

```

=>
=>
=>
=> #EX1-UNCONSTRAINED OPTIMIZATION
=>
=> #Function
> f :=  $\frac{x \cdot y^3}{3} - x^2 - 4 \cdot x \cdot y$ ;
                                 $f := \frac{1}{3} x y^3 - x^2 - 4 x y$  (1)
=>
=> #Gradient
> dfx := diff(f, x);
                                 $dfx := \frac{1}{3} y^3 - 2 x - 4 y$  (2)
=>
> dfy := diff(f, y);
                                 $dfy := x y^2 - 4 x$  (3)
=>
=> #Critical Points
> CP := solve( {dfx=0, dfy=0}, {x, y}, allsolutions, explicit);
                                 $CP := \{x=0, y=0\}, \left\{x=-\frac{8}{3}, y=2\right\}, \left\{x=\frac{8}{3}, y=-2\right\}, \{x=0, y=2\sqrt{3}\}, \{x=0, y=-2\sqrt{3}\}$  (4)
=>
=>
=> with(VectorCalculus) :
> Hf := Hessian(f, [x, y]);
                                 $Hf := \begin{bmatrix} -2 & y^2 - 4 \\ y^2 - 4 & 2 x y \end{bmatrix}$  (5)
=>
> with(LinearAlgebra) :
> detHf := Determinant(Hf);
                                 $detHf := -y^4 - 4 x y + 8 y^2 - 16$  (6)
=>
> trHf := Trace(Hf);
                                 $trHf := 2 x y - 2$  (7)
=>
> eigf := Eigenvalues(Hf);
                                 $eigf := \begin{bmatrix} x y - 1 + \sqrt{x^2 y^2 + y^4 + 2 x y - 8 y^2 + 17} \\ x y - 1 - \sqrt{x^2 y^2 + y^4 + 2 x y - 8 y^2 + 17} \end{bmatrix}$  (8)
=>
=> #Detailed analysis for each point
=>
> CP[1];
                                 $\{x=0, y=0\}$  (9)
=>
> Hf1 := eval(Hf, CP[1]);

```

$$Hf1 := \begin{bmatrix} -2 & -4 \\ -4 & 0 \end{bmatrix} \quad (10)$$

> *detHf1* := *Determinant*(*Hf1*);

$$detHf1 := -16 \quad (11)$$

> *trHf1* := *Trace*(*Hf1*);

$$trHf1 := -2 \quad (12)$$

> *eigf1* := *Eigenvalues*(*Hf1*); *evalf*(*eigf1*);

$$eigf1 := \begin{bmatrix} -1 + \sqrt{17} \\ -1 - \sqrt{17} \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} 3.123105626 \\ -5.123105626 \end{bmatrix}$$

> *CP*[2];

$$\left\{ x = -\frac{8}{3}, y = 2 \right\} \quad (14)$$

> *Hf2* := *eval*(*Hf*, *CP*[2]);

$$Hf2 := \begin{bmatrix} -2 & 0 \\ 0 & -\frac{32}{3} \end{bmatrix} \quad (15)$$

> *detHf2* := *Determinant*(*Hf2*);

$$detHf2 := \frac{64}{3} \quad (16)$$

> *trHf2* := *Trace*(*Hf2*);

$$trHf2 := -\frac{38}{3} \quad (17)$$

> *eigf2* := *Eigenvalues*(*Hf2*); *evalf*(*eigf2*);

$$eigf2 := \begin{bmatrix} -2 \\ -\frac{32}{3} \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} -2. \\ -10.66666667 \end{bmatrix}$$

> *CP*[3];

$$\left\{ x = \frac{8}{3}, y = -2 \right\} \quad (19)$$

> *Hf3* := *eval*(*Hf*, *CP*[3]);

$$Hf3 := \begin{bmatrix} -2 & 0 \\ 0 & -\frac{32}{3} \end{bmatrix} \quad (20)$$

> *detHf3* := *Determinant*(*Hf3*);

$$detHf3 := \frac{64}{3} \quad (21)$$

> $trHf3 := Trace(Hf3);$

$$trHf3 := -\frac{38}{3} \quad (22)$$

> $eigf3 := Eigenvalues(Hf3); evalf(eigf3);$

$$eigf3 := \begin{bmatrix} -2 \\ -\frac{32}{3} \end{bmatrix} \quad (23)$$

> $CP[4];$

$$\{x=0, y=2\sqrt{3}\} \quad (24)$$

> $Hf4 := eval(Hf, CP[4]);$

$$Hf4 := \begin{bmatrix} -2 & 8 \\ 8 & 0 \end{bmatrix} \quad (25)$$

> $detHf4 := Determinant(Hf4);$

$$detHf4 := -64 \quad (26)$$

> $trHf4 := Trace(Hf4);$

$$trHf4 := -2 \quad (27)$$

> $eigf4 := Eigenvalues(Hf4); evalf(eigf4);$

$$eigf4 := \begin{bmatrix} -1 + \sqrt{65} \\ -1 - \sqrt{65} \end{bmatrix} \quad (28)$$

> $CP[5];$

$$\{x=0, y=-2\sqrt{3}\} \quad (29)$$

> $Hf5 := eval(Hf, CP[5]);$

$$Hf5 := \begin{bmatrix} -2 & 8 \\ 8 & 0 \end{bmatrix} \quad (30)$$

> $detHf5 := Determinant(Hf5);$

$$detHf5 := -64 \quad (31)$$

> $trHf5 := Trace(Hf5);$

$$trHf5 := -2 \quad (32)$$

> $eigf5 := Eigenvalues(Hf5); evalf(eigf5);$

$$eigf5 := \begin{bmatrix} -1 + \sqrt{65} \\ -1 - \sqrt{65} \end{bmatrix}$$

$$\begin{bmatrix} 7.062257748 \\ -9.062257748 \end{bmatrix}$$

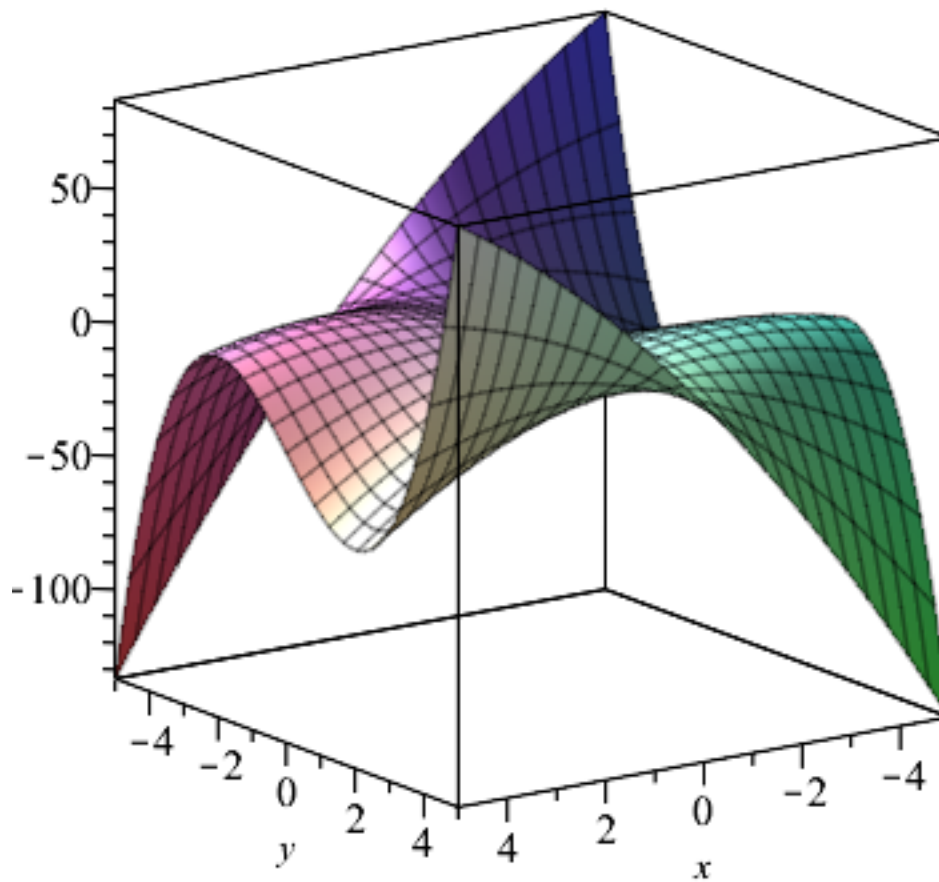
(33)

>

>

> *with(plots) :*

> $a := \text{plot3d}\left(\frac{x \cdot y^3}{3} - x^2 - 4 \cdot x \cdot y, x = -5 \dots 5, y = -5 \dots 5, \text{contours} = 10\right);$



>

> $b := \text{contourplot}\left(\frac{x \cdot y^3}{3} - x^2 - 4 \cdot x \cdot y, x = -5 \dots 5, y = -5 \dots 5, \text{contours} = 70\right);$

