Image Processing for Physicists

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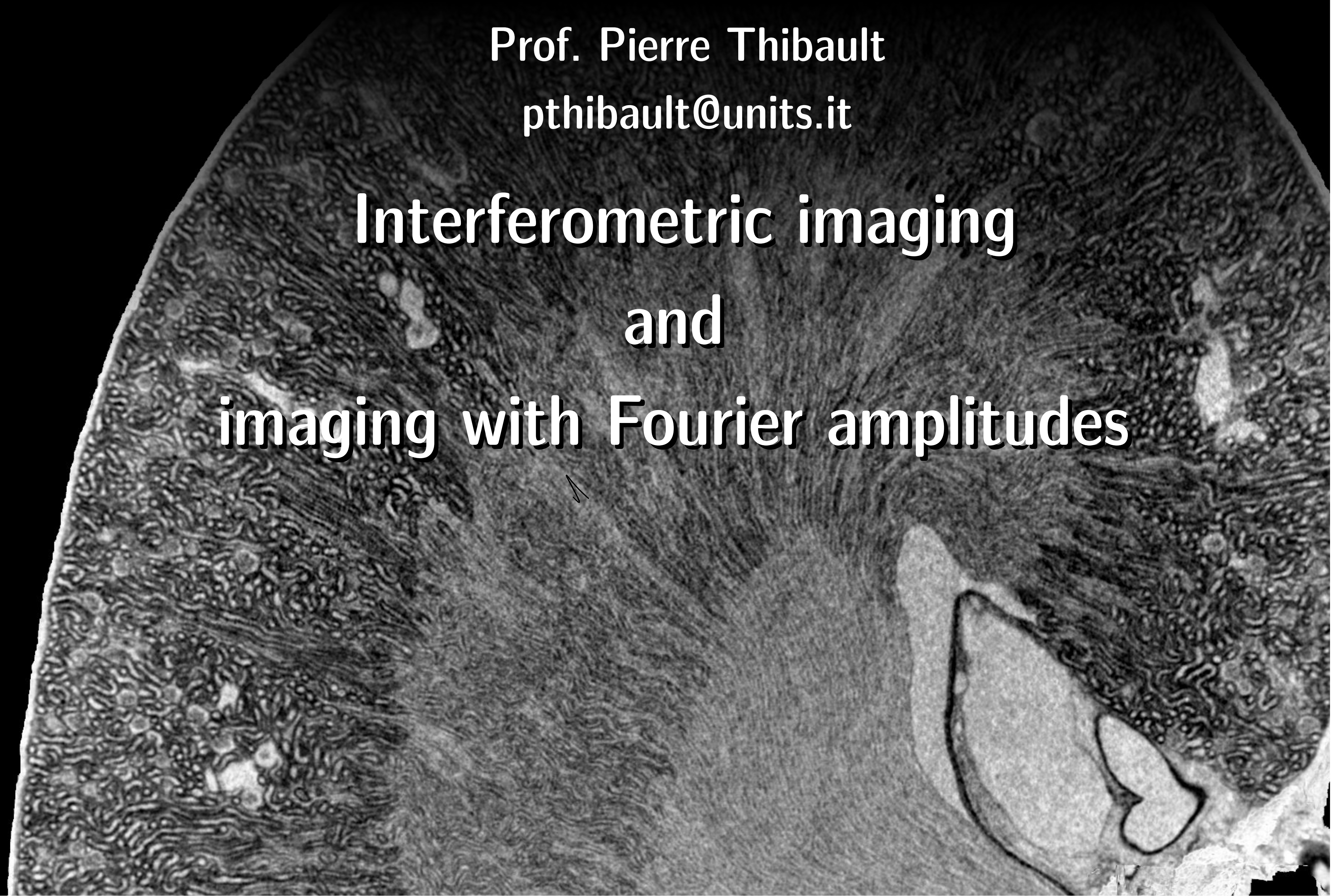


Interferometric imaging
and
imaging with Fourier amplitudes

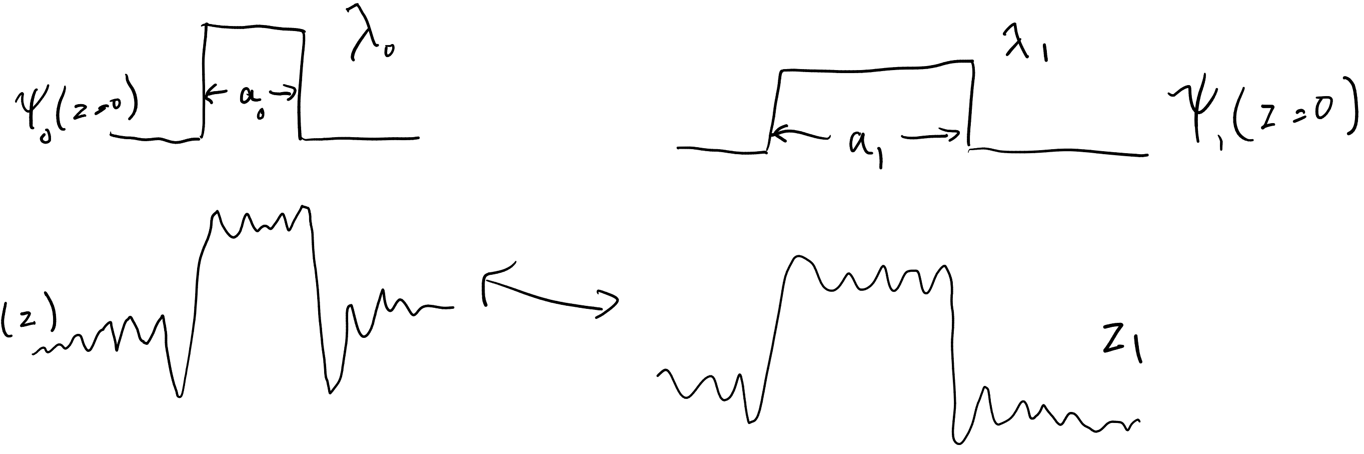
Overview

- The phase problem
- Holography: on/off-axis
- Grating interferometric imaging
- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

Wave propagation

Near-field / far-field

$$\Psi(\vec{r}; z) = \mathcal{F}^{-1}\left\{\mathcal{F}\left\{\psi(r; z=0)\exp(-i\pi \underbrace{u^2\lambda z}_{\text{unitless}})\right\}\right\}$$



$$\frac{a^2}{\lambda z} := f$$

"Fresnel number"

$f \ll 1$: far-field

$f \gg 1$: near-field

mathematically identical if

$$\frac{1}{a_0}\sqrt{\lambda_0 z_0} = \frac{1}{a_1}\sqrt{\lambda_1 z_1}$$

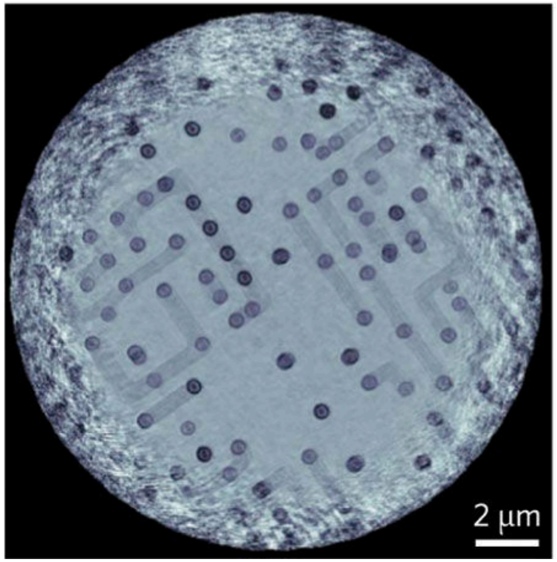
$$\frac{\lambda z}{a2}$$

$\sqrt{\lambda z}$: characteristi
length of oscillations caused by propagation

"first Fresnel zone"

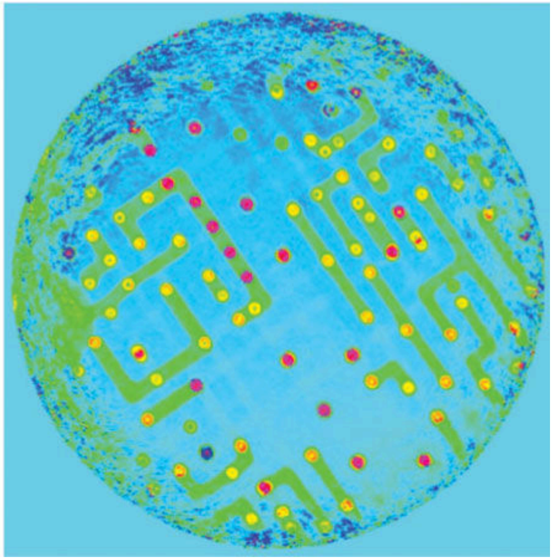
Complex-valued images

X-ray transmission function



Amplitude

= attenuation of the wave as it travels through the object

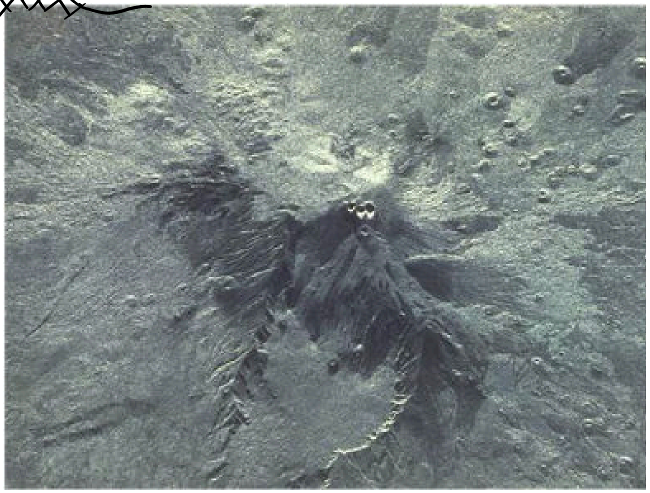


Phase

= delay of the wave as it travel through the object

→ refraction

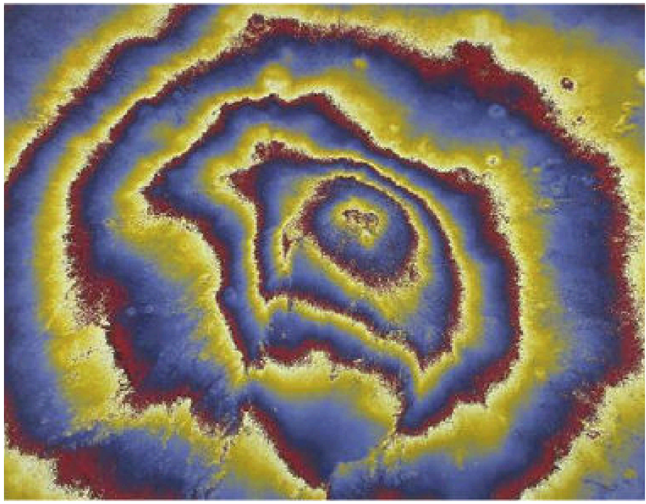
Synthetic aperture radar
SAR



unwrapped phase

phase

phase is "wrapped"



raw data

Etna

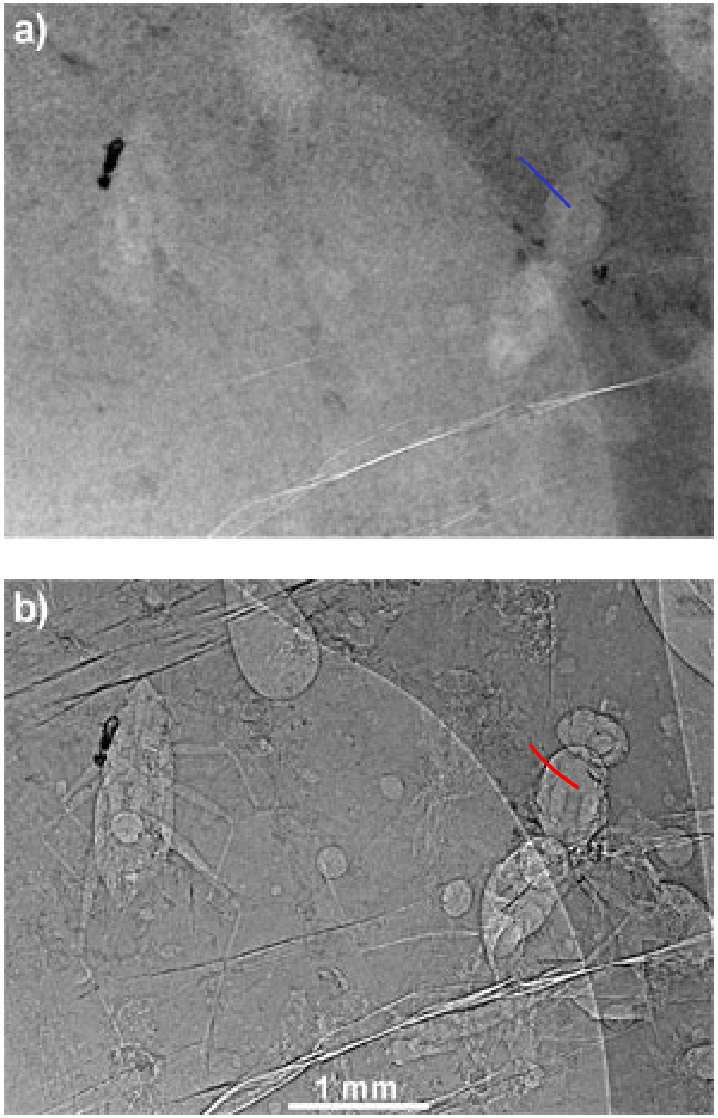
"phase unwrapping" requires spatial information

Phase-contrast

examples for short-wavelength radiation

Hard X-ray propagation-based "Very" near-field phase contrast

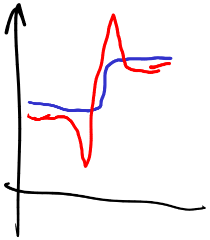
Neutron phase contrast



Source: www.esrf.eu/news/general/amber/amber/

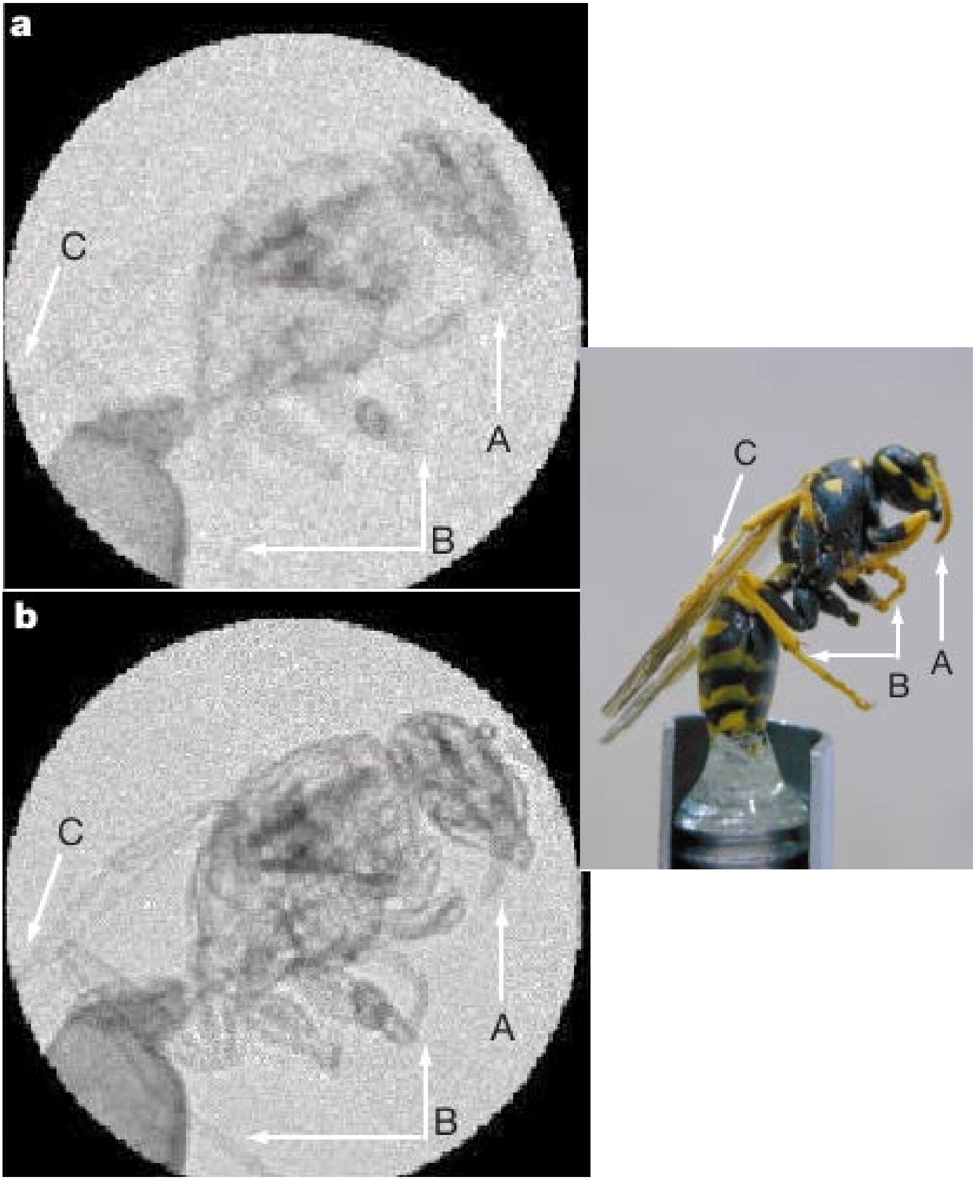
"contact-mode images"

propagation-based phase-contrast



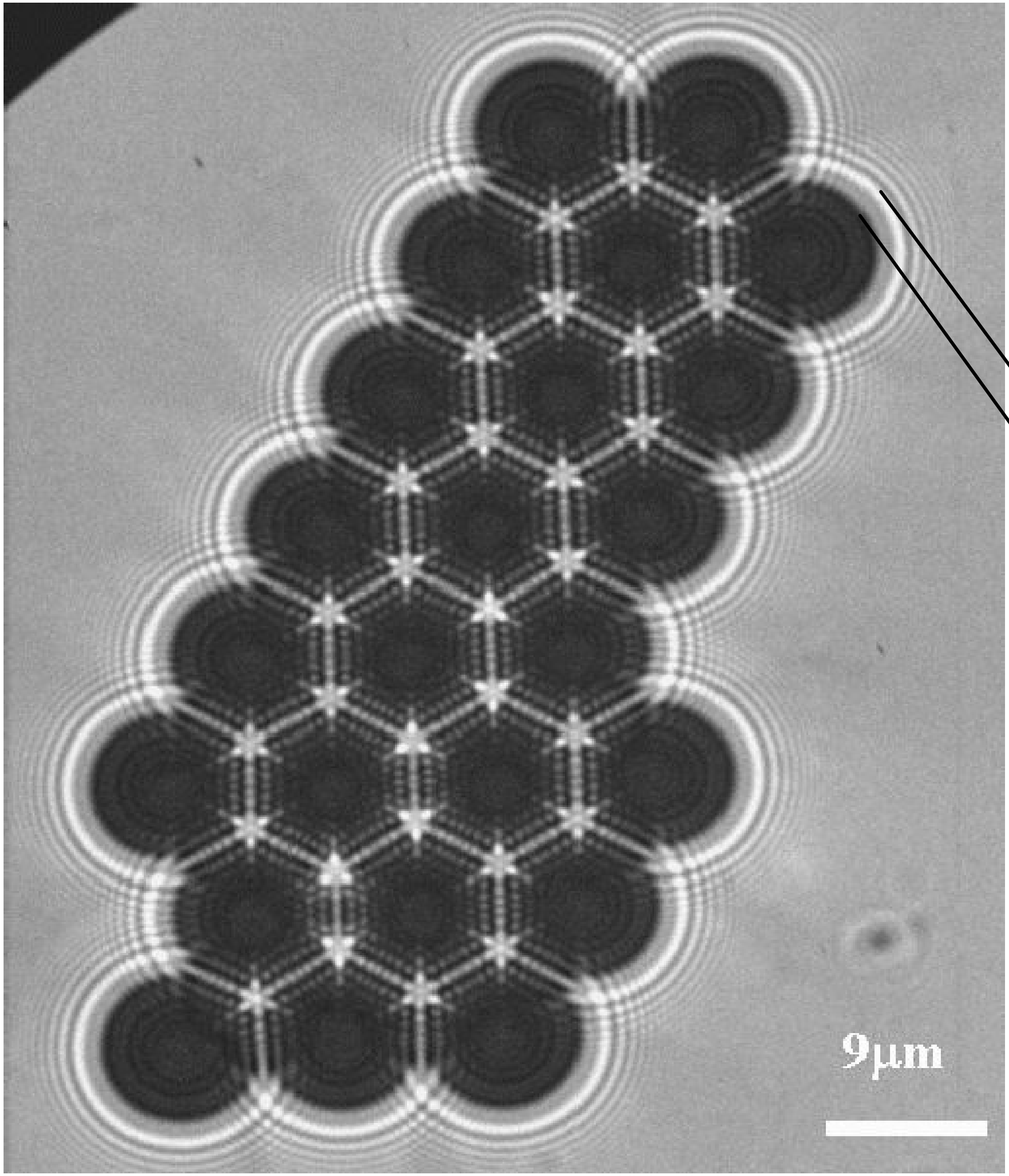
"edge-enhancement"

= Laplace filter



Source: Allman et al. Nature **408** (2000).

In-line holography



Source: Mayo et al. Opt Express **11** (2003).

- Going towards smaller Fresnel numbers

$\sim \sqrt{\lambda z}$

if a is the diameter of the spheres,

Fresnel number $\simeq 10-20$

$\frac{a^2}{\lambda z} = \left(\frac{a}{\sqrt{\lambda z}}\right)^2$

In-line holography

Requirement:
monochromatic & plane incident wave

Measured $I(\vec{r}) = |\psi(\vec{r};z)|^2$

Common model: $\psi(\vec{r};z=0) = A\underbrace{(1+\varepsilon(\vec{r}))}_{\text{transmission function}}$ weak object

$$\psi(\vec{r};z) = A(1+\varepsilon(\vec{r};z))$$

$$I(\vec{r}) = |A|^2\Big(\underbrace{1}_{\text{uniform background}} + \underbrace{\underbrace{\varepsilon(\vec{r};z)}_{\text{propagated image by z}} + \underbrace{\varepsilon^*(\vec{r};z)}_{\text{propagated image by -z}}}_{\text{twin image problem}} + \underbrace{O(\varepsilon^2)}_{\text{negligible}}\Big)$$

The phase problem

The problem: we can measure only the squared amplitude of a wave (E.M. / matter)

measurement $I = |\Psi|^2$

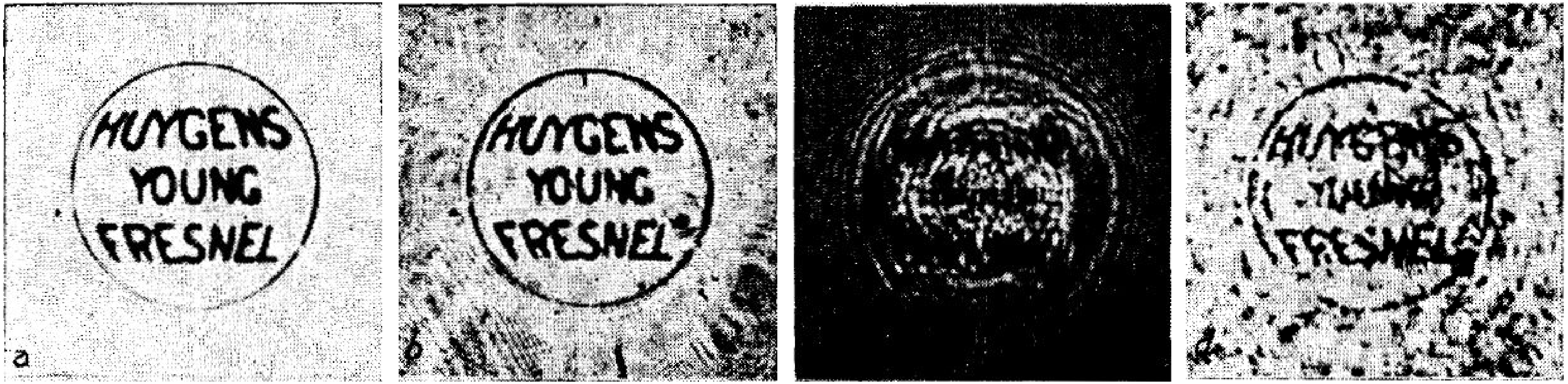
Sometimes:
- phase is the quantity of interest

e.g.

$$\exp(ik\underbrace{(n-1)}_{}\overset{\text{thickness}}{t})$$

- phase is an auxiliary quantity required to extract the sought information (e.g. through propagation)

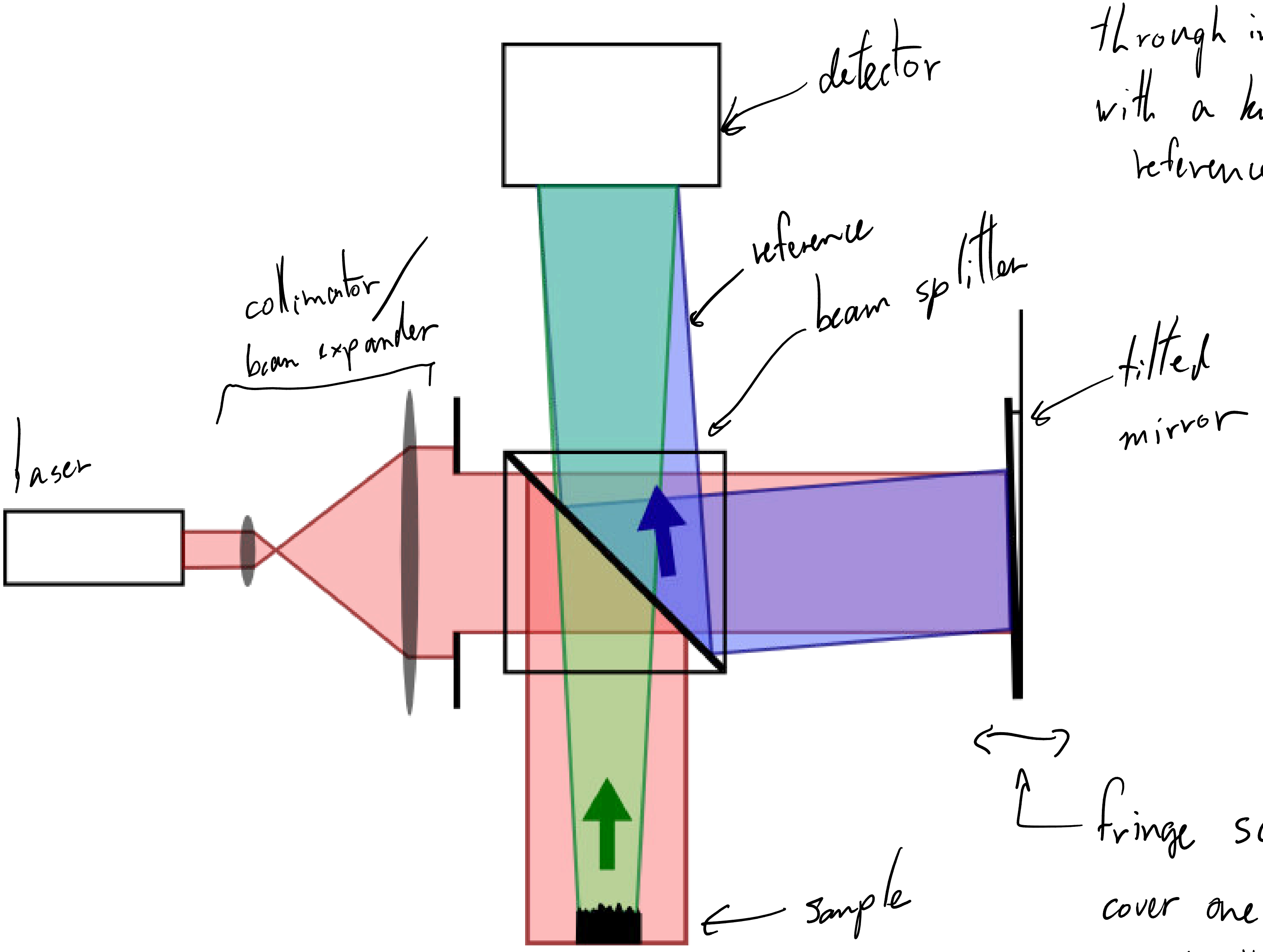
In-line holography



D. Gabor, Nature **161**, 777-778 (1948).

Fringe interferometry

phase measurement through interference with a known reference.

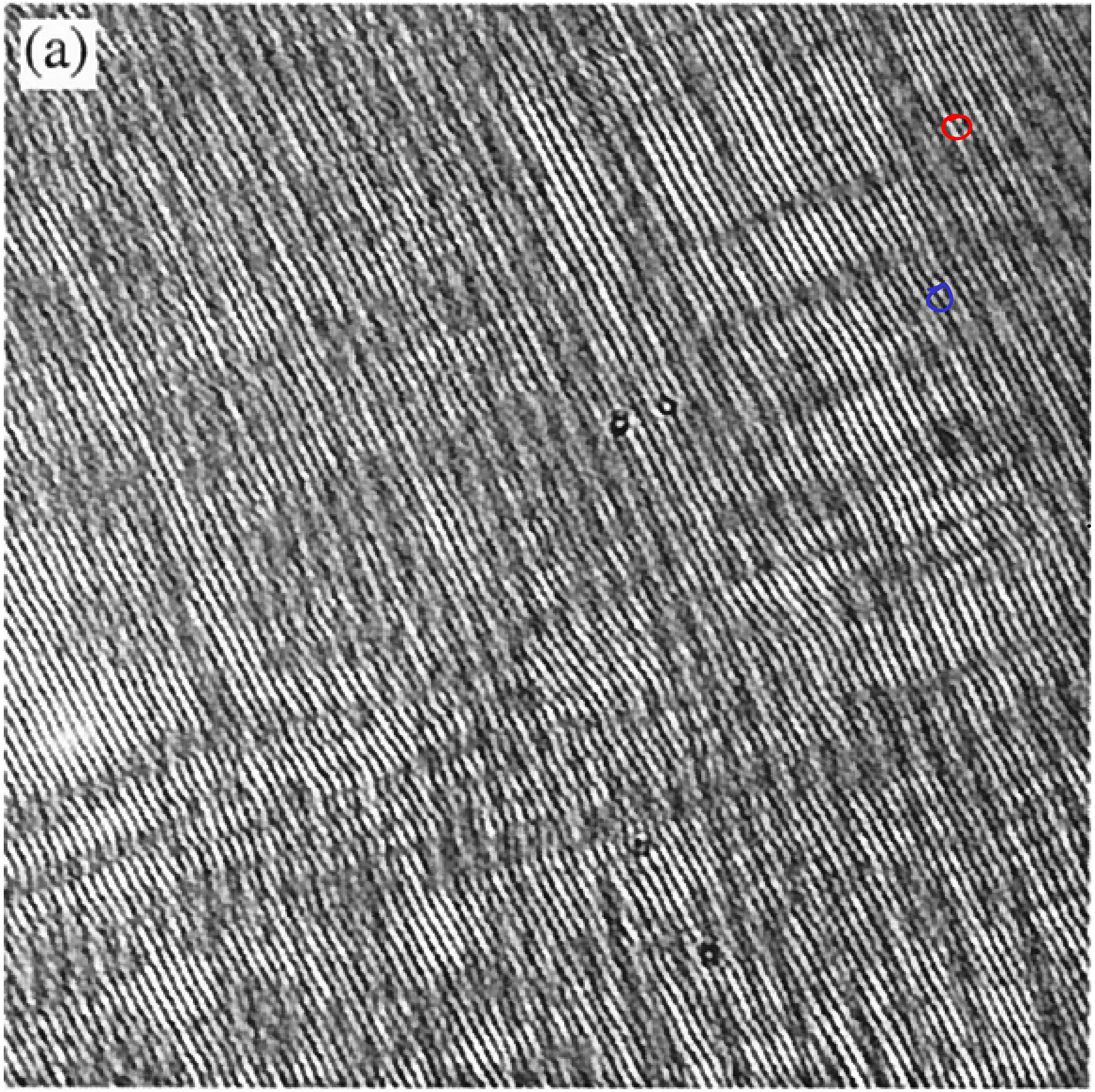


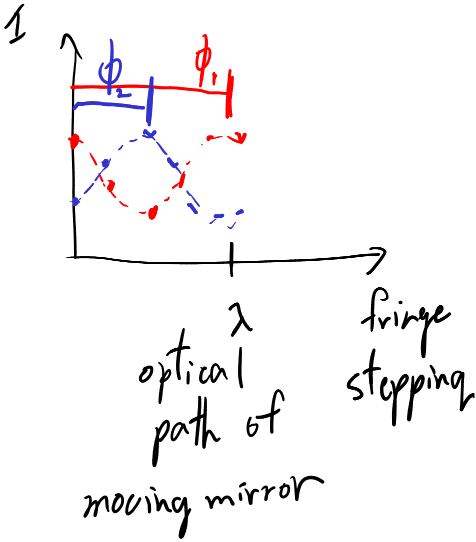
fringe scanning: cover one full wavelength in path difference

Twyman-Green interferometer

Fringe interferometry

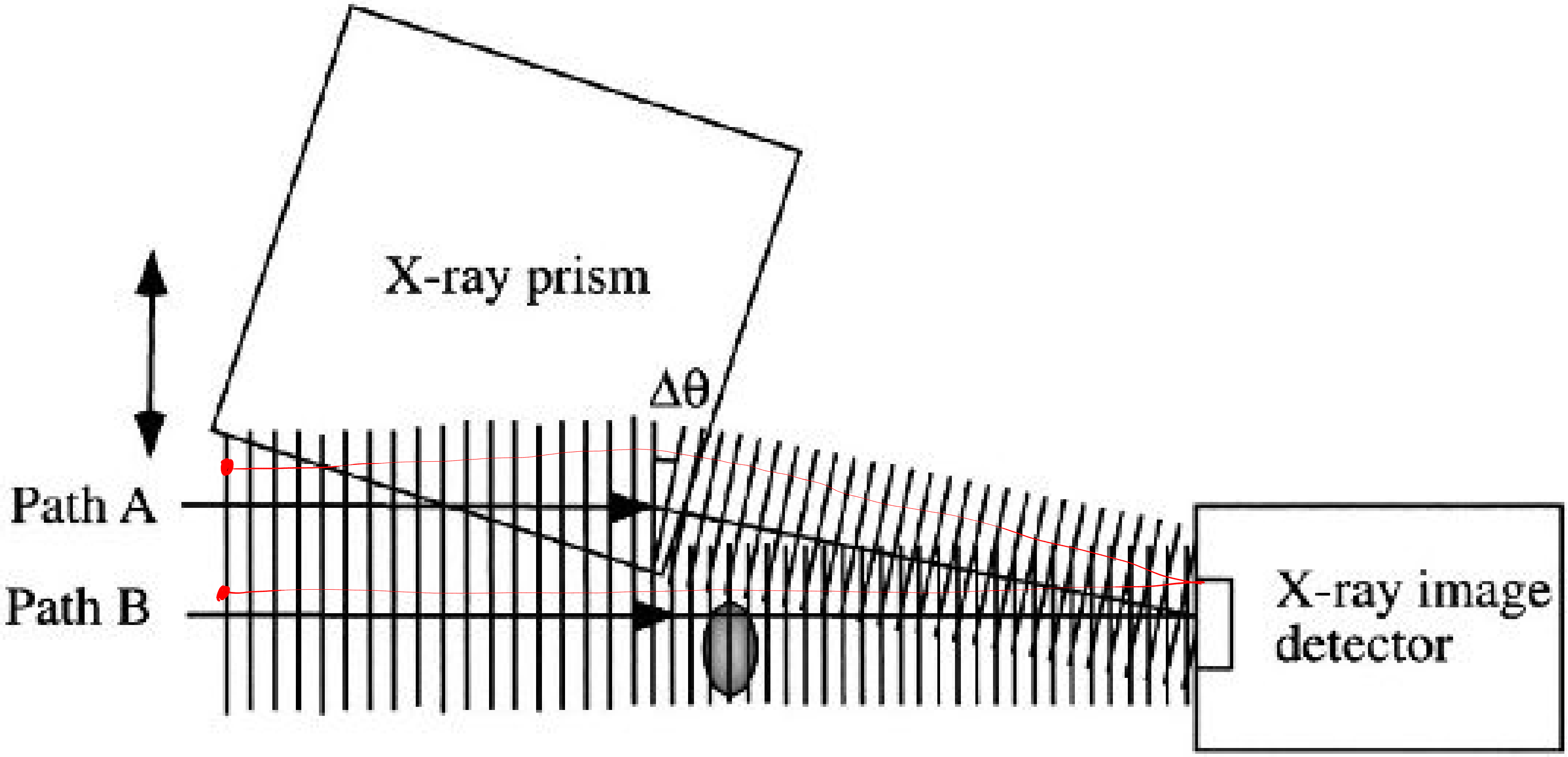
$\cos\left(\frac{2\pi x}{\lambda} + \phi\right)$ ↑





Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

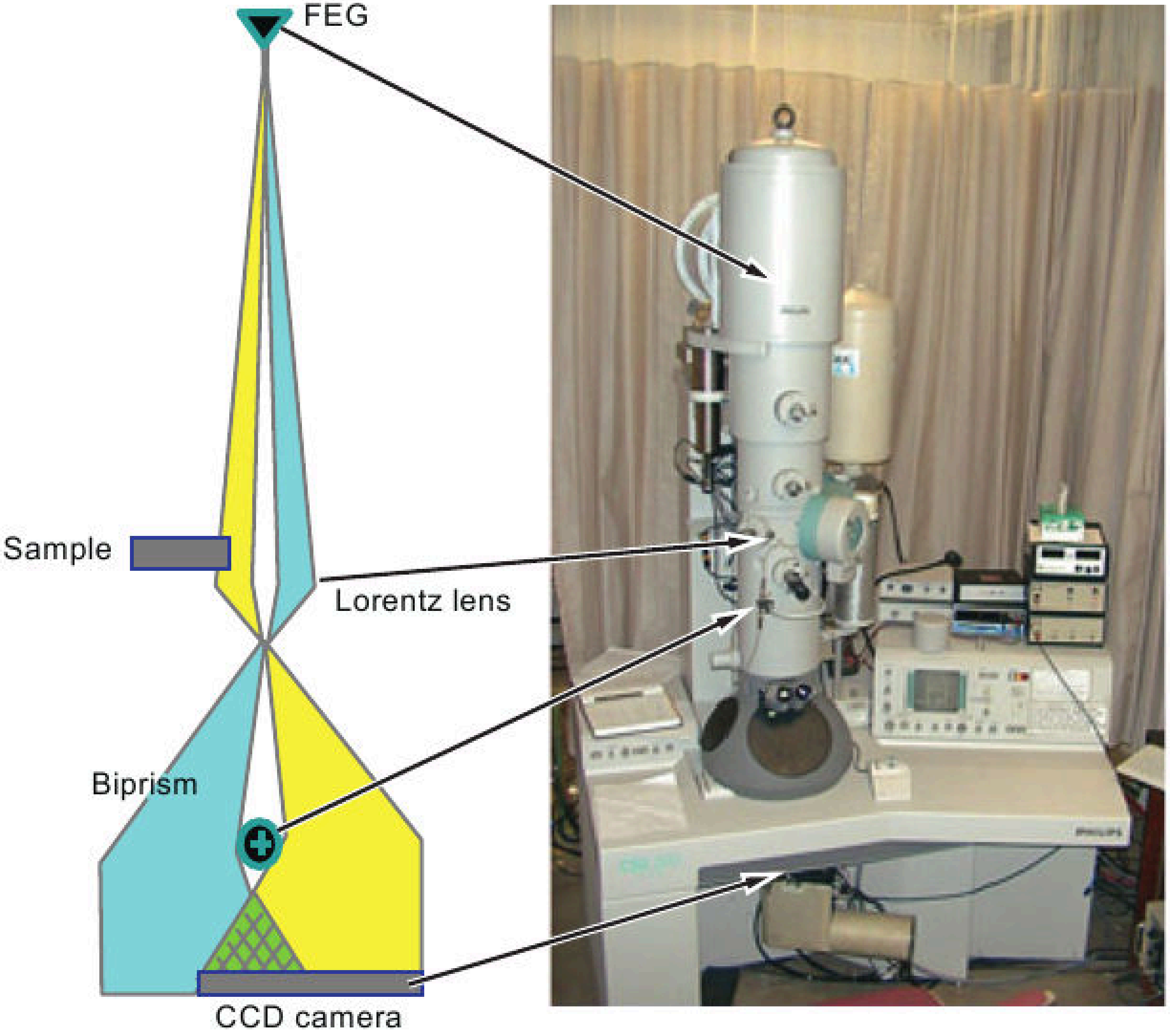
Off-axis X-ray holography



Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

Off-axis electron holography

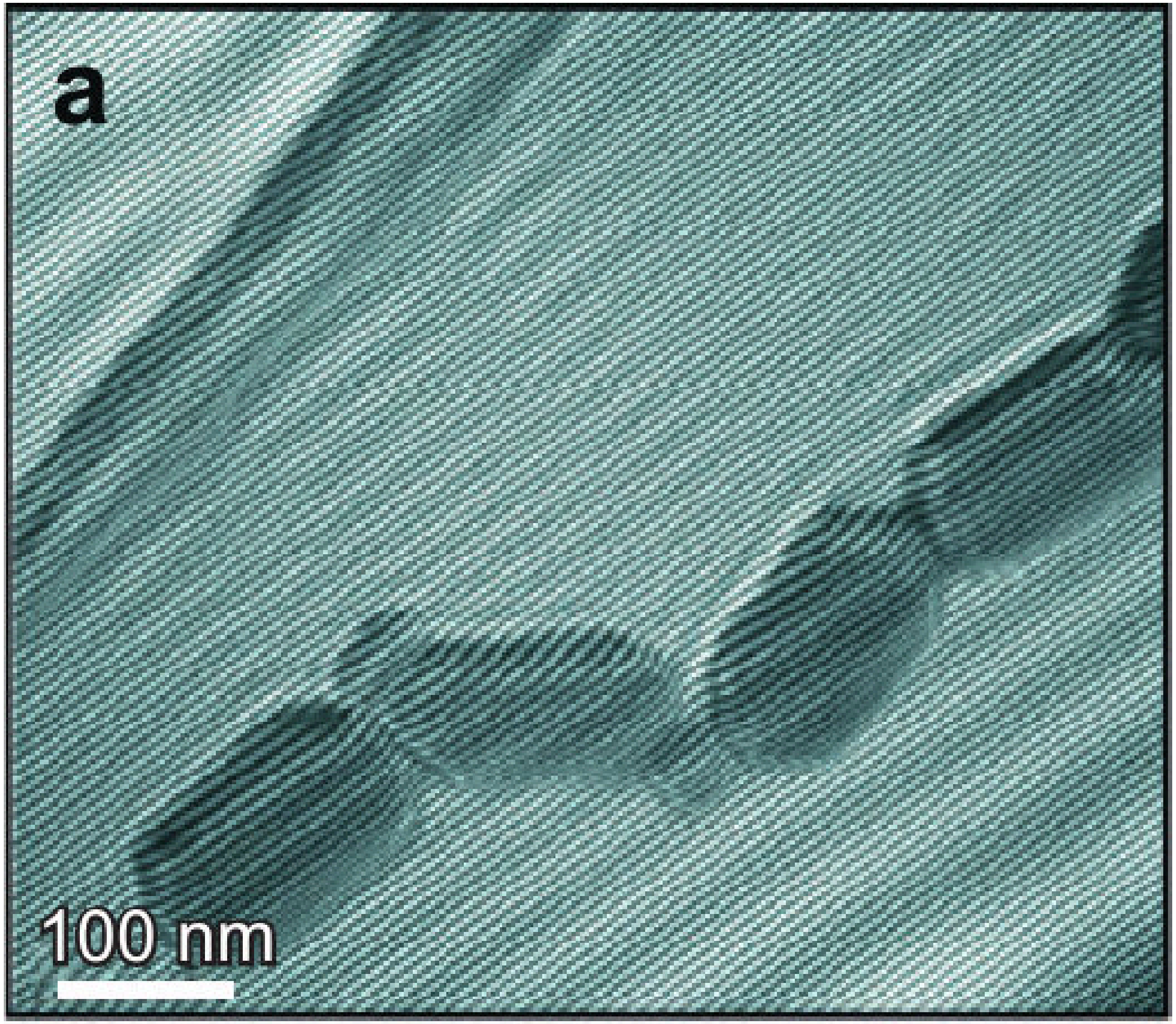
Electron microscopy

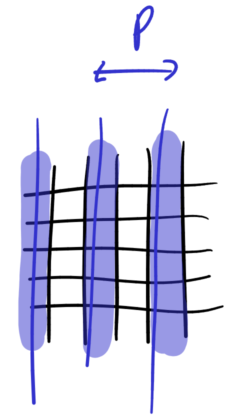


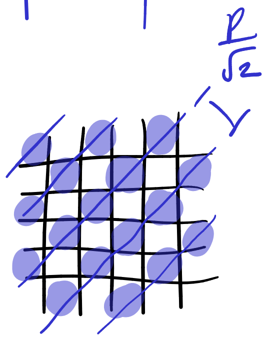
Hologram **Philips CM200 FEG**

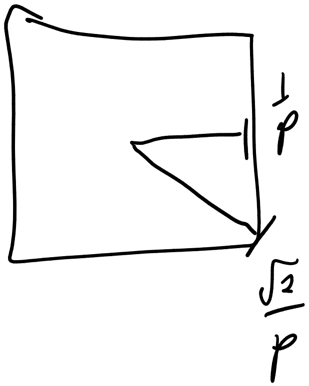
Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis electron holography



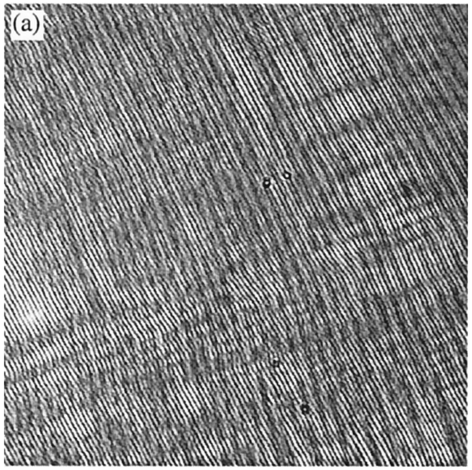


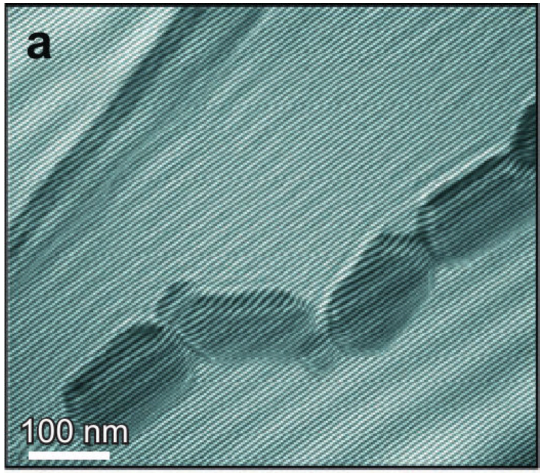




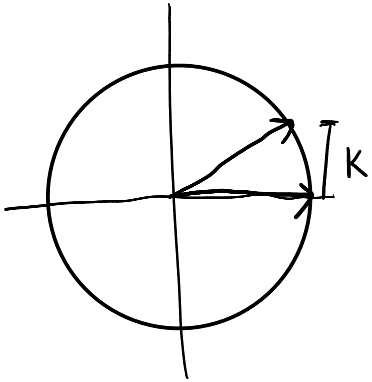
Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Fringe interferometry





Ewald sphere



$$\psi = \underset{\text{object}}{\psi_o} + \underset{\text{reference}}{\psi_r}$$

$$\psi_r(\vec{r}) = A e^{i\vec{K}\cdot\vec{r}}$$

$$\psi_o(\vec{r}) = A \underset{\text{absorption}}{a(\vec{r})} e^{i\overset{\text{phase shift (refraction)}}{\varphi(\vec{r})}}$$

complex-valued transmission function

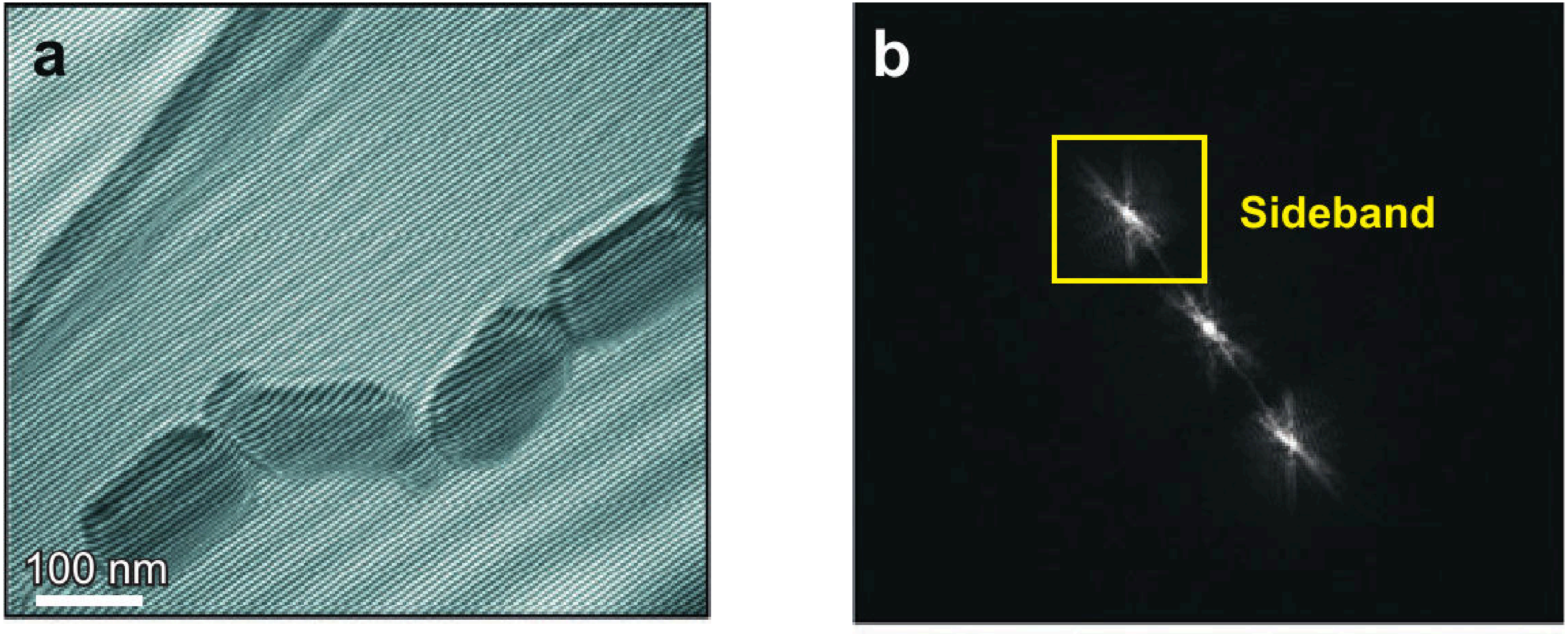
Measured:

$$|\psi(\vec{r})|^2 = |\psi_o(\vec{r}) + \psi_r(\vec{r})|^2 = \left(A e^{i\vec{K}\cdot\vec{r}} + A a(\vec{r}) e^{i\varphi(\vec{r})}\right)\left(A^* e^{-i\vec{K}\cdot\vec{r}} + A^* a(\vec{r}) e^{-i\varphi(\vec{r})}\right)$$

$$= |A|^2 \left(1 + a^2(\vec{r}) + a(\vec{r}) e^{-i(\vec{K}\cdot\vec{r} - \varphi)} + \vec{a}(\vec{r}) e^{i(\vec{K}\cdot\vec{r} - \varphi)}\right)$$

$$= |A|^2 \left(\underbrace{1 + a^2(\vec{r})}_{\text{smooth}} + 2a(\vec{r}) \underbrace{\cos\left(\vec{K}\cdot\vec{r} - \varphi(\vec{r})\right)}_{\text{fringes}}\right)$$

Off-axis holography

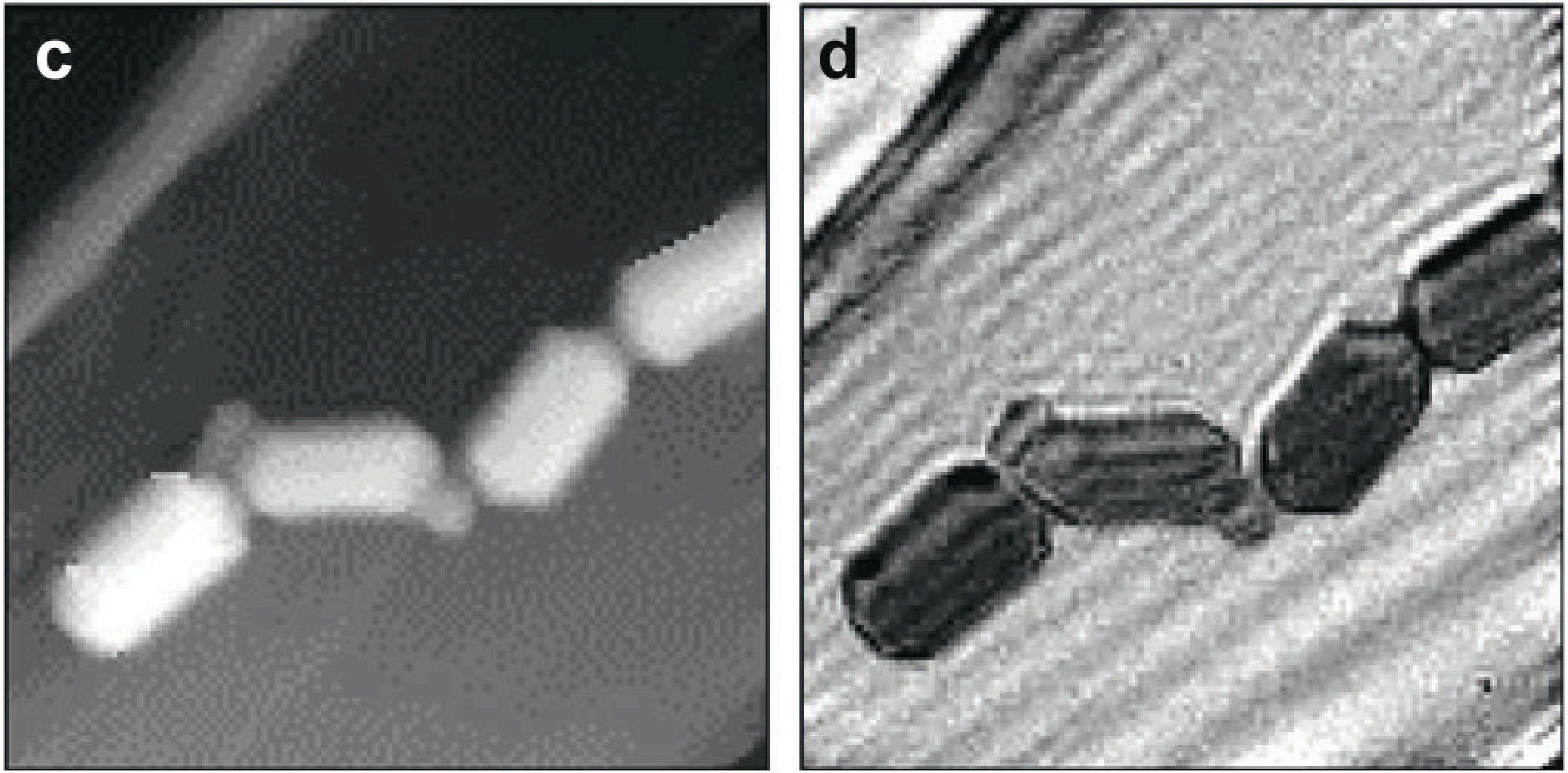


Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis holography

$\phi(\vec{r})$

$a(\vec{r})$ ↴



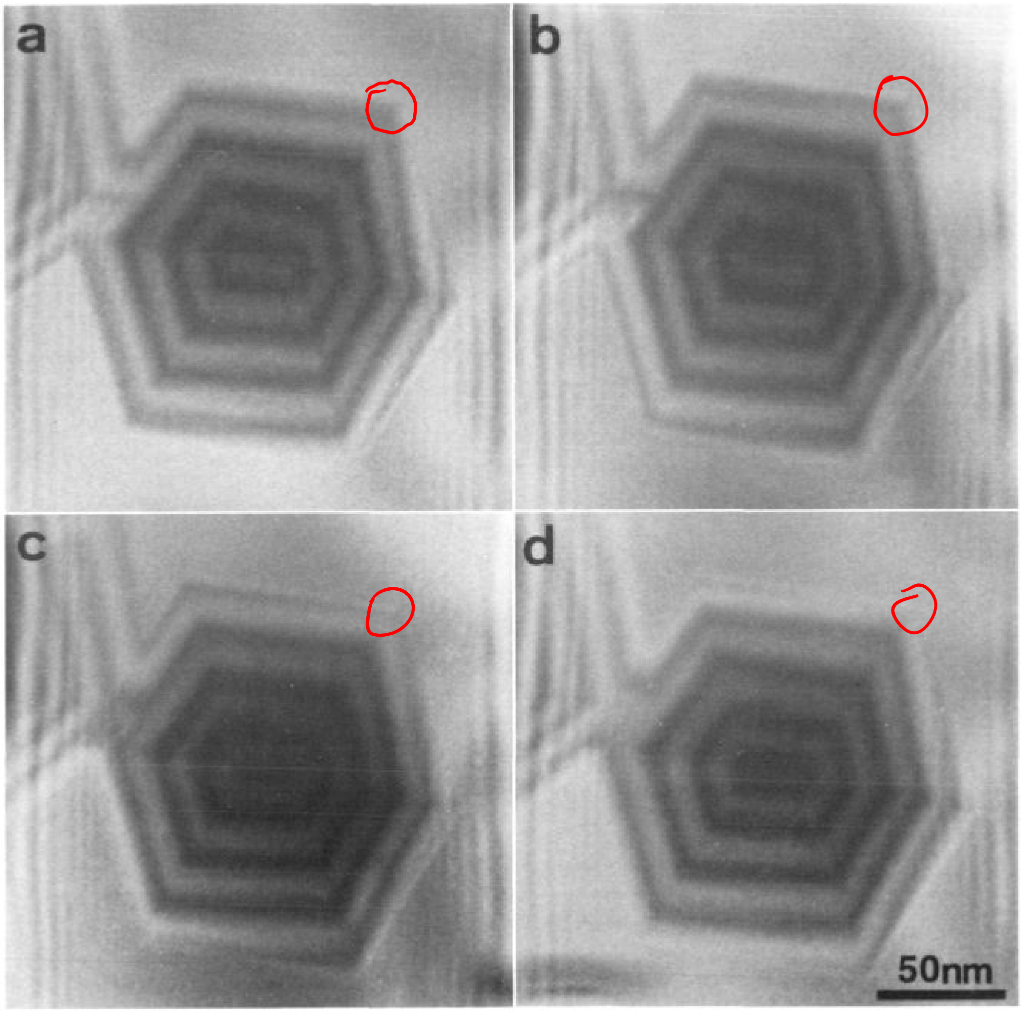
Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

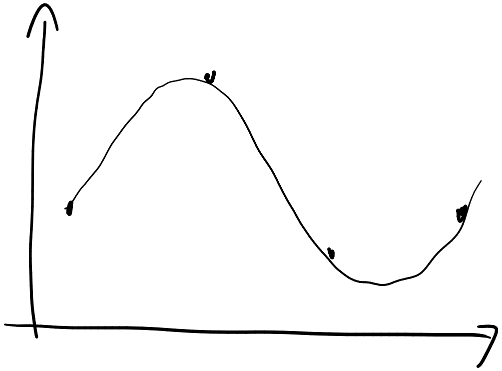
Phase stepping

- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.

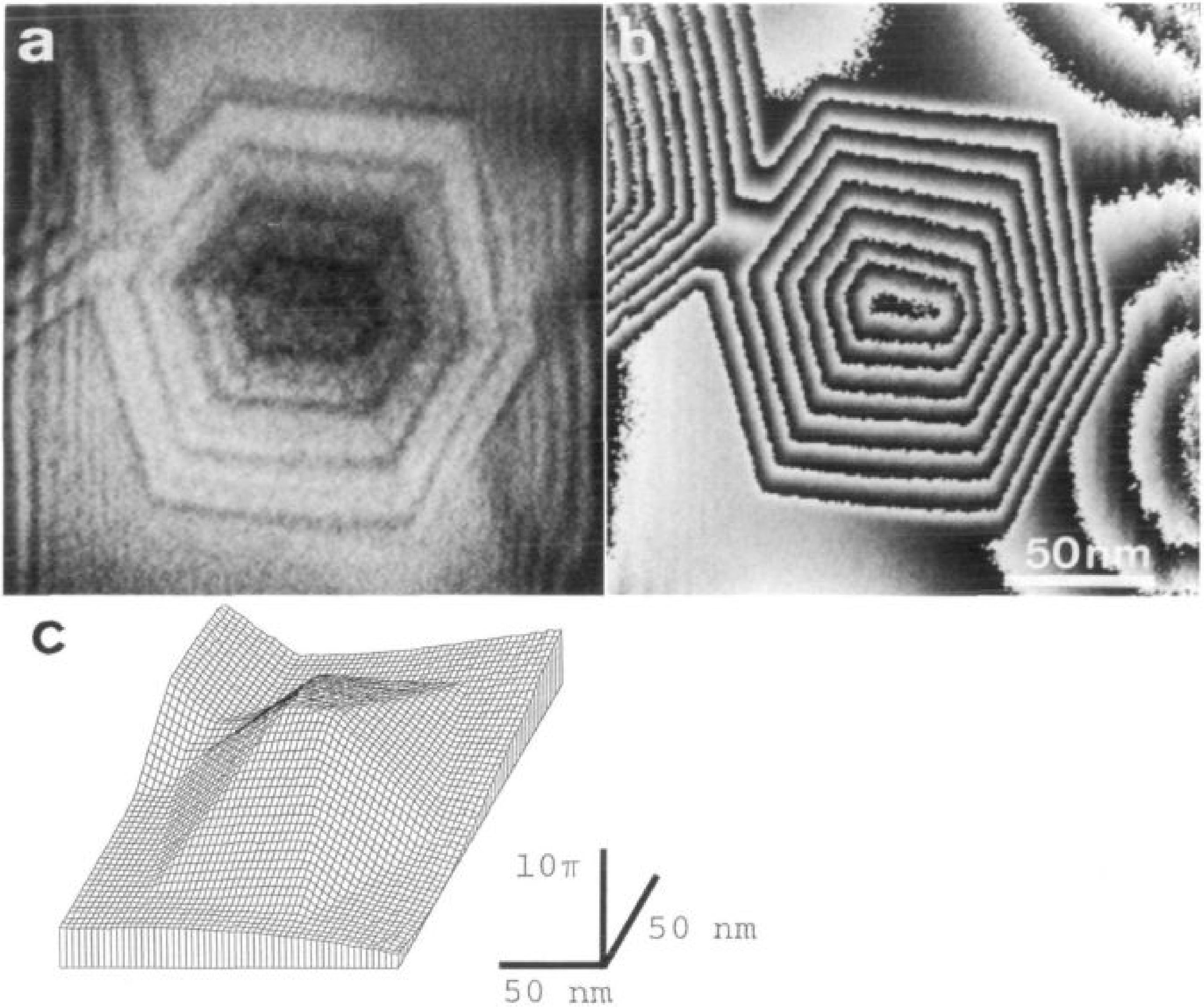
Fringe scanning





Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

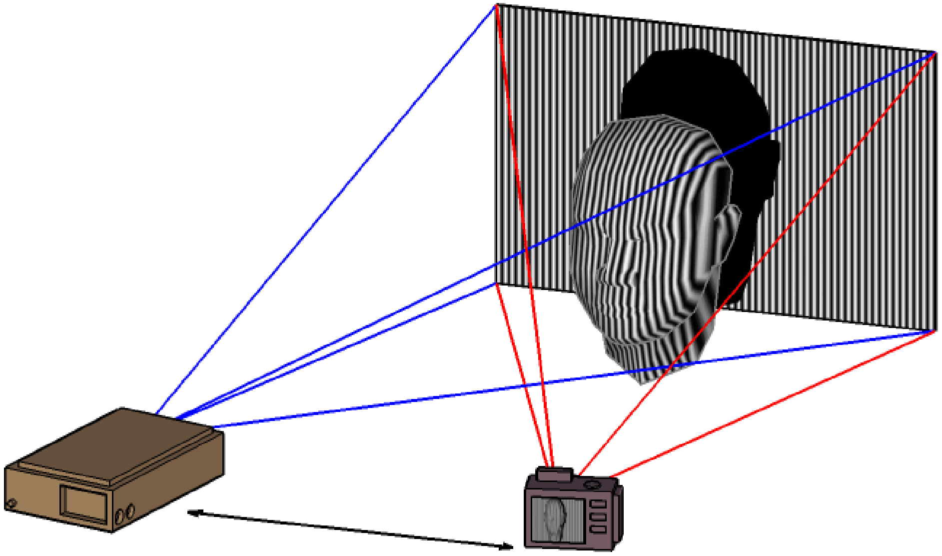
Fringe scanning

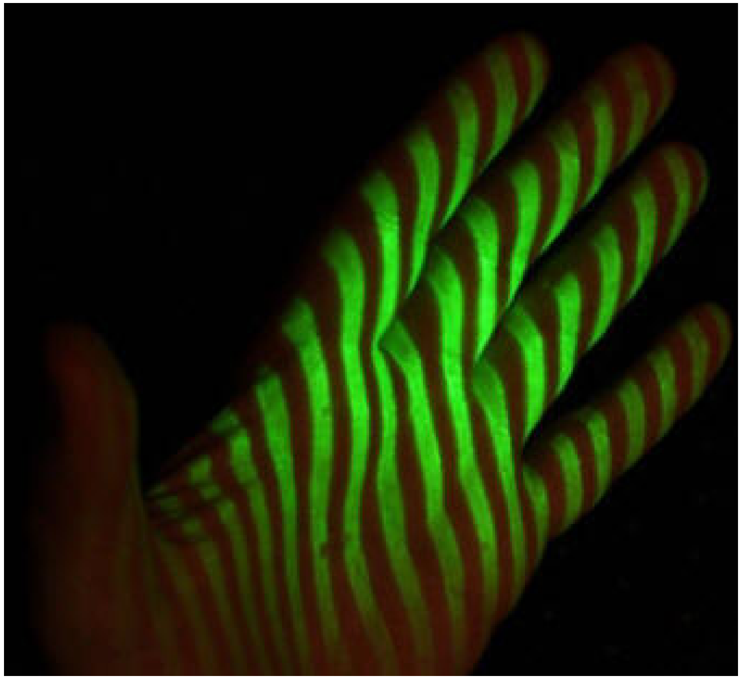


Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





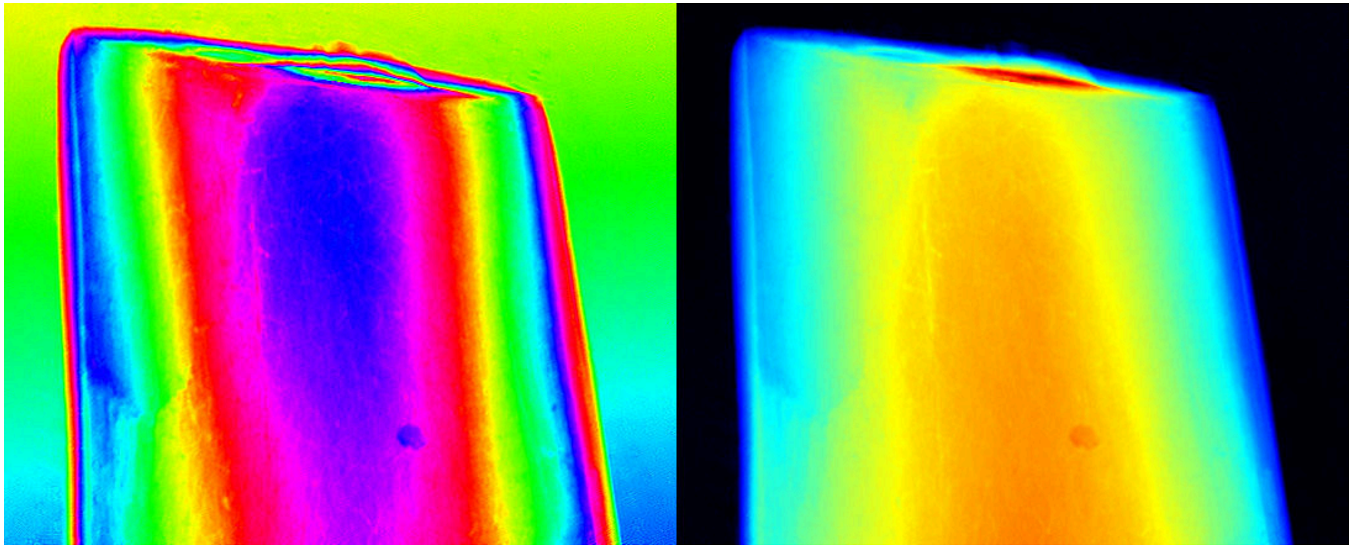
Phase unwrapping

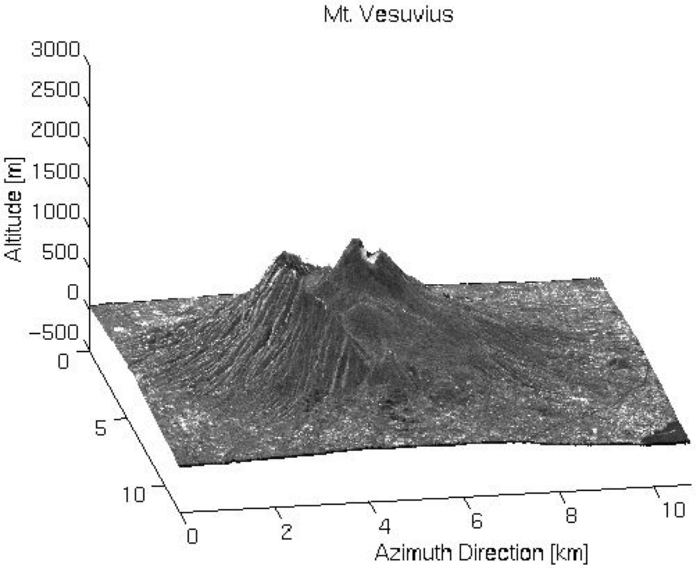
- Phase is measured only in the interval [0, 2π)
- Physical phase shifts (which can be larger) are wrapped on this interval

  → Any multiple of 2π is possible
- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

Complex-valued images

Phase unwrapping

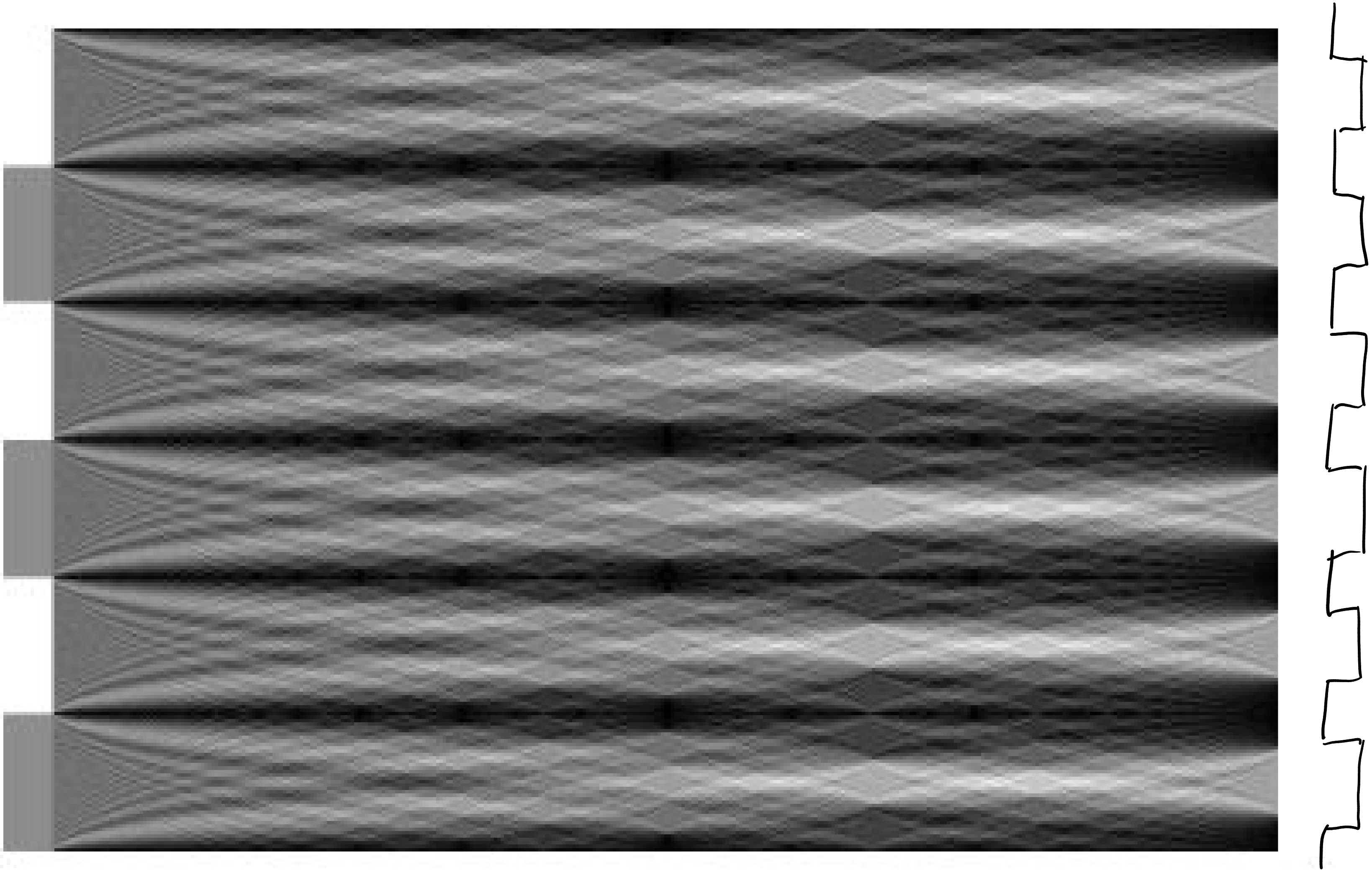




Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

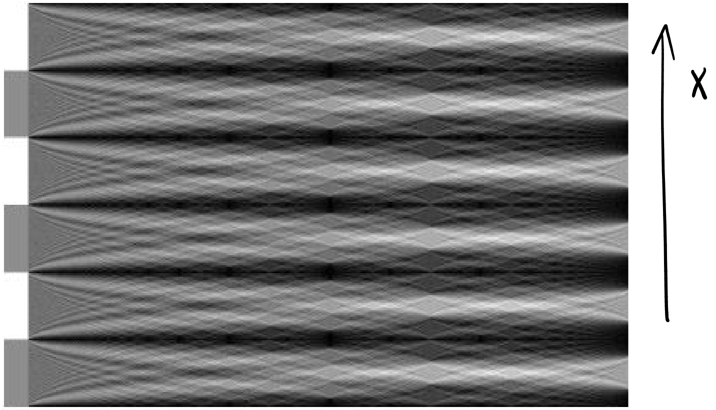
Grating interferometry

Diffraction from a grating

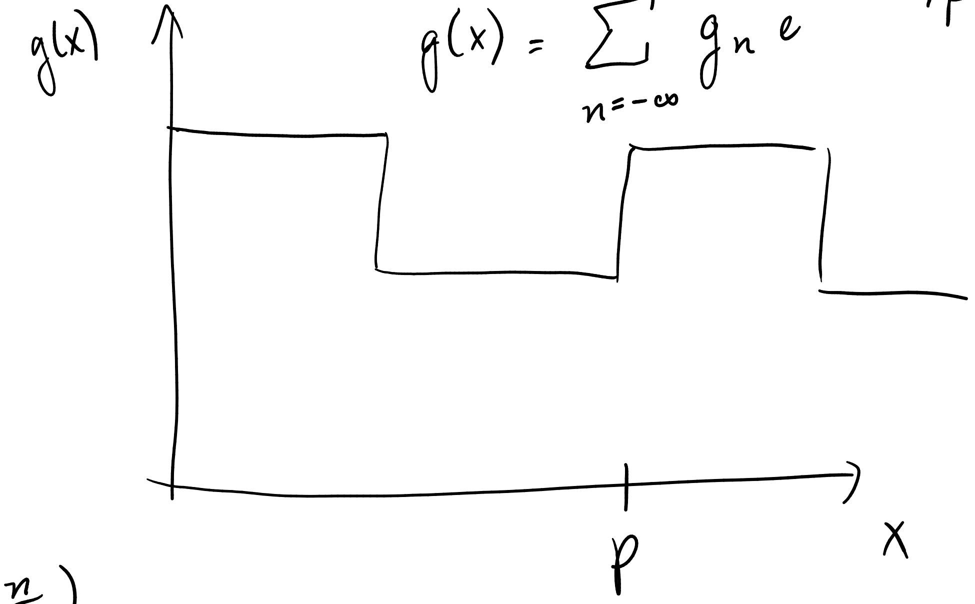


Grating interferometry

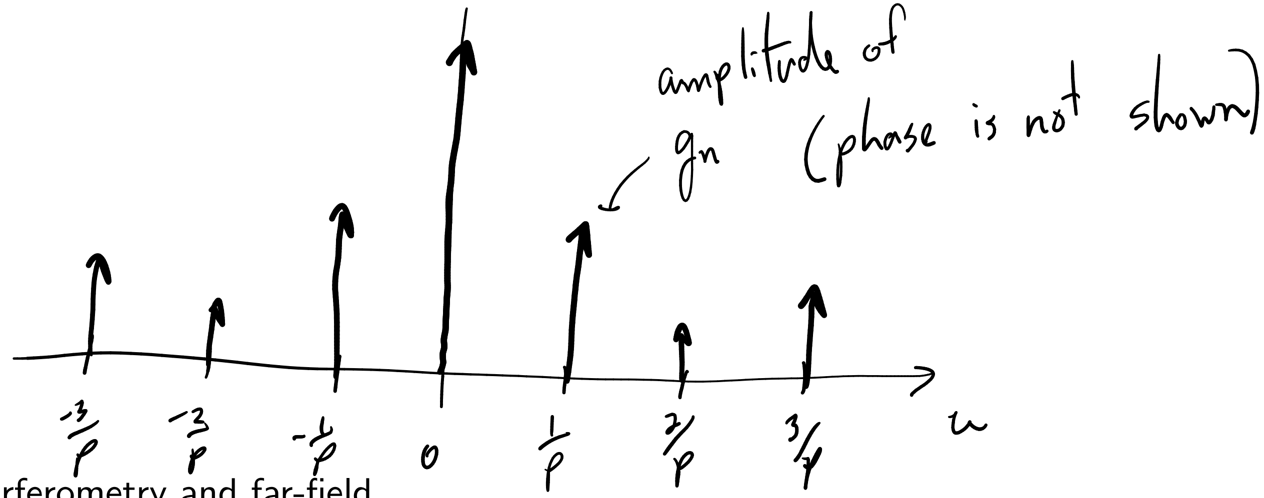
Diffraction from a grating

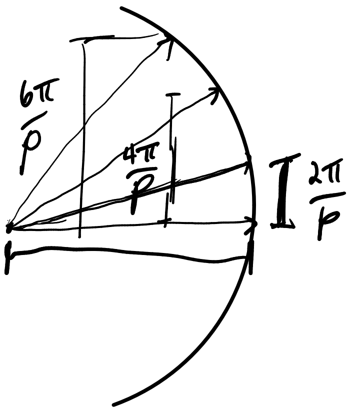


$$g(x) = \sum_{n=-\infty}^{\infty} g_n e^{2\pi i x n/p}$$



$$G(u) = \sum_{n=-\infty}^{\infty} g_n \, \delta(u - \frac{n}{p})$$

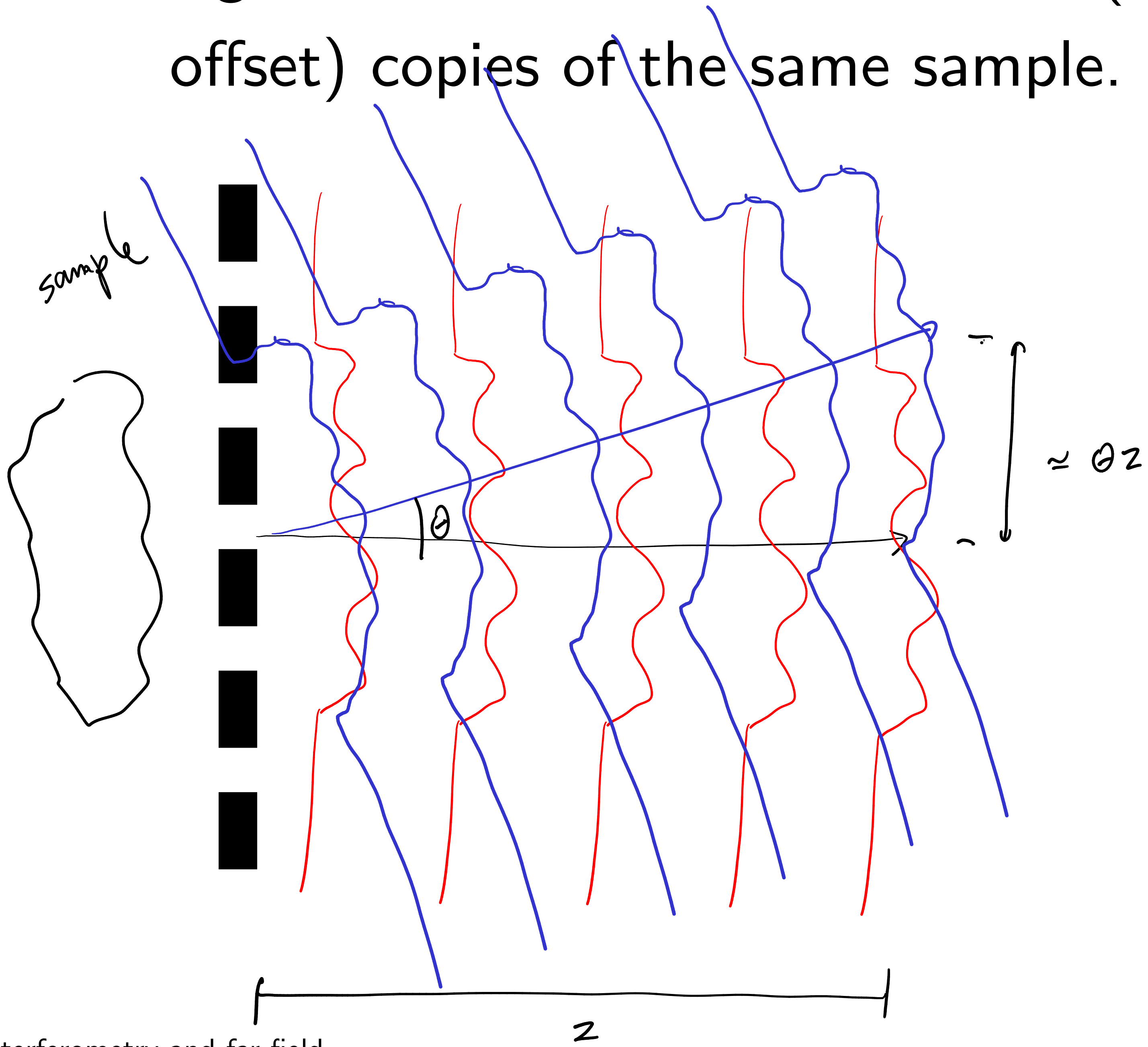




Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

$$\theta = \frac{2\pi/p}{2\pi/\lambda} = \frac{\lambda}{p}$$



Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

e.g. if only orders ± 1 are relevant.

$$\psi(\vec{r};z) = \psi_0\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right)e^{2\pi i x/p} + \psi_0\left(\vec{r} - \frac{z\lambda}{p}\hat{x}\right)e^{-2\pi i x/p}$$

$$\vdots$$

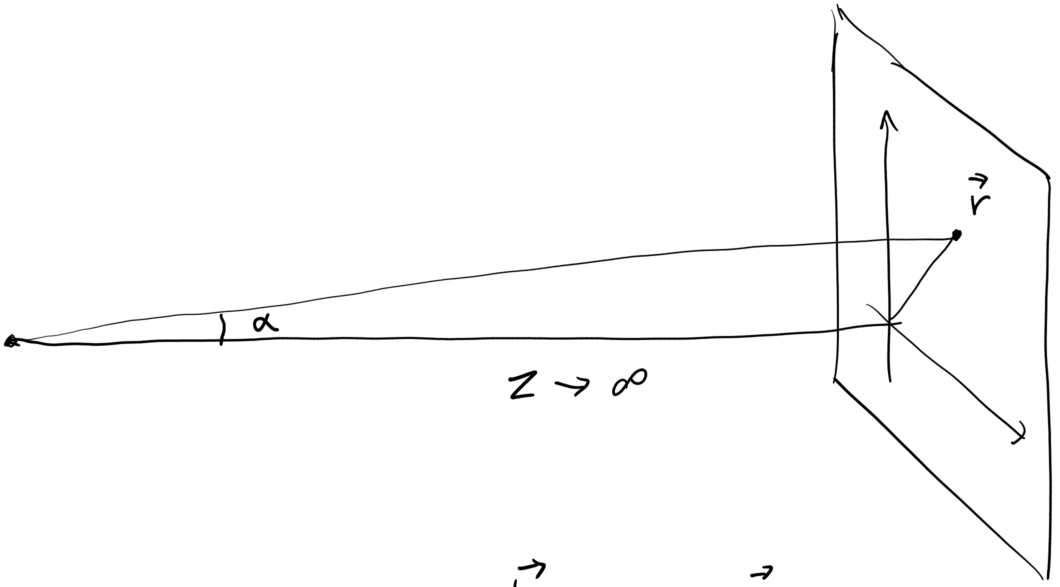
$\psi_0 \sim a e^{i\varphi}$

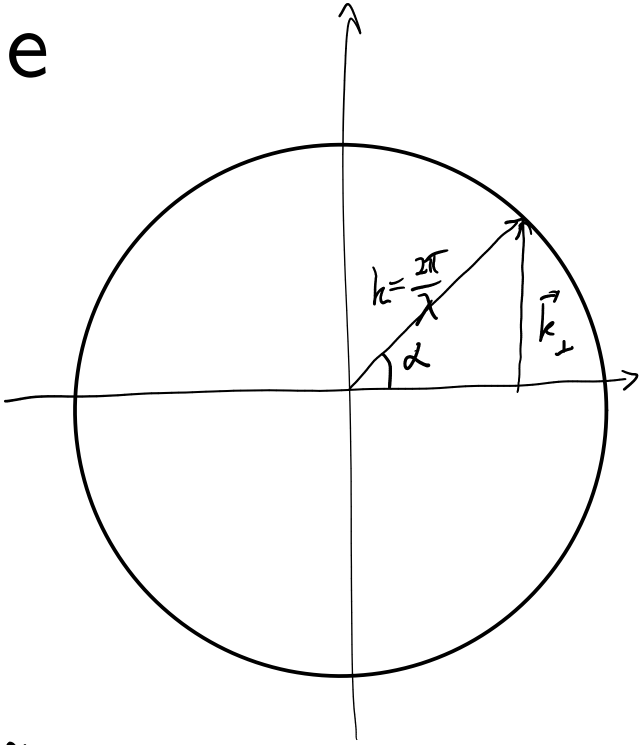
"differential phase contrast"

$$I(\vec{r};z) = 2a^2(\vec{r}) + 2\underbrace{a\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right)a\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}_{\sim a^2(\vec{r})}\cos\left[\overbrace{\varphi\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right) - \varphi\left(\vec{r} - \frac{z\lambda}{p}\hat{x}\right)}^{\simeq 2\frac{z\lambda}{p}\boxed{\frac{\partial}{\partial x}\varphi}} + 4\pi x/p\right]$$

Far-field diffraction

The Fraunhofer regime



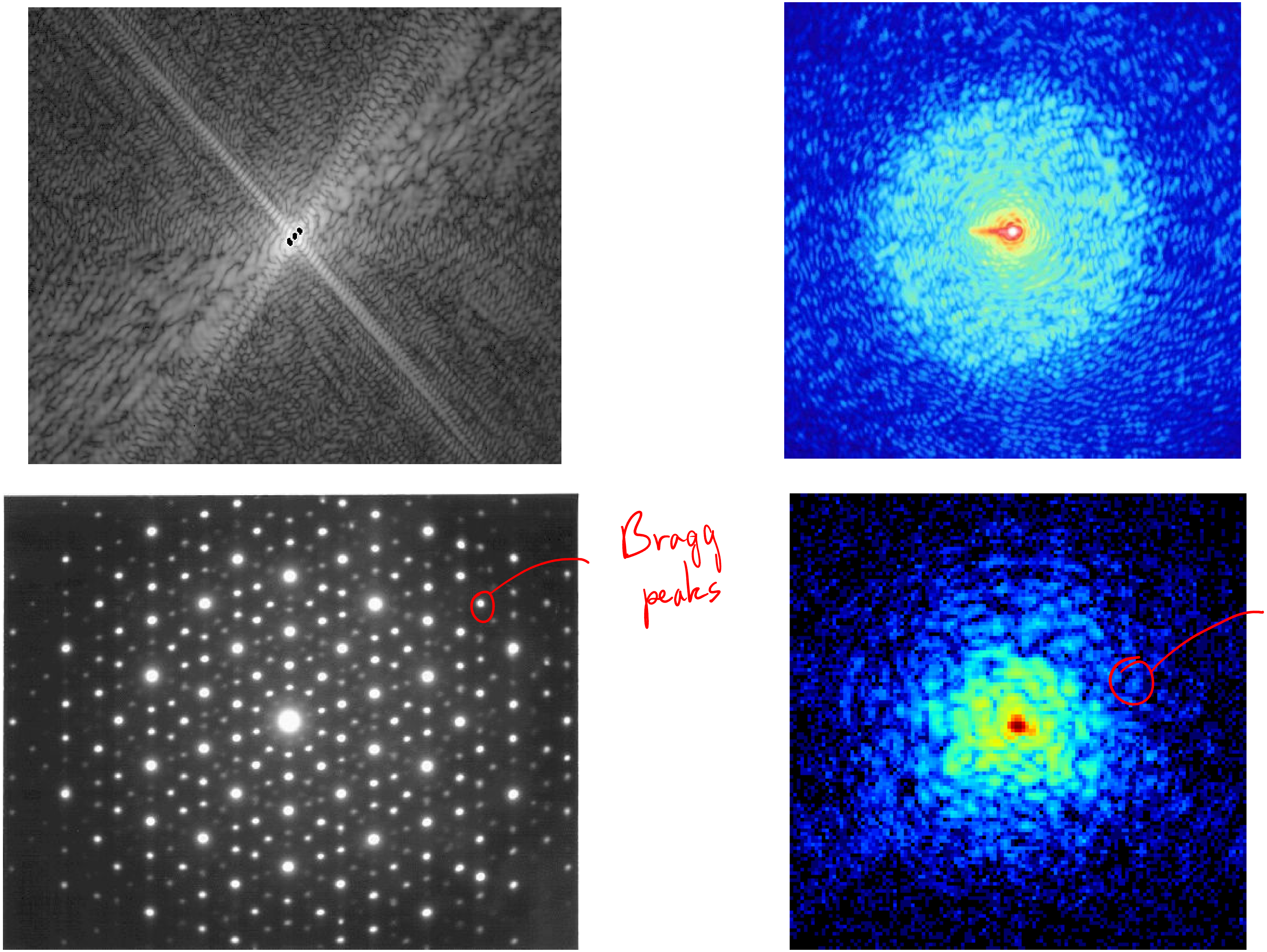


$$\frac{\vec r}{z} = \frac{\vec k_\perp}{k} = \frac{2\pi \vec u}{2\pi/\lambda} = \lambda \vec u = \sin\alpha$$

$$\psi(\vec r) = \frac{-2\pi i}{\lambda z} \exp\left(\frac{i\pi r^2}{\lambda z}\right) \int d^2 r' \, \psi(\vec r') \exp\left(-2\pi i \, \vec r' \cdot \underbrace{\frac{\vec r}{\lambda z}}_{\vec u}\right)$$

$$|\psi(r)|^2 \propto |\mathcal{F}\psi|^2 = I(\vec u)$$

Diffraction patterns



Bragg peaks

speckles

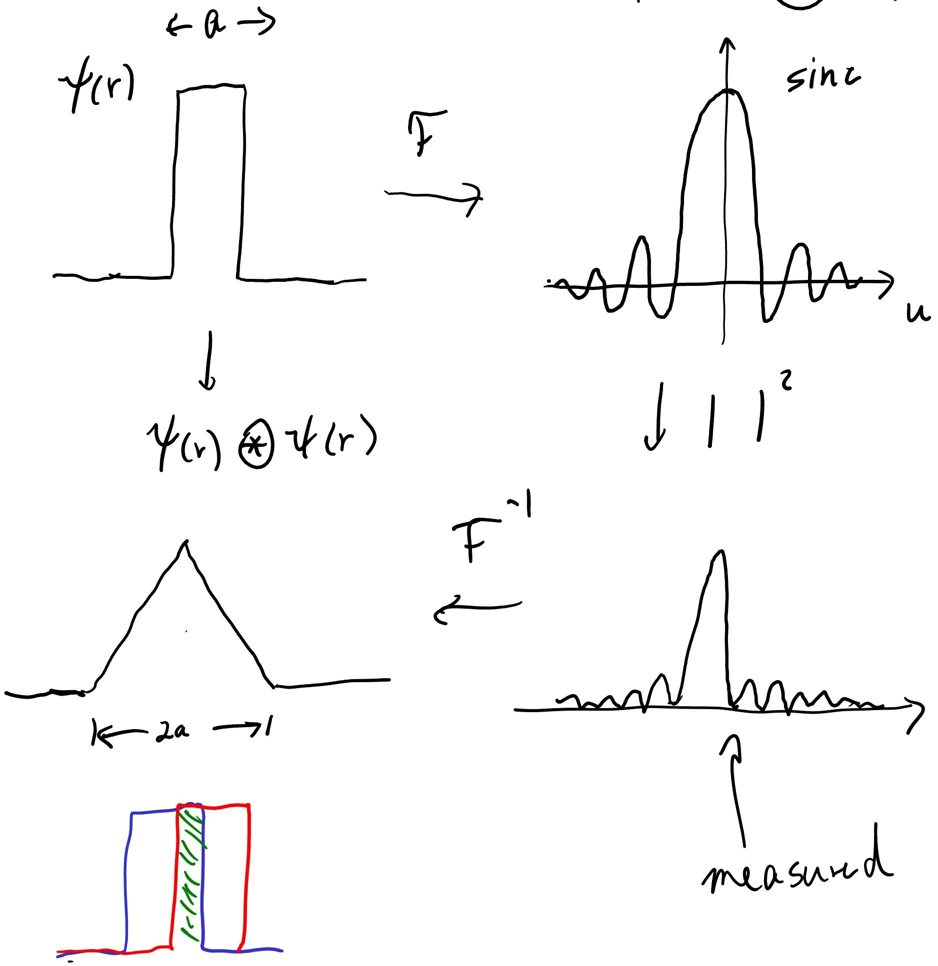
Diffraction and autocorrelation

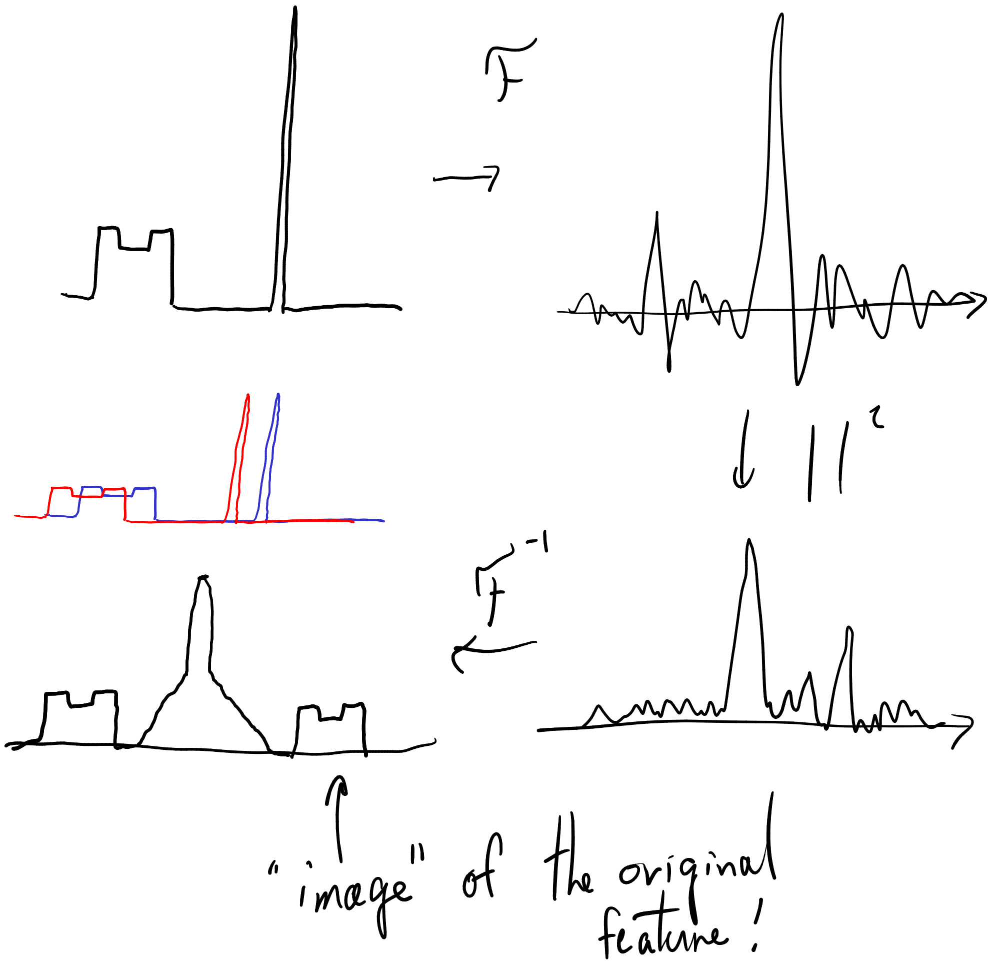
$\mathcal{F}^{-1}\{I(\vec{u})\} \longleftarrow$ not $\psi(\vec{r})$!

$= \mathcal{F}^{-1}\{\psi(\vec{u}) \cdot \psi^*(\vec{u})\}$

$\mathcal{F}^{-1}\{F \cdot G^*\}$ = correlation

$= \psi(\vec{r}) \circledast \psi(\vec{r})$ autocorrelation





Fourier transform holography

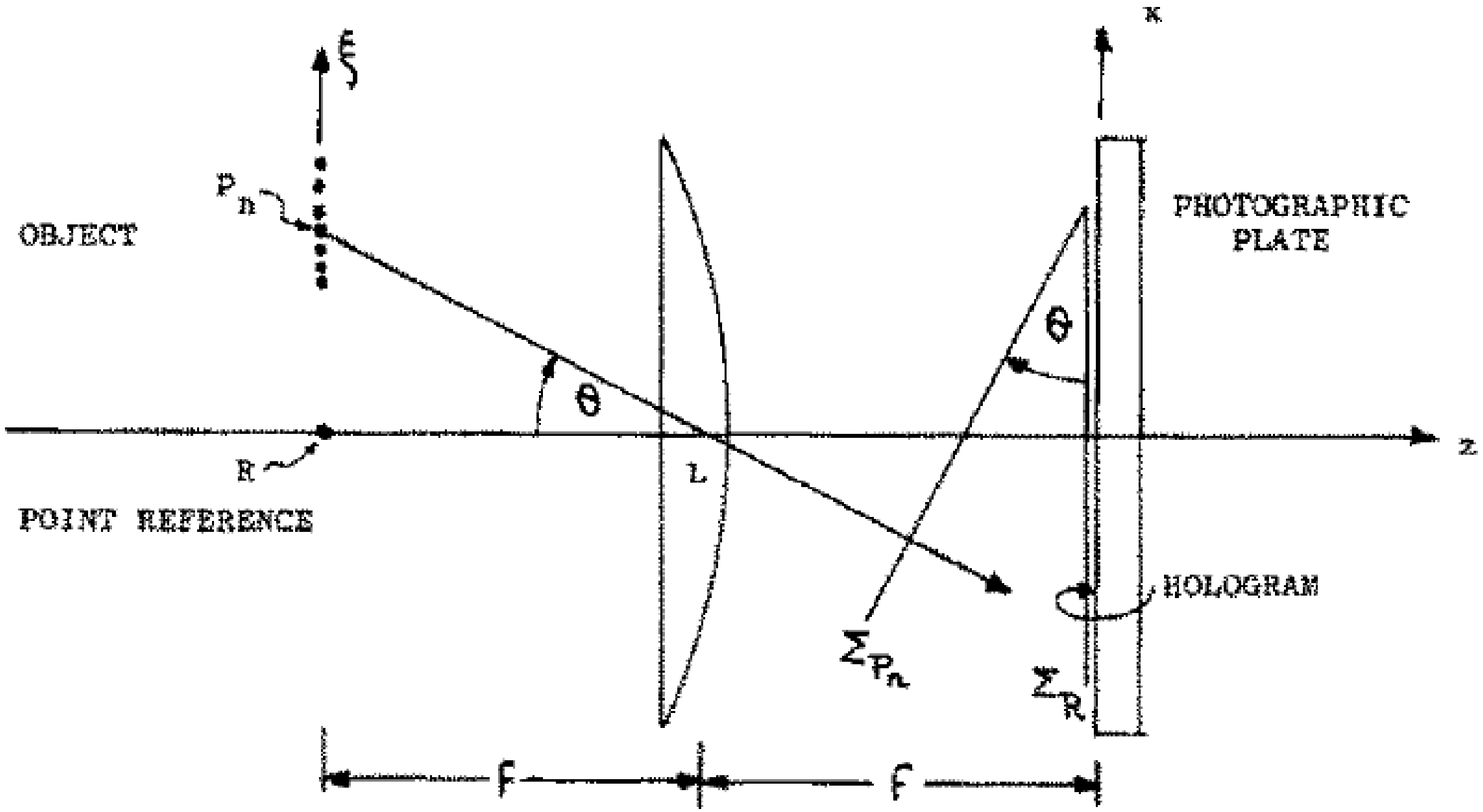
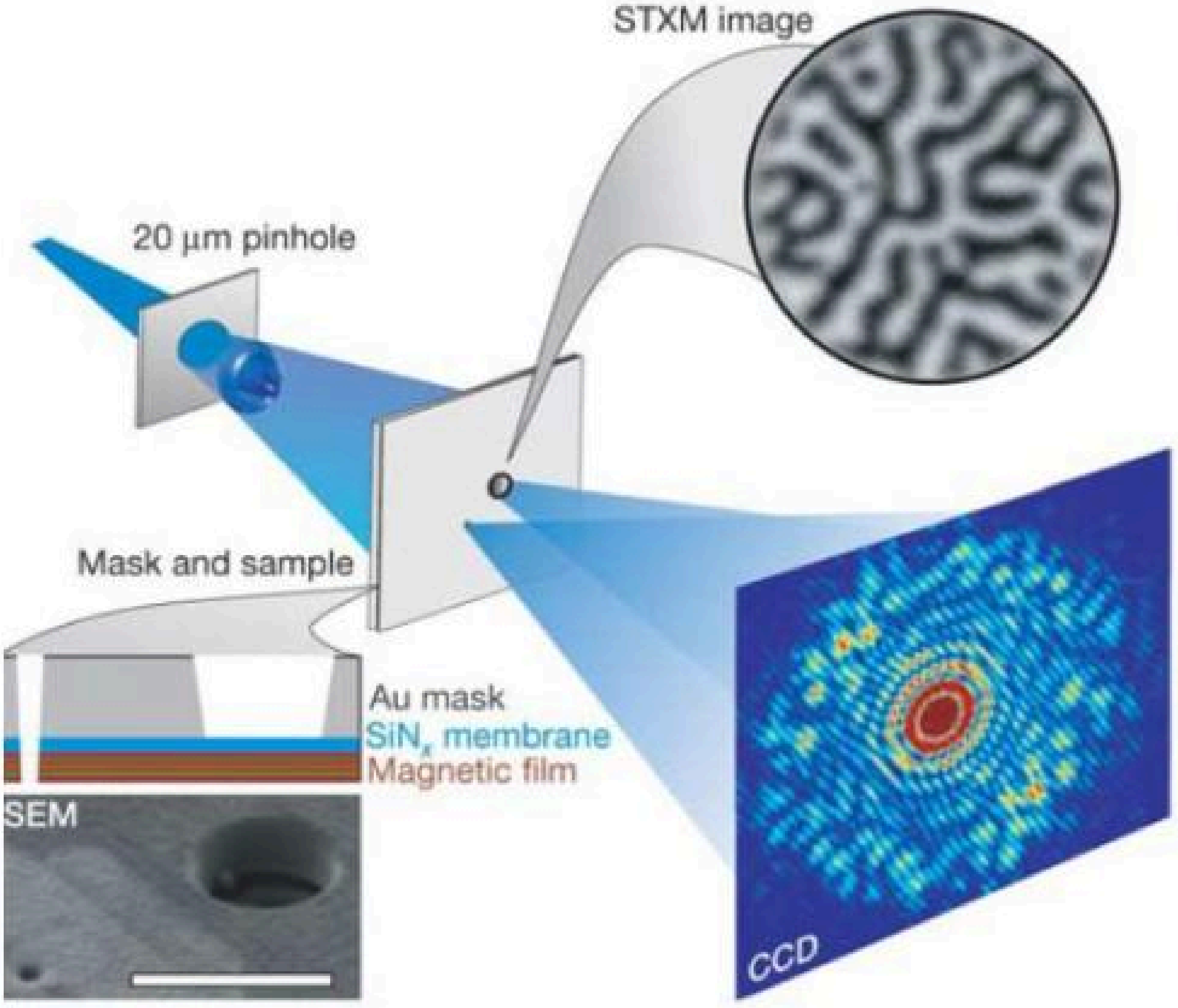
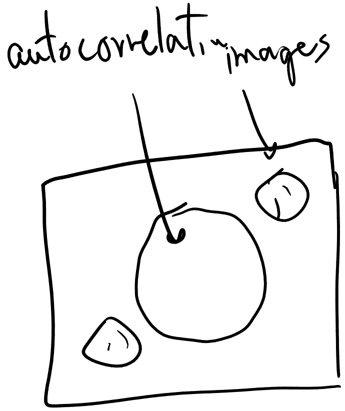


Fig. 1. Recording of a Fourier-transform hologram with a lens *L*. Σ_R = reference wavefront.

Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

Fourier transform holography





Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

Fourier transform holography

$$\psi(\vec{r}) = \psi_R(\vec{r}) + \psi_o(\vec{r})$$

$$\tilde{\psi}(\vec{u}) = \tilde{\psi}_R(\vec{u}) + \tilde{\psi}_o(\vec{u}) \leftarrow \text{Fourier transform}$$

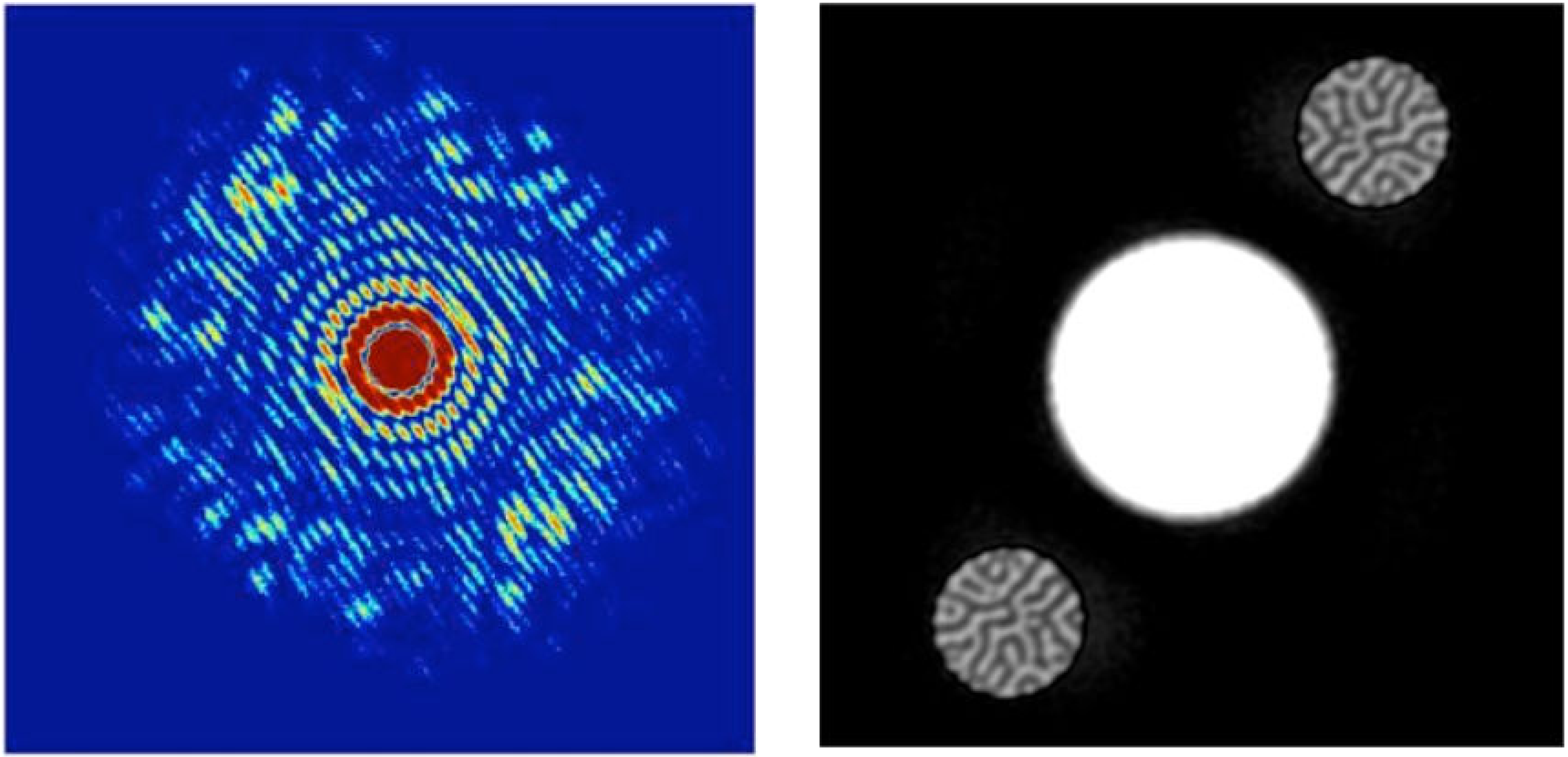
(far-field or back focal plane of a lens)

$$I(\vec{u}) = |\psi_R(\vec{u})|^2 + |\psi_o(\vec{u})|^2 + \tilde{\psi}_R(\vec{u})\, \psi_o^*(\vec{u}) + \text{c.c}$$

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \underbrace{\psi_R \circledast \psi_R + \psi_o \circledast \psi_o}_{\text{autocorrelations superimposed at the origin}} + \underbrace{\psi_R \circledast \psi_o + \psi_o \circledast \psi_R}_{\text{cross-correlations}}$$

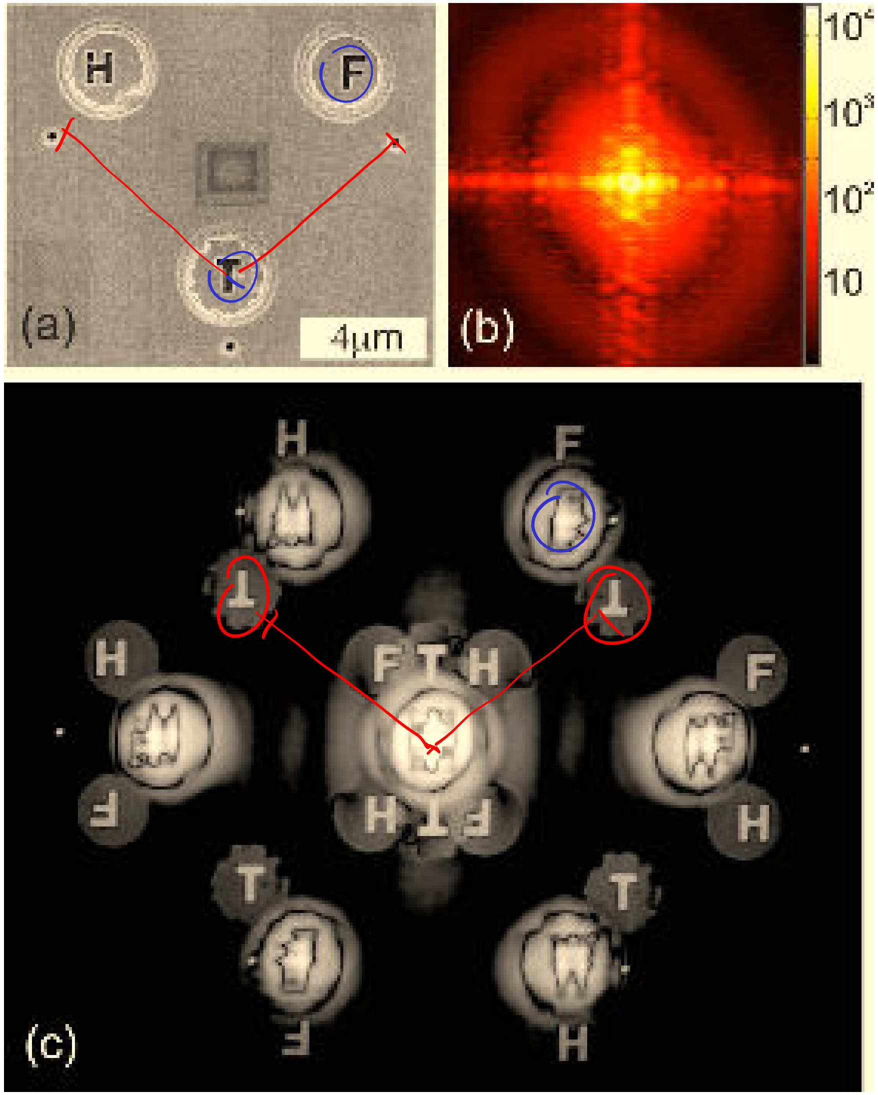
Fourier transform holography

autocorrelation is always centrosymmetric
because F.T. of a real quantity



Fourier transform holography

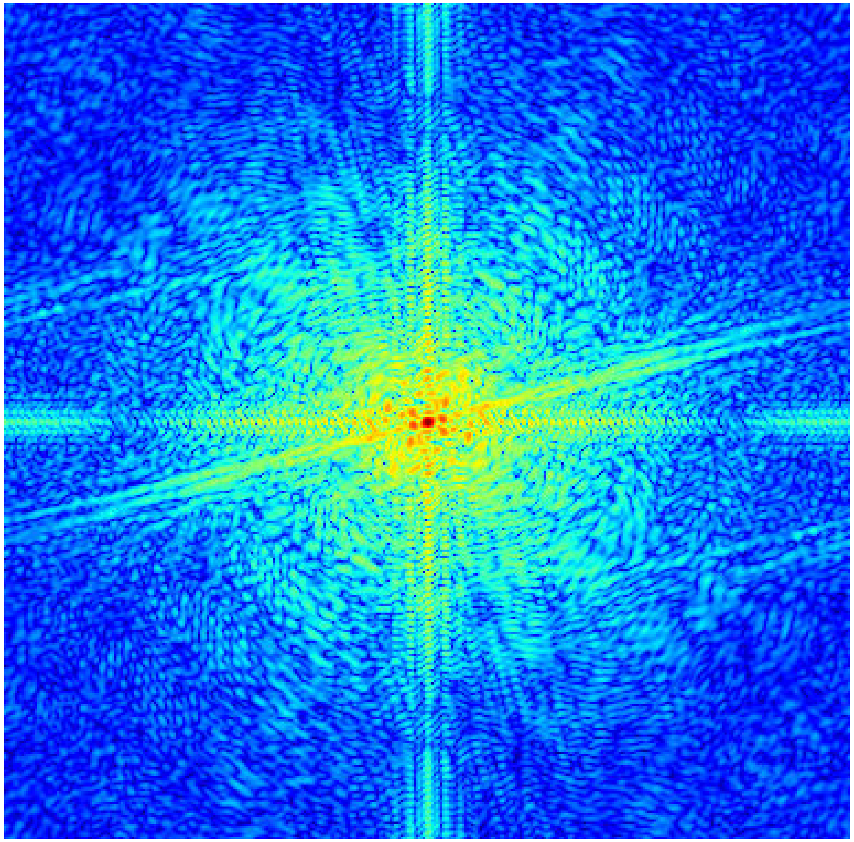
Multiple references



Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

Coherent diffractive imaging

Diffraction pattern of an isolated sample

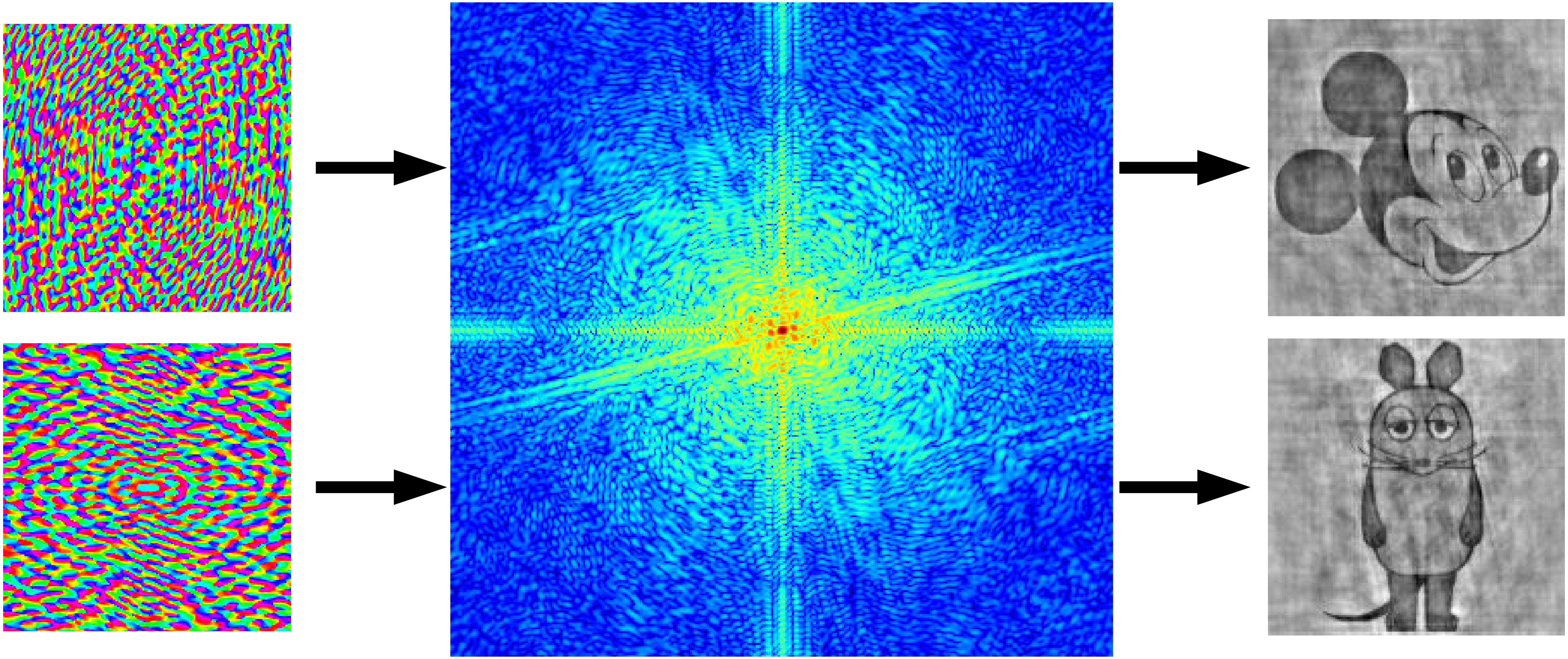


The phase problem

$e^{i\varphi(\vec{u})}$

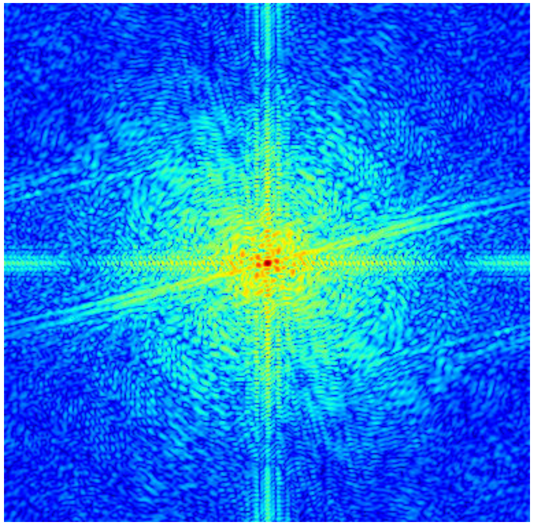
$I(\vec{u})$

$\mathcal{F}^{-1}\left\{\sqrt{I(\vec{u})}\, e^{i\varphi(\vec{u})}\right\}$

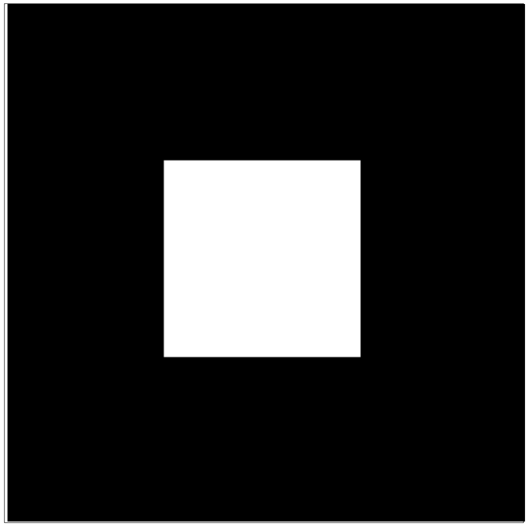


Finding the true $\varphi(\vec{u})$ is a difficult problem

Coherent diffractive imaging



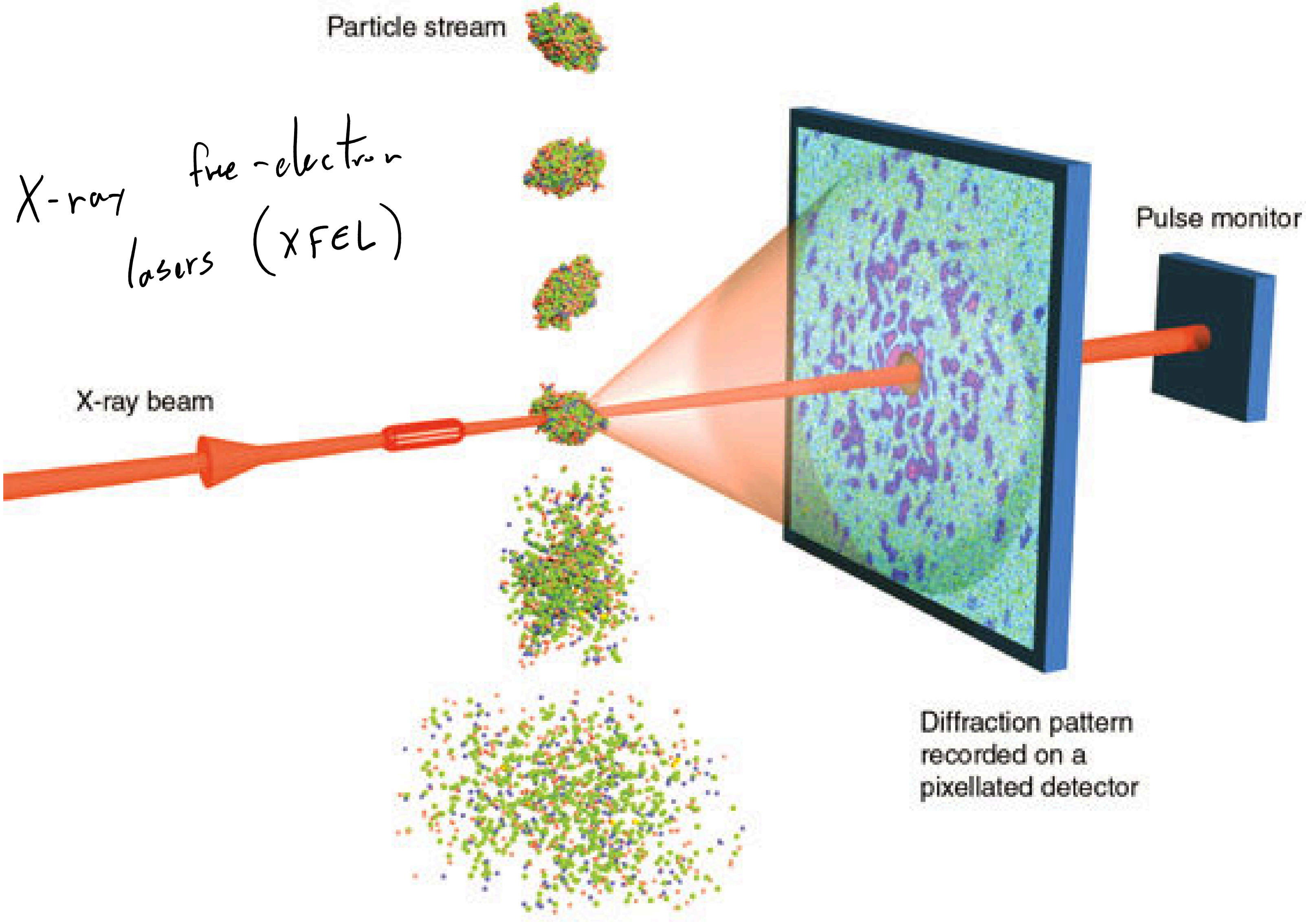
1. Solution has to be consistent with the measured Fourier amplitudes



2. Solution is isolated

Radiation damage limits on radiation

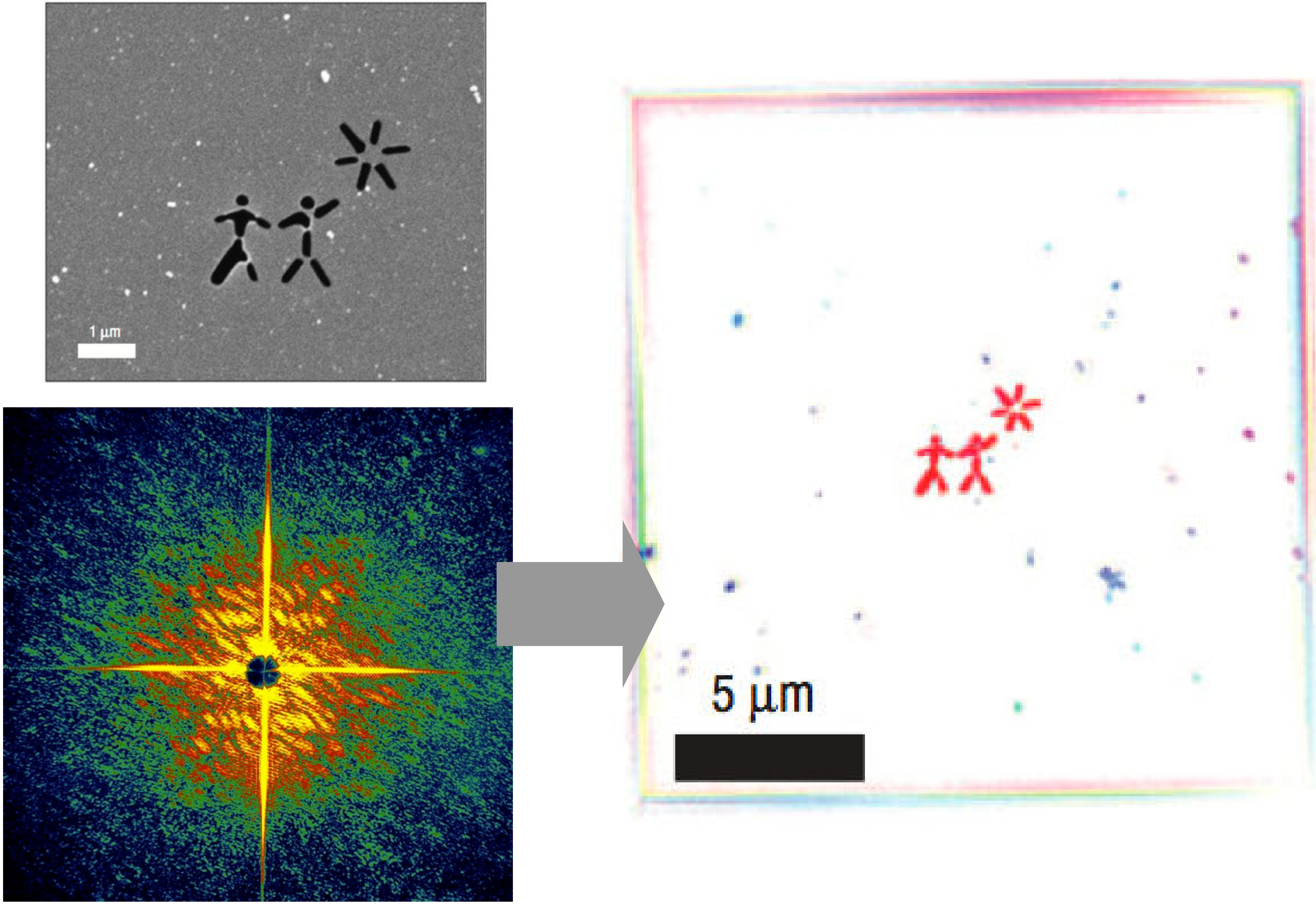
X-ray free-electron lasers (XFEL)



R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

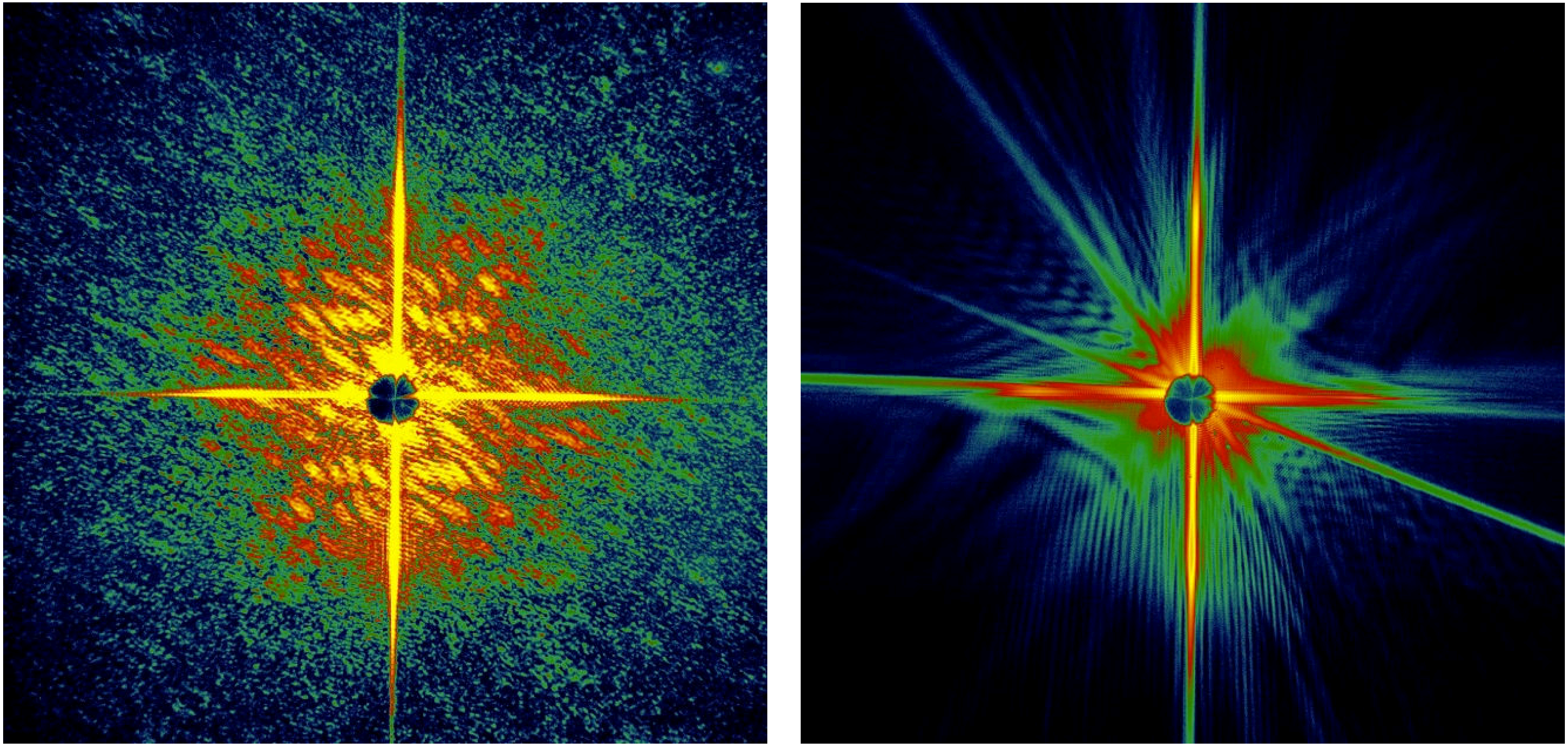
“Diffraction before destruction”



H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

“Diffraction before destruction”

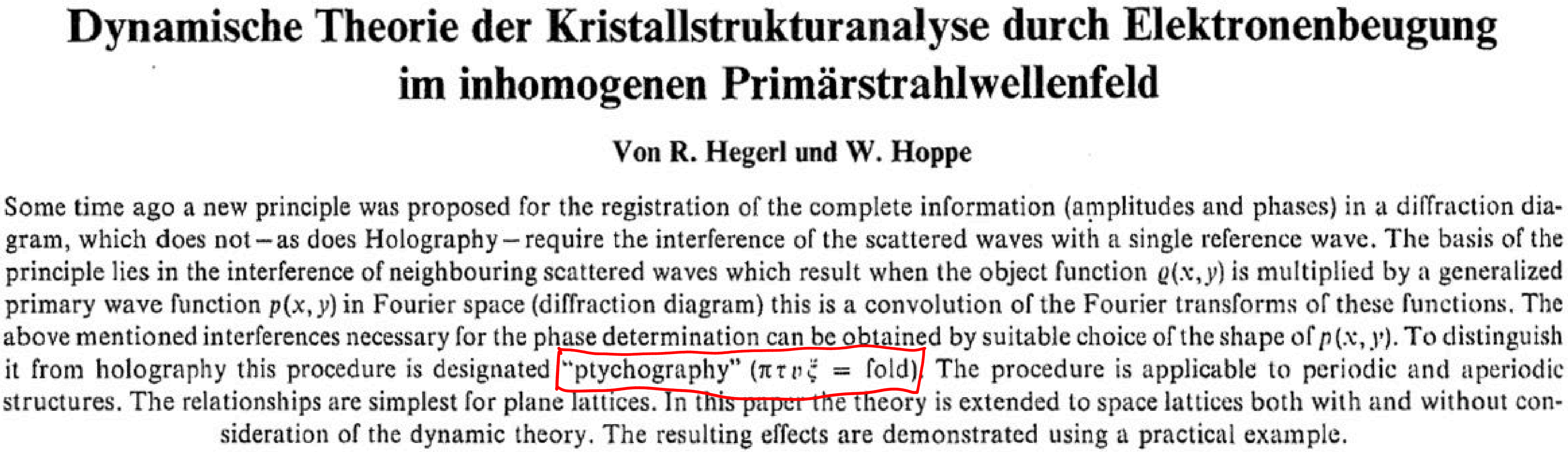
The imaging pulse vaporized the sample



H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction



Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld

Von R. Hegerl und W. Hoppe

Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" ($\pi\tau\upsilon\xi$ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

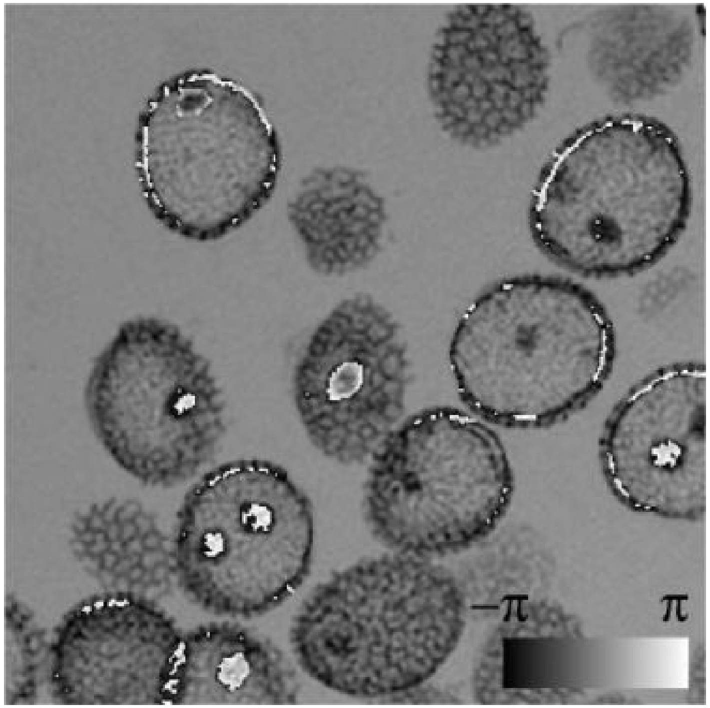
Ptychography

A few examples

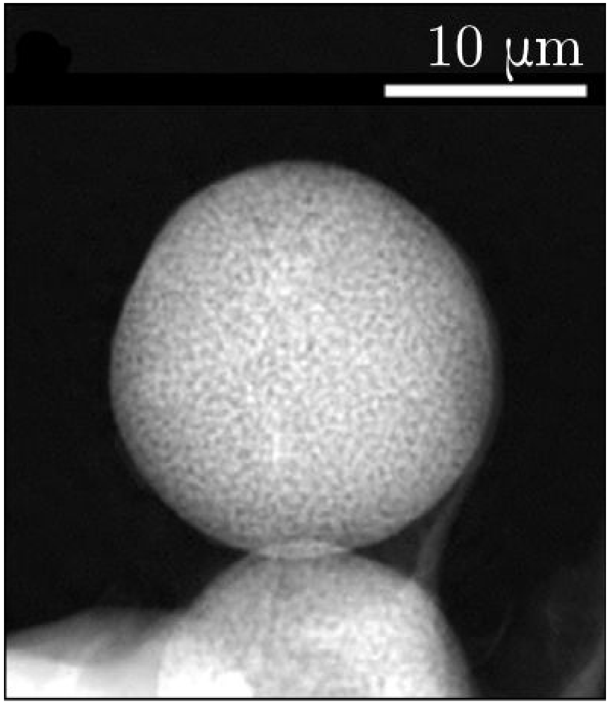
Visible light

X-rays

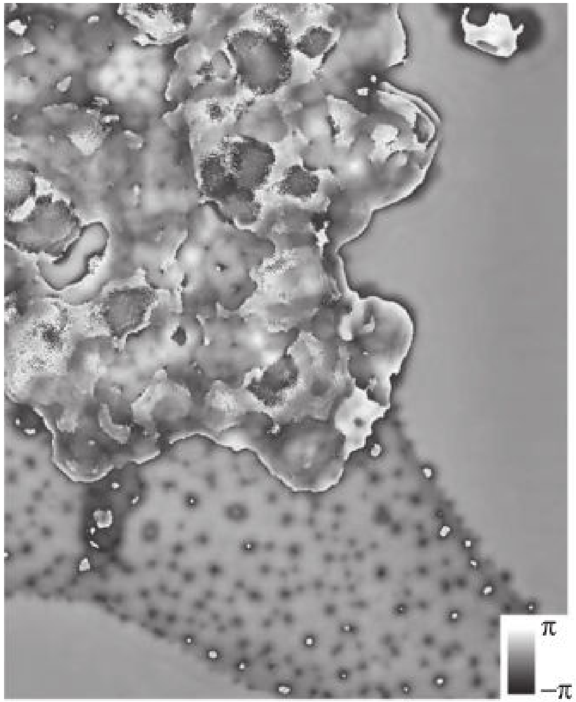
electrons



A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

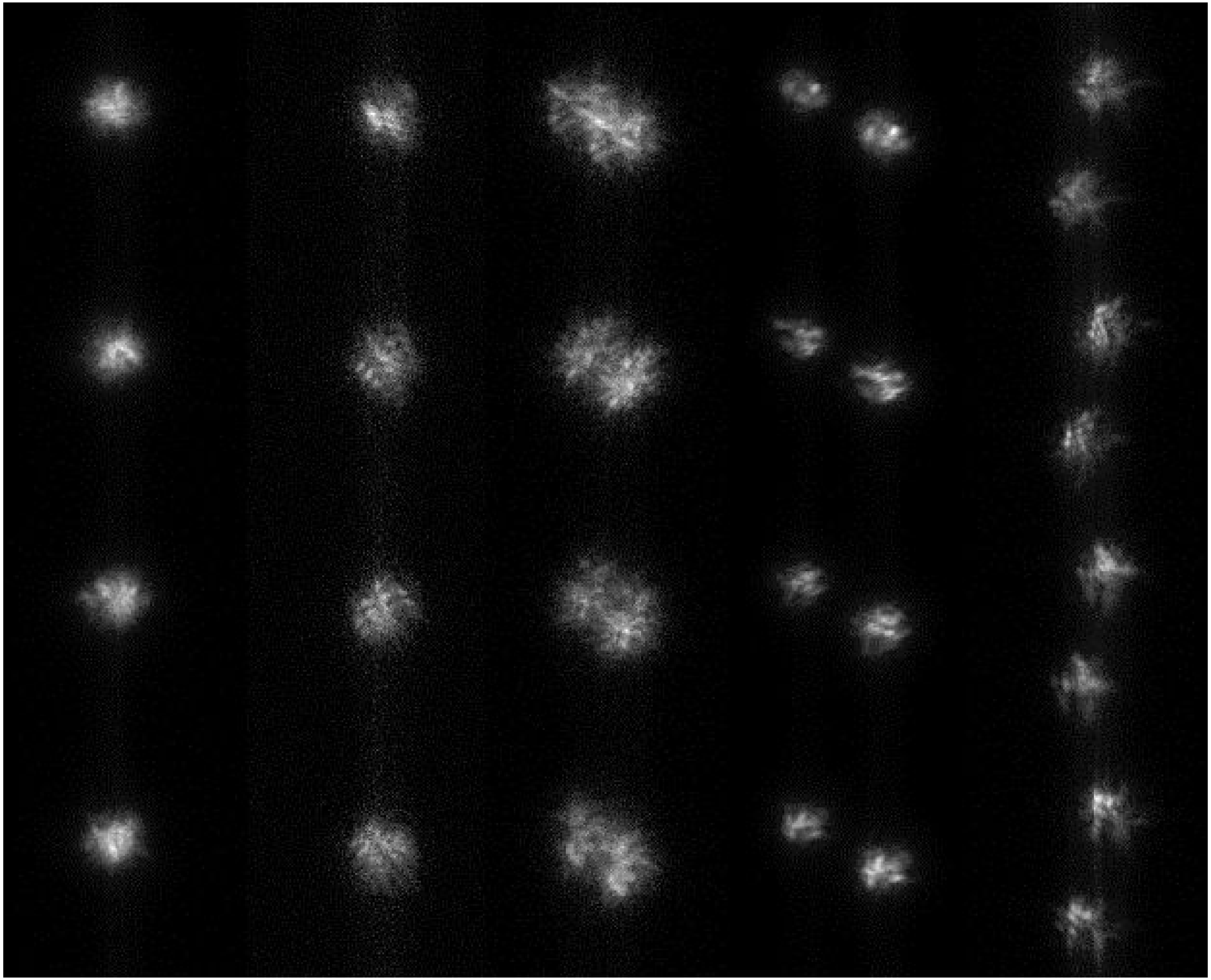


P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).



M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

Speckle imaging in astronomy



Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

Speckle imaging in astronomy

Model

object

$$I(\vec{r}) = O * |P|^2$$

"instaneous PSF"

$$\tilde{I} = \tilde{O} \cdot P_A$$

autocorrelation of PSF

$$|\tilde{I}|^2 = |\tilde{O}|^2 \cdot |P_A|^2$$

known quantity (from understanding of turbulence in atmosphere)

average over multiple independent measurements

$$\langle |\tilde{I}|^2 \rangle = |\tilde{O}|^2 \langle |P_A|^2 \rangle$$

$$|\tilde{O}|^2 = \left. \frac{\langle |I|^2 \rangle}{\langle |P_A|^2 \rangle} \right]$$

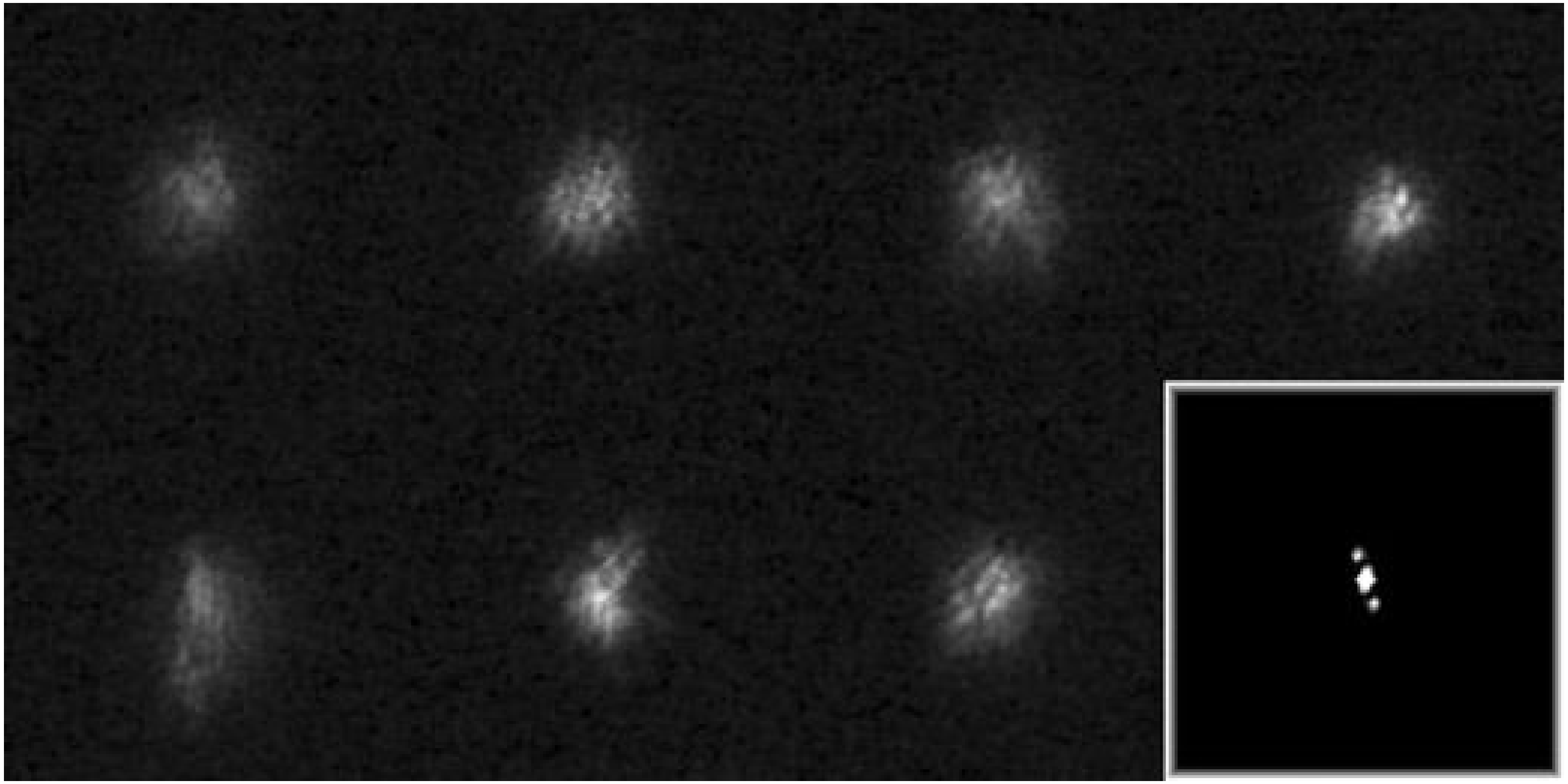
known

reconstruct O from $|\tilde{O}|^2$

same problem as before!

Speckle imaging in astronomy

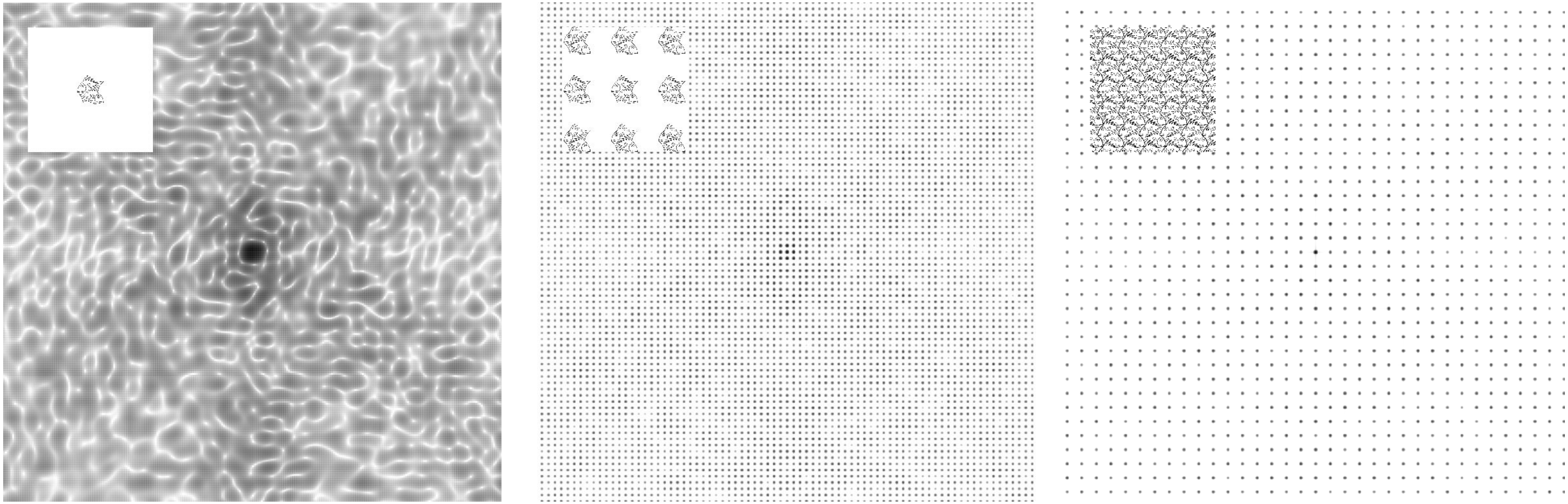
Retrieval of the autocorrelation



Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

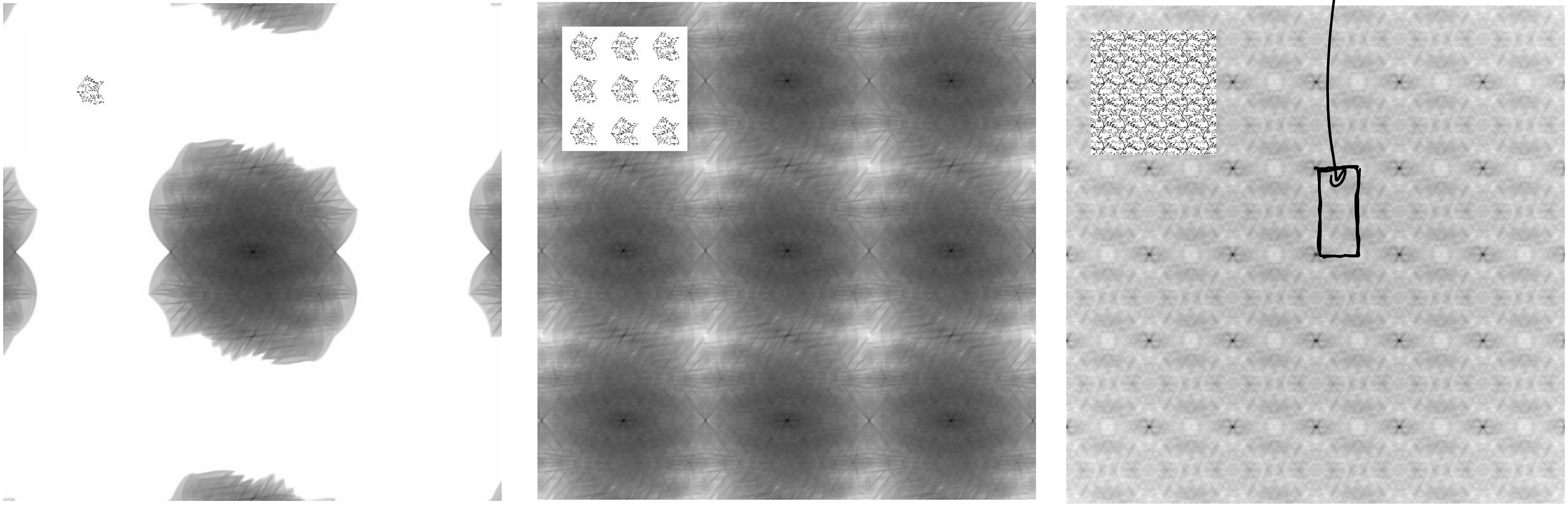
Crystallography

Diffraction by a crystal: Bragg peaks



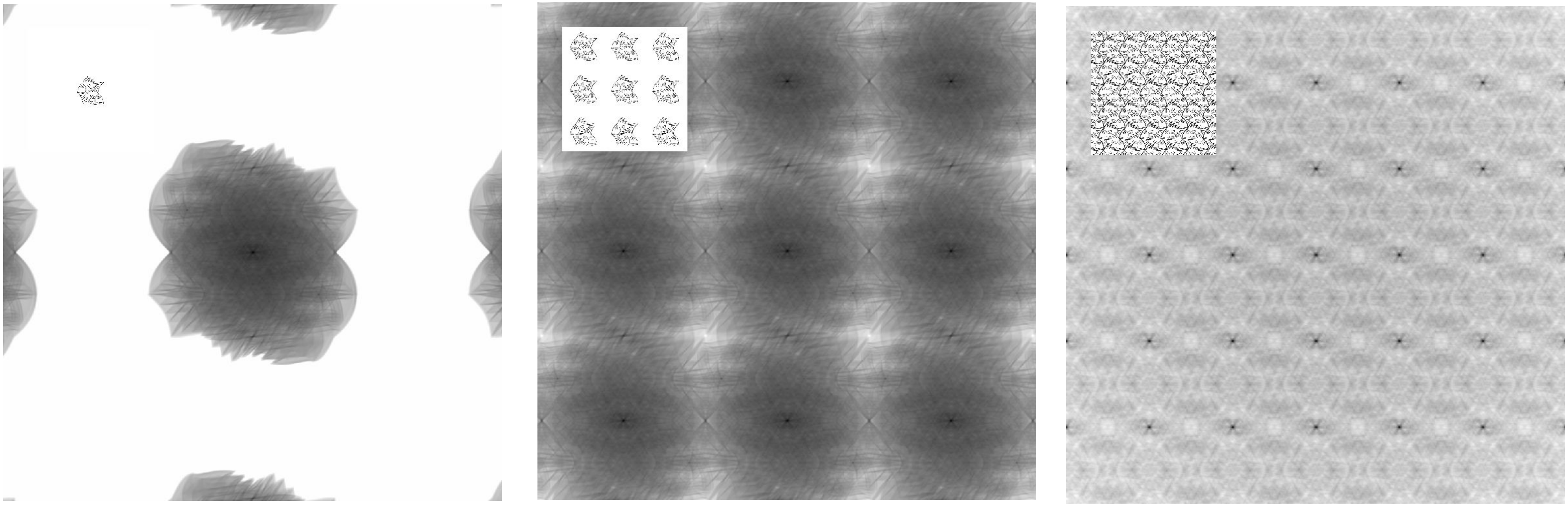
Crystallography

Fourier transform of intensity: autocorrelation



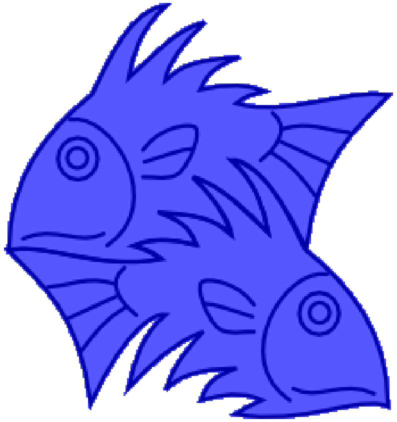
only this information is encoded in the measurements

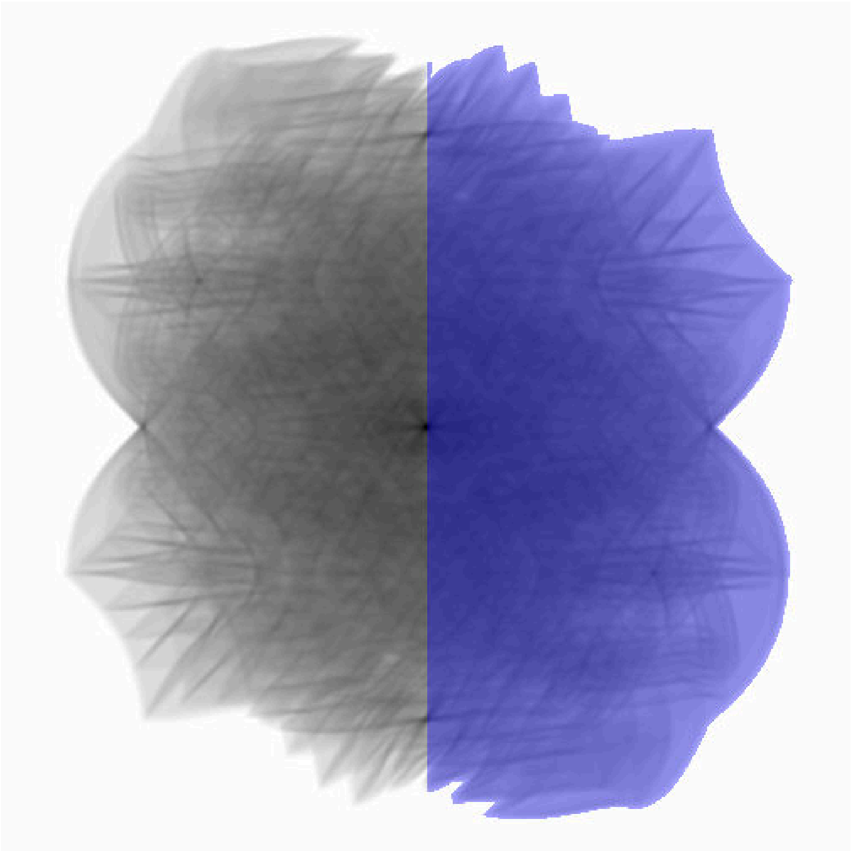
Crystallography



Crystallography

Problem is overconstrained with an isolated sample





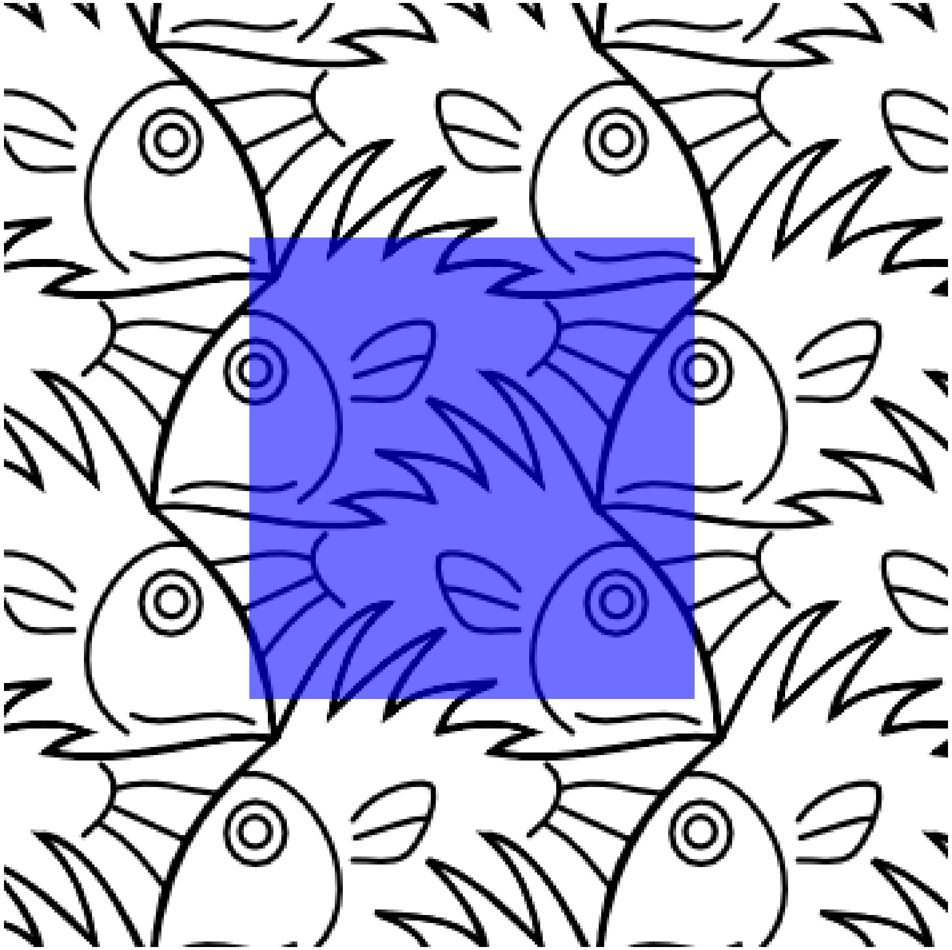
unknowns = N

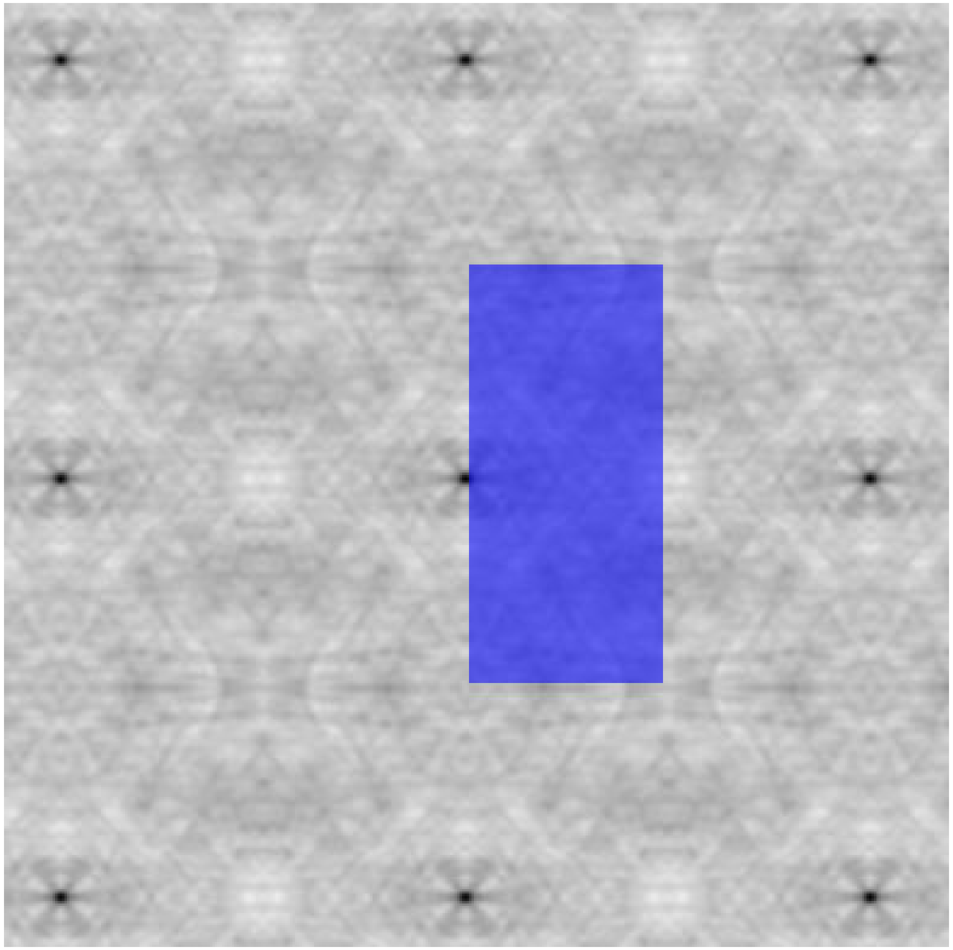
constraints ≥ 2N

Overconstrained

Crystallography

Problem is **under**constrained with a crystal





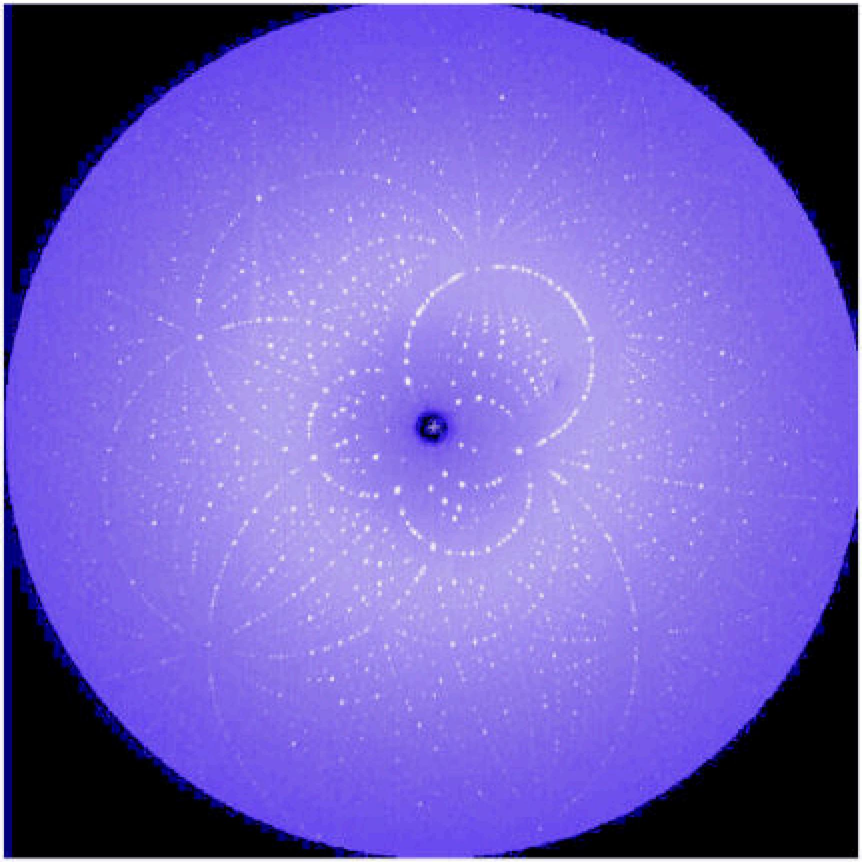
unknowns = N

constraints = N/2

Underconstrained

Crystallography

Structure determination



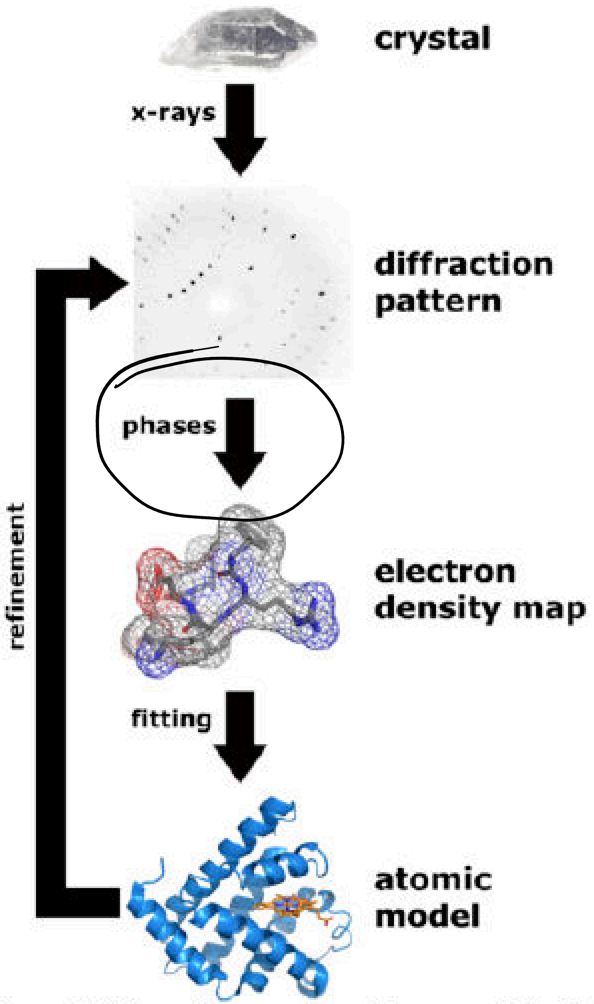


Image from Wikimedia courtesy Thomas Splettstoesser

Crystallography

Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

Summary

Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)