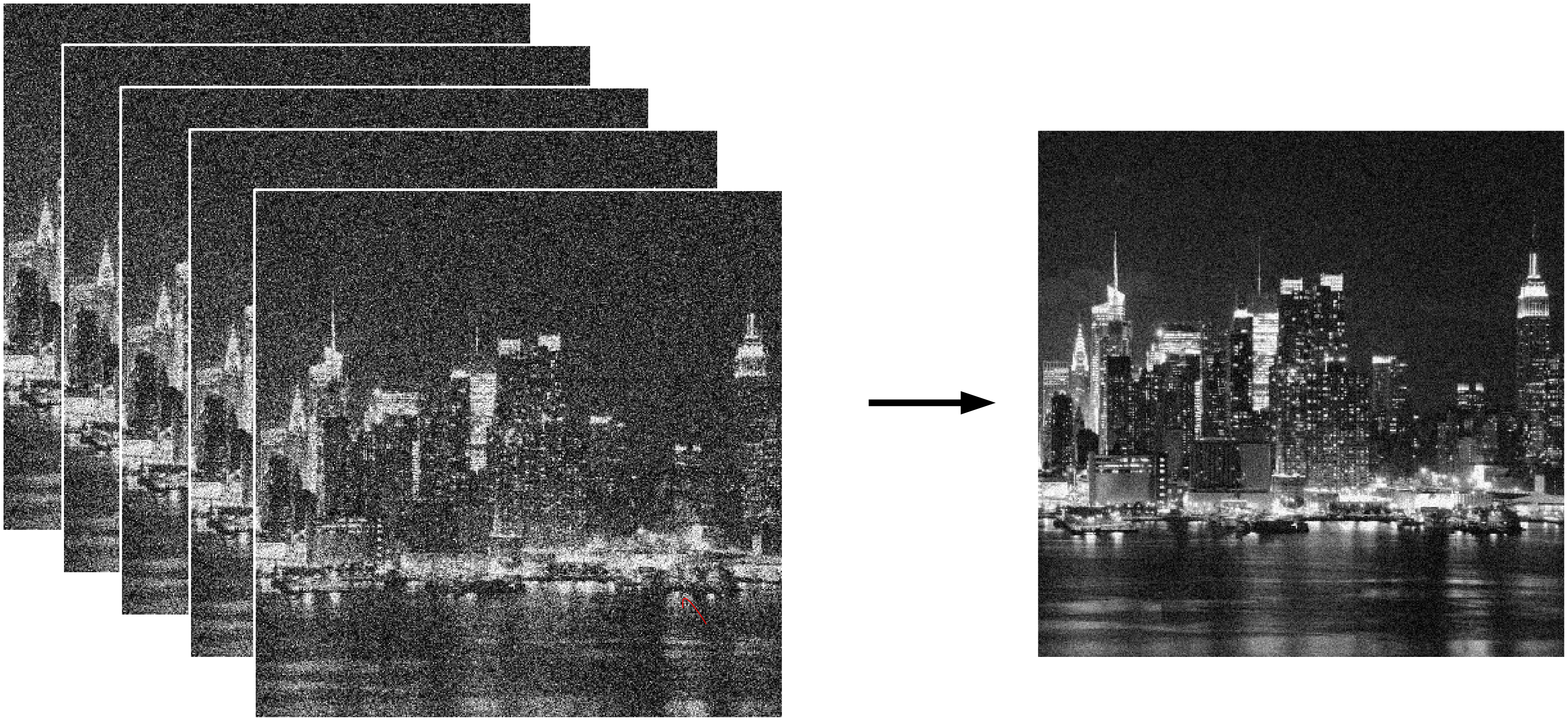
- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

Signal power vs. noise power



Noise reduction by averaging

- Average multiple images



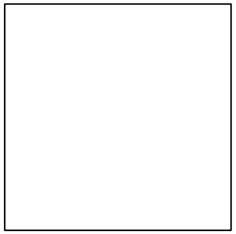
- requirement: additive noise, zero mean

Denoising by linear filtering

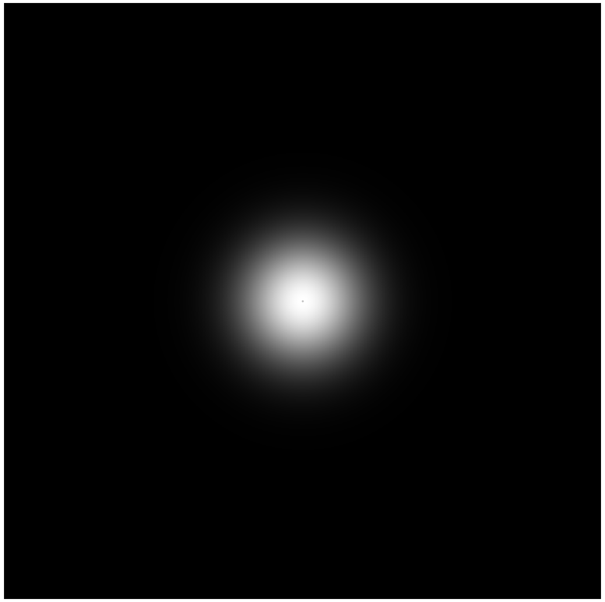
- use spatial convolution or frequency filtering to reduce noise
- noise reduction possible, but at cost of sharpness
- trade-off between noise reduction and resolution
- need fancier methods

original

convolution kernel

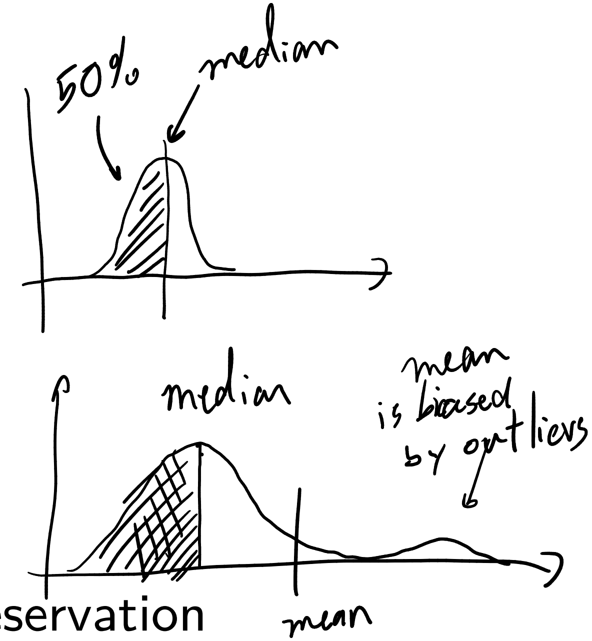


frequency filter



Resulting image

Median filtering



- Use median as estimator for fat tail distributions

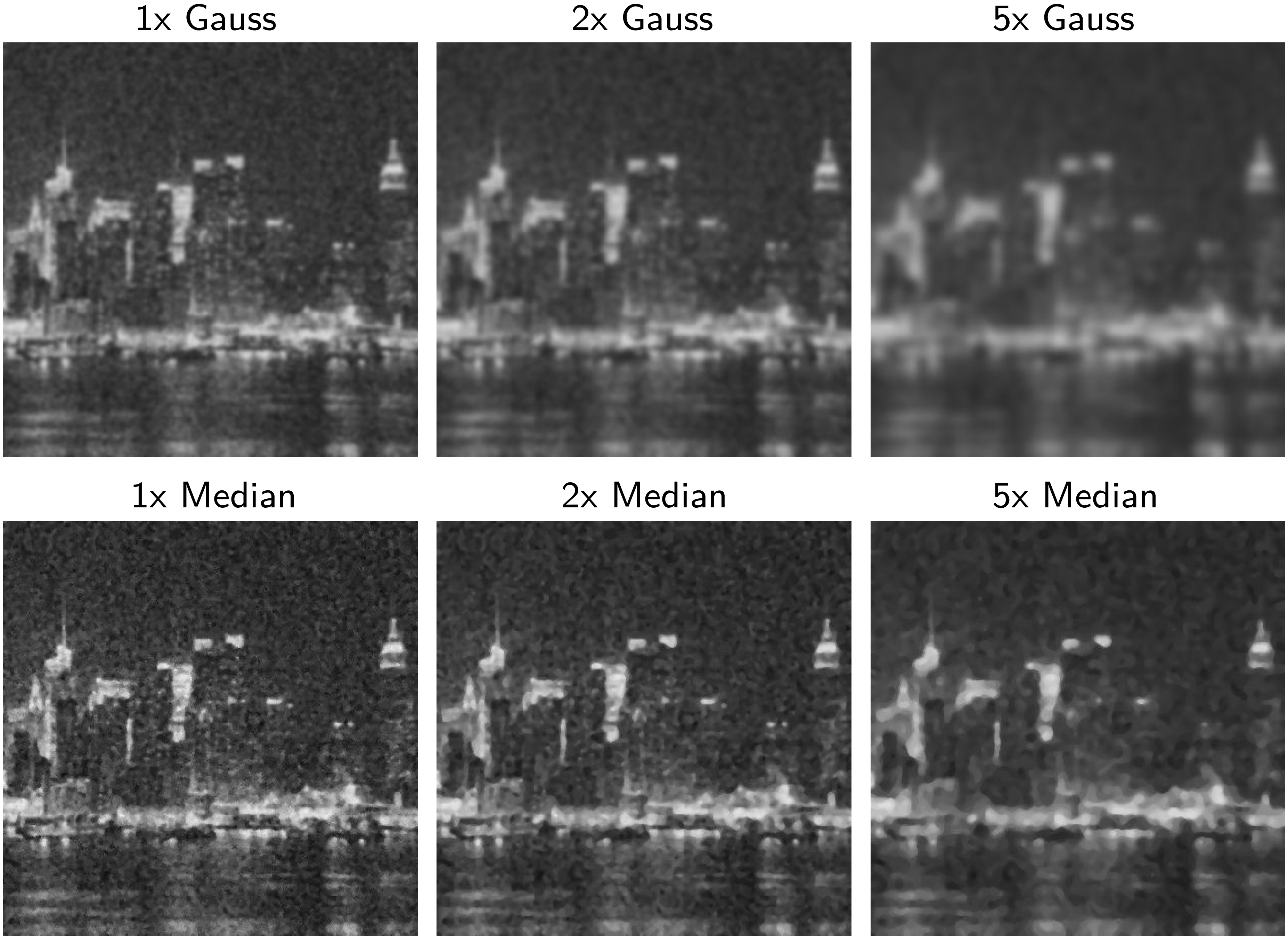
- less sensitive to outliers in pixel ensemble, better edge preservation

Salt and pepper noise

Gauss sigma=1 pixel

Median 1 pixel

Median filtering



Common abbreviations

Abbreviation	Name	Definition
IRF	Impulse response function	Linear operator map of delta function
PSF	Point spread function	Image of point object (optical IRF)
OTF	Optical transfer function	Fourier transform of PSF
PTF	Phase transfer function	Phase part of OTF
MTF	Modulation transfer function	Amplitude of OTF
CTF	Contrast transfer function	MTF for non-sinusoidal objects
PDF	Probability density function	Probability distribution for a given random variable
SPS	Signal power spectrum	Amplitude squared of signal F.T.
NPS	Noise power spectrum	Amplitude squared of noise F.T.
SNR	Signal to noise ratio	Mean signal / mean noise
CNR	Contrast to noise ratio	Mean contrast / mean noise