Corso di Laurea in Fisica - UNITS

ISTITUZIONI DI FISICA

PER IL SISTEMA TERRA

SEISMIC BODY WAVES

FABIO ROMANELLI

Department of Mathematics & Geosciences

University of Trieste

romanel@units.it

Some basic definitions - 1

Principles of mechanics applied to bulk matter:

Mechanics of fluids   Mechanics of solids

Continuum Mechanics

A material can be called solid (rather than -perfect- fluid) if it can support a shearing force over the time scale of some natural process.

Shearing forces are directed parallel, rather than perpendicular, to the material surface on which they act.

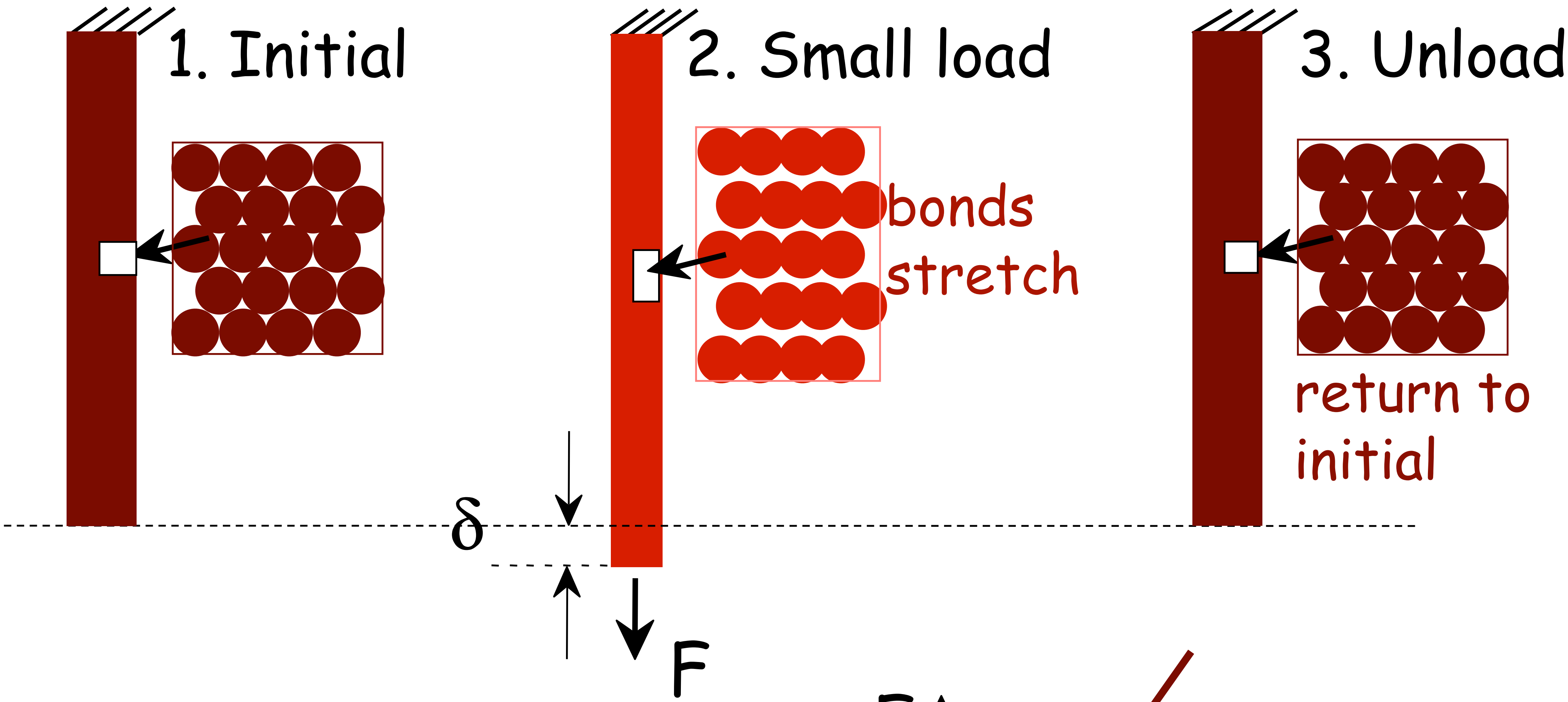
Some basic definitions - 2

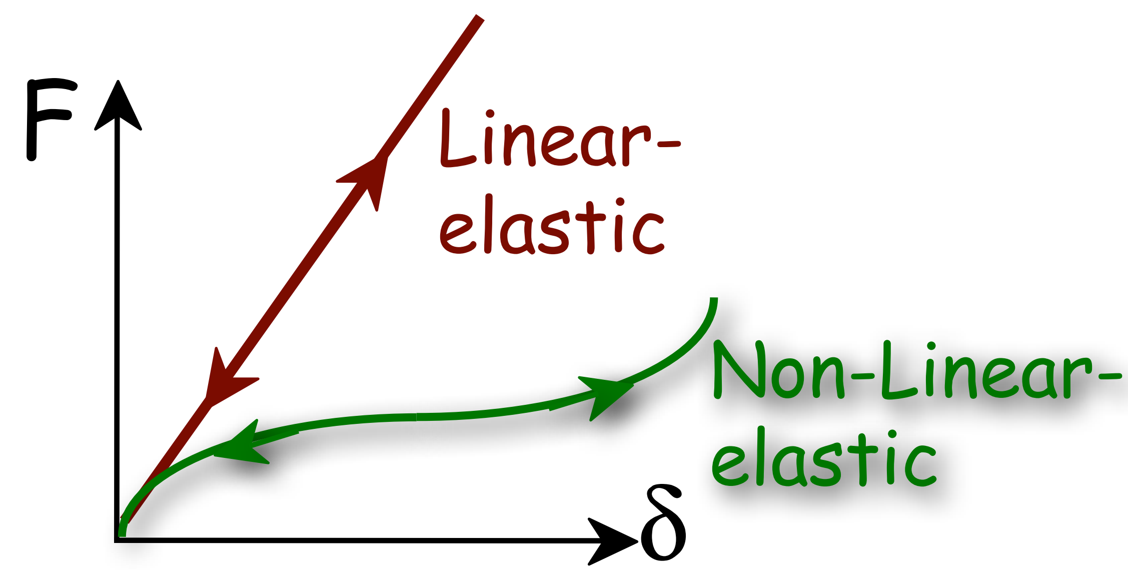
When a material is loaded at sufficiently low temperature, and/or short time scale, and with sufficiently limited stress magnitude, its deformation is fully recovered upon uploading: the material is **elastic**

If there is a permanent (plastic) deformation due to exposition to large stresses: the material is **elastic-plastic**

If there is a permanent deformation (viscous or creep) due to time exposure to a stress, and that increases with time: the material is **viscoelastic** (with elastic response), or the material is **visco-plastic** (with partial elastic response)

Elastic Deformation

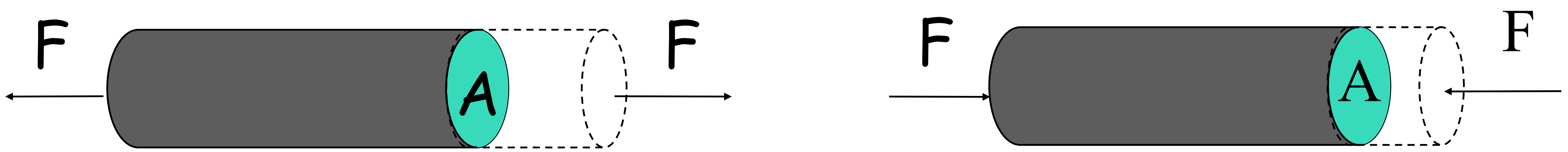




Elastic means reversible!
It goes back to its original state
once the loading is removed.

Normal stress acts perpendicular to the surface

(F=normal force)

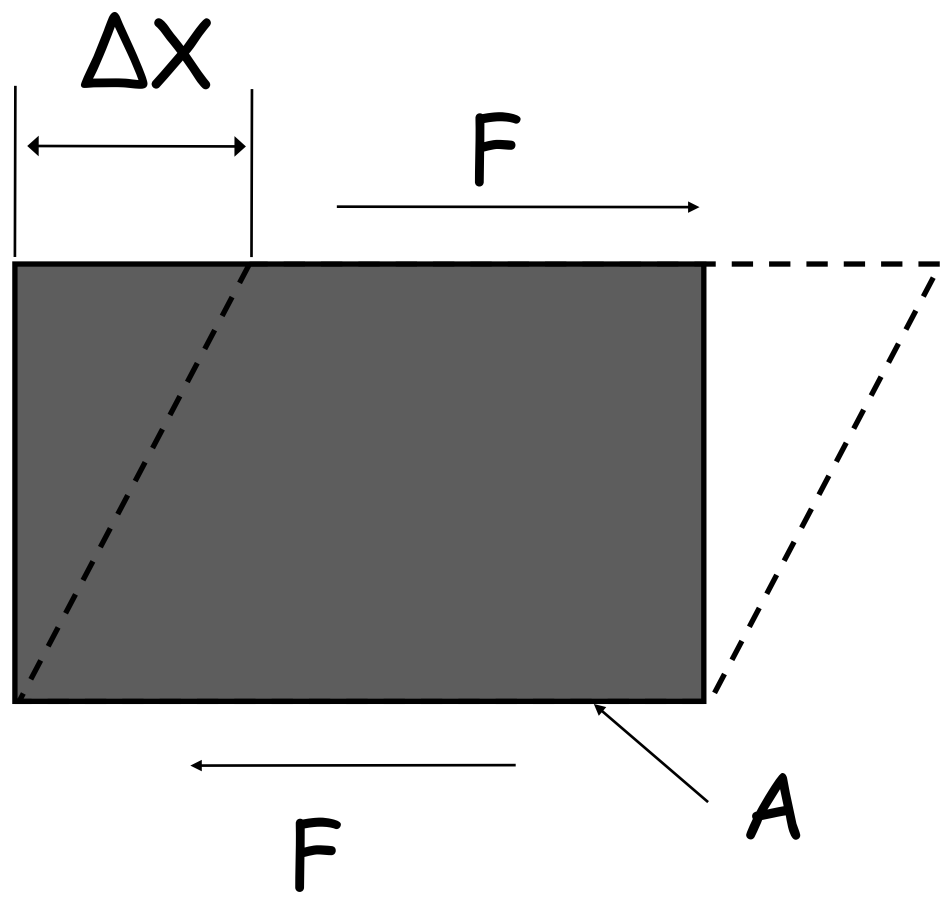


Tensile causes elongation

Compressive causes shrinkage

$$\sigma = \frac{\text{stretching force}}{\text{cross sectional area}}$$

Shear Stress as a measure of Force



$$\tau = \frac{\text{shear force}}{\text{tangential area}}$$

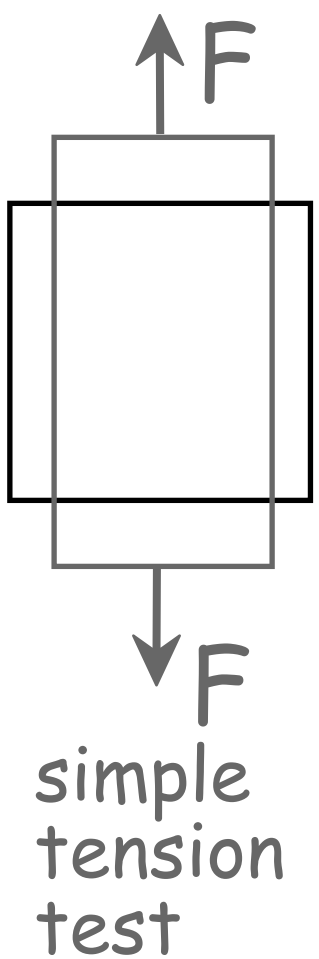
Modulus of Elasticity, E:

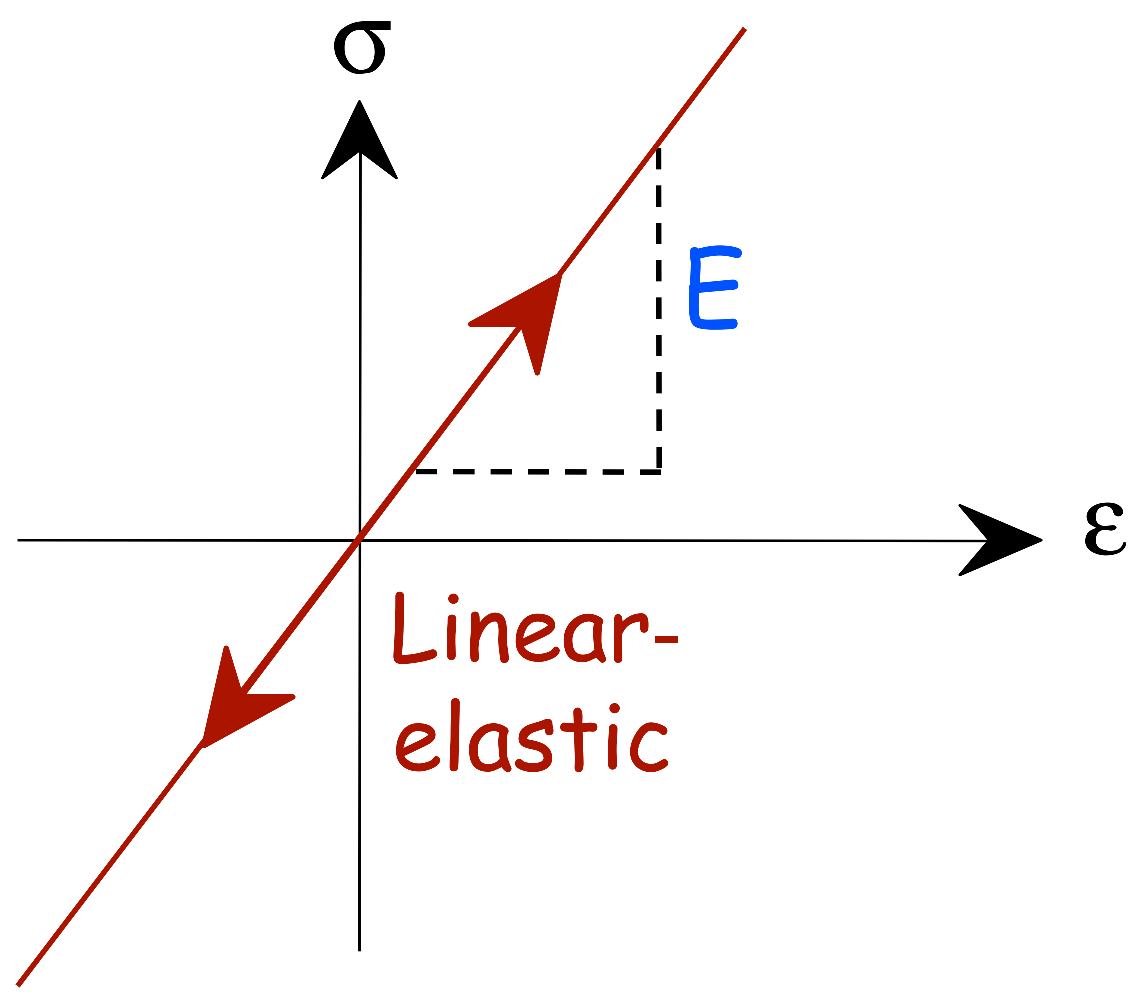
(also known as Young's modulus)

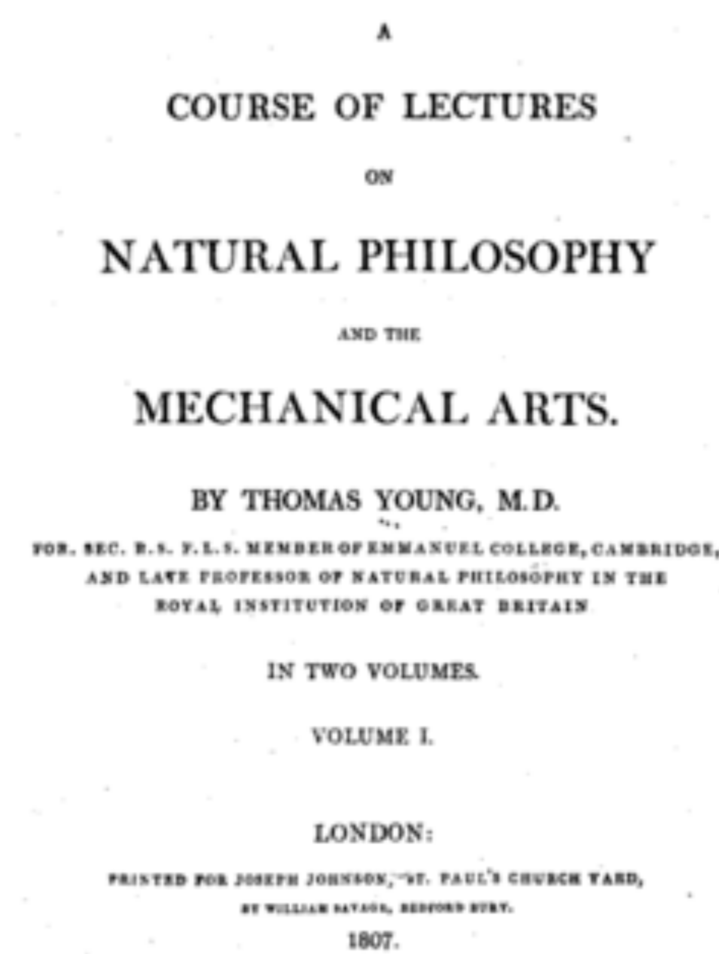
- **Hooke's Law:**

$$\sigma = E\,\varepsilon$$

E: **stiffness** (material's resistance to elastic deformation)






A

COURSE OF LECTURES

ON

NATURAL PHILOSOPHY

AND THE

MECHANICAL ARTS.

BY THOMAS YOUNG, M.D.

FOR. SEC. R.S. F.L.S. MEMBER OF EMMANUEL COLLEGE, CAMBRIDGE, AND LATE PROFESSOR OF NATURAL PHILOSOPHY IN THE ROYAL INSTITUTION OF GREAT BRITAIN

IN TWO VOLUMES.

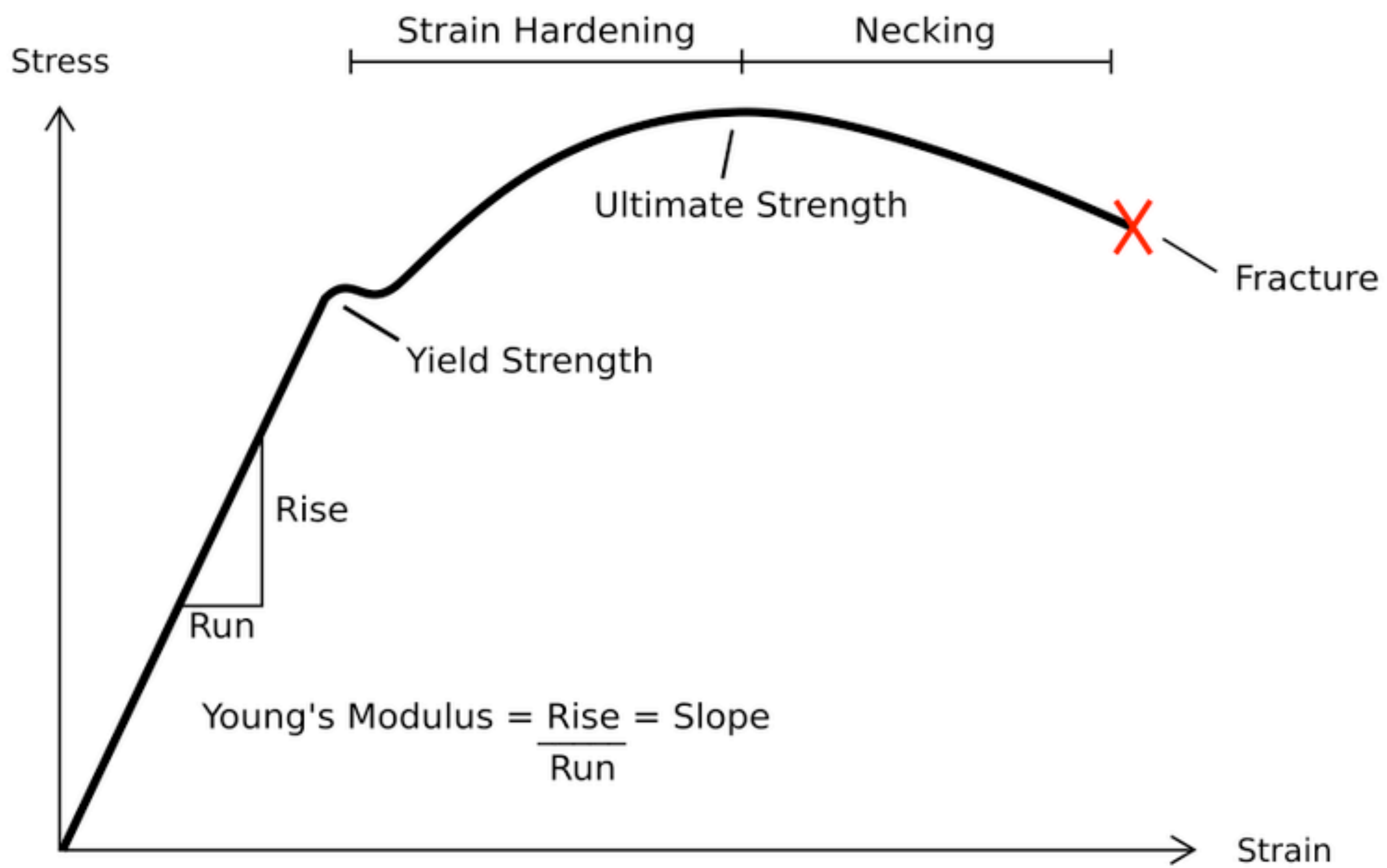
VOLUME I.

LONDON:

PRINTED FOR JOSEPH JOHNSON, ST. PAUL'S CHURCH YARD, BY WILLIAM SAVAGE, BEDFORD BURY.

1807.

Young's modulus



Elasticity Theory

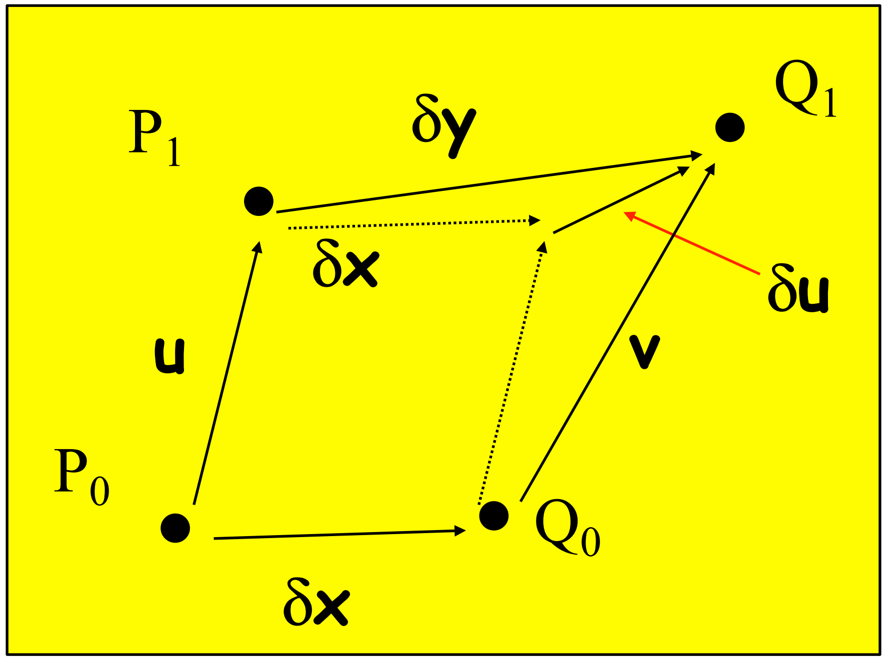
A time-dependent perturbation of an elastic medium (e.g. a rupture, an earthquake, a meteorite impact, a nuclear explosion etc.) generates elastic waves emanating from the source region. These disturbances produce local changes in stress and strain.

To understand the propagation of elastic waves we need to describe kinematically the deformation of our medium and the resulting forces (stress). The relation between deformation and stress is governed by elastic constants.

The time-dependence of these disturbances will lead us to the elastic wave equation as a consequence of conservation of energy and momentum.

The symmetric part is called the **strain tensor**

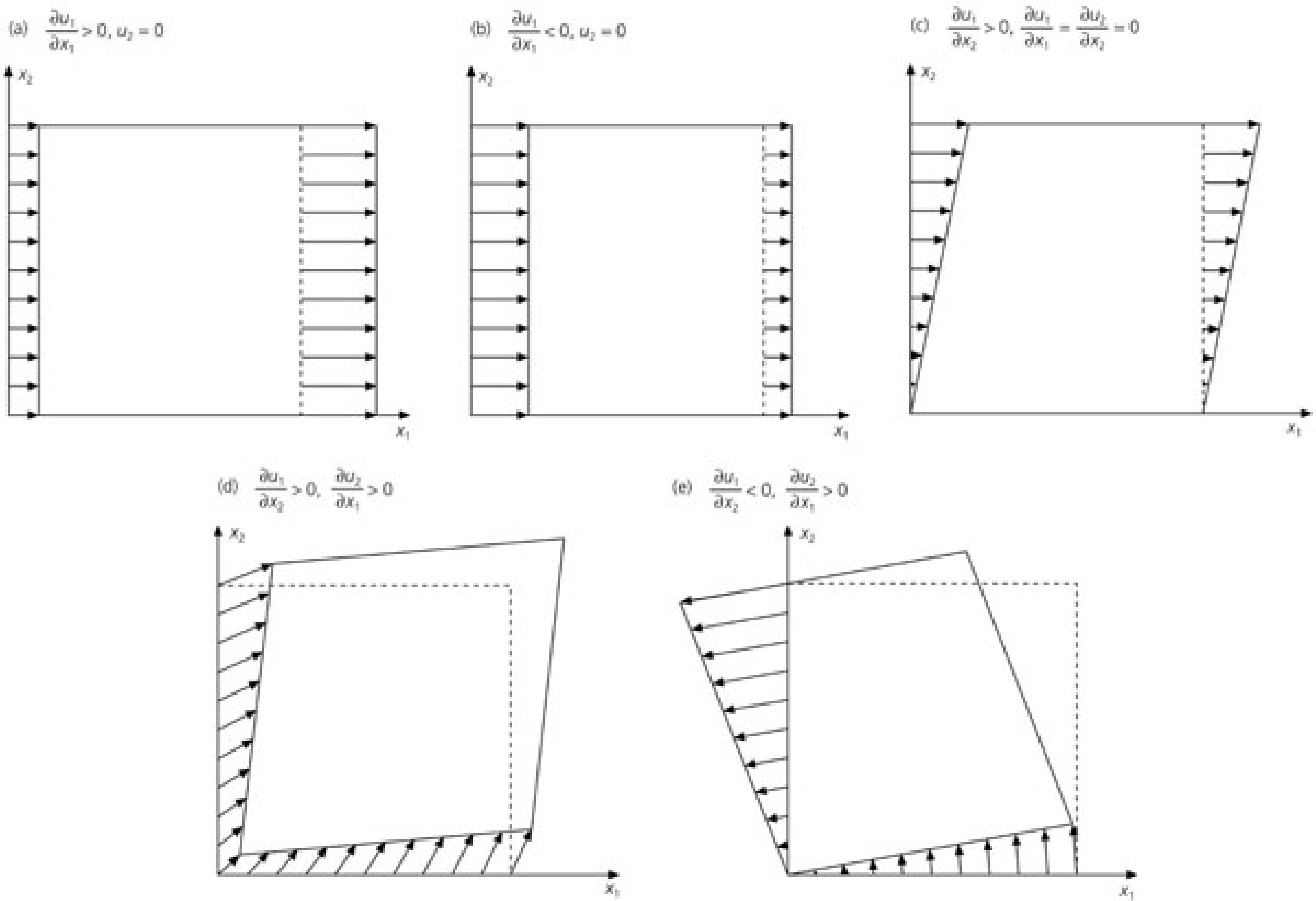
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



and describes the relation between deformation and displacement in linear elasticity. In 2-D this tensor looks like

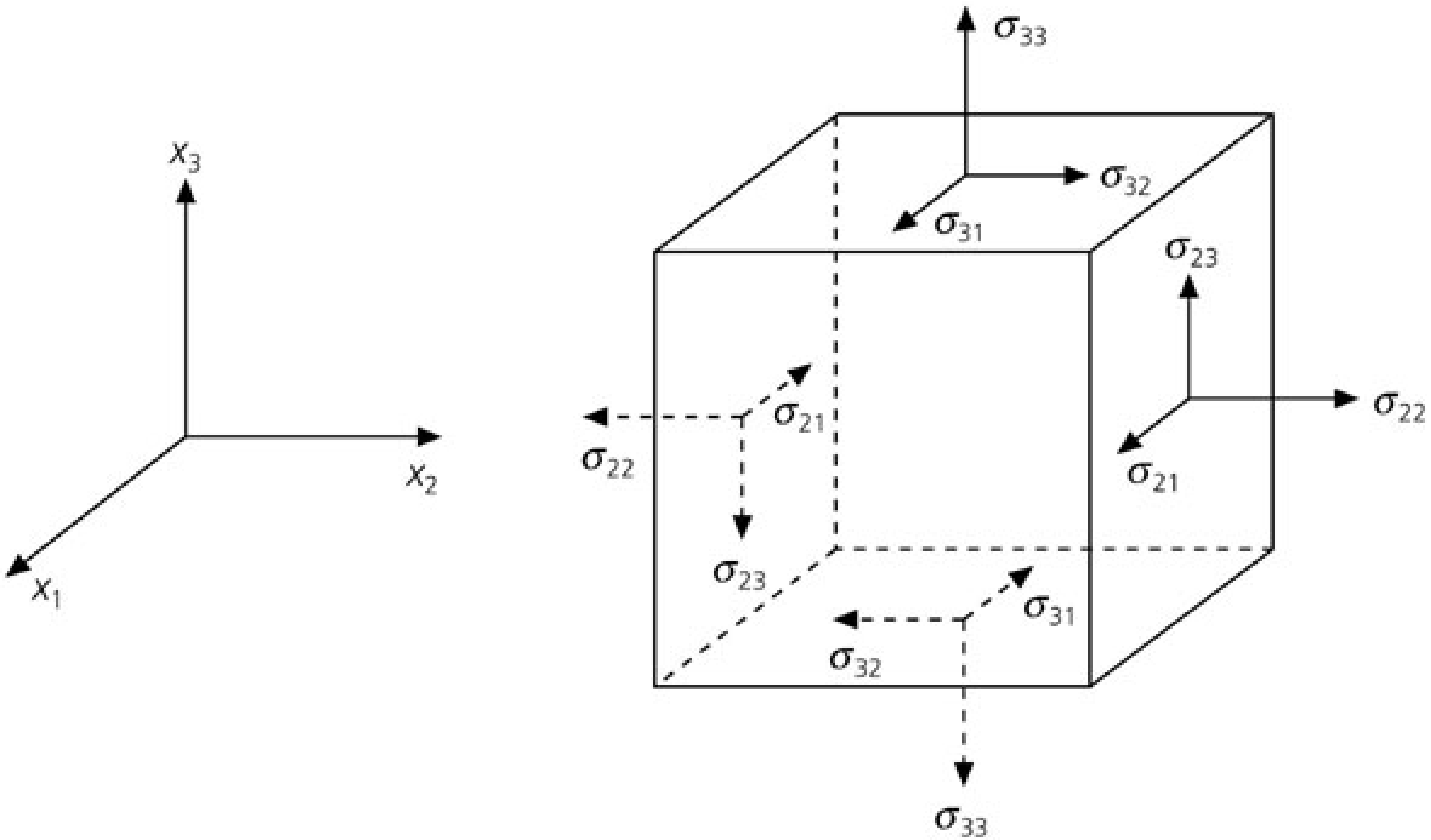
$$\varepsilon_{ij}= \begin{bmatrix} \frac{\partial u_1}{\partial x} & \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) & \frac{\partial u_2}{\partial y} \end{bmatrix}$$

Figure 2.3-12: Some possible strains for a two-dimensional element.



...and the stress state in a point of the material can be expressed with:

Figure 2.3-4: Stress components on the faces of a volume element.



Stress-strain relation - 1

The relation between stress and strain in general is described by the tensor of elastic constants c_{ijkl}

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

Generalised Hooke's Law

From the symmetry of the stress and strain tensor and a thermodynamic condition if follows that the maximum number if independent constants of c_{ijkl} is 21. In an isotropic body, where the properties do not depend on direction, the relation reduces to

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$$

Hooke's Law

where λ and μ are the Lame parameters, θ is the dilatation and δ_{ij} is the Kronecker delta.

$$\theta\delta_{ij} = \varepsilon_{kk}\delta_{ij} = \left(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\right)\delta_{ij}$$

Elastic parameters

Rigidity is the ratio of pure shear strain and the applied shear stress component

$$\mu = \frac{\sigma_{ij}}{2\varepsilon_{ij}}$$

Bulk modulus of incompressibility is defined the ratio of pressure to volume change. Ideal fluid means no rigidity ($\mu = 0$), thus λ is the incompressibility of a fluid.

$$K = -\frac{P}{\theta} = \lambda + \frac{2}{3}\mu$$

Consider a stretching experiment where tension is applied to an isotropic medium along a principal axis (say x).

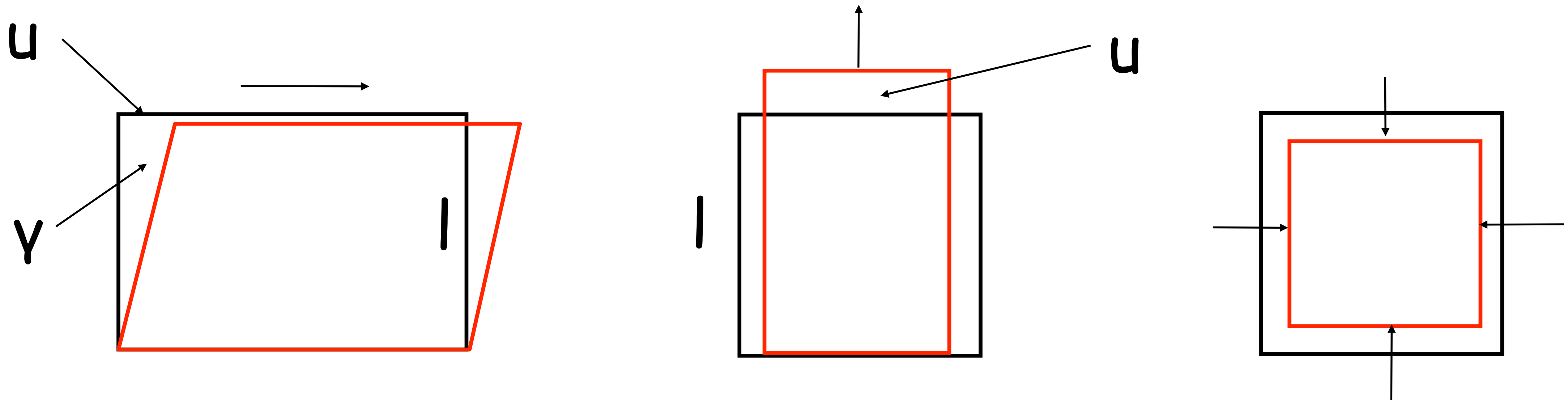
Poisson's ratio $\equiv \nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = \frac{\lambda}{2(\lambda + 2\mu)}$ **Young's modulus** $\equiv E = -\frac{\sigma_{11}}{\varepsilon_{11}} = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

For Poisson's ratio we have $0<\nu<0.5$.

A useful approximation is $\lambda=\mu$ (Poisson's solid), then $\nu=0.25$ and for fluids $\nu=0.5$

As in the case of deformation the stress-strain relation can be interpreted in simple geometric terms:



$\sigma_{12} = \mu\gamma$ $\sigma_{22} = E\frac{u}{l}$ $P = K\frac{\Delta V}{V} = K\varepsilon_{ii}$

Remember that these relations are a generalization of Hooke's Law:

F= Kx

D being the spring constant and s the elongation.

Elastic constants

Let us look at some examples for elastic constants:

Rock	K 10^{12} dynes/cm^2	E 10^{12} dynes/cm^2	μ 10^{12} dynes/cm^2	ν
Limestone		0.621	0.248	0.251
Granite	0.132	0.416	0.197	0.055
Gabbro	0.659	1.08	0.438	0.219
Dunite		1.52	0.60	0.27

We now have a complete description of the forces acting within an elastic body. Adding the inertia forces with opposite sign leads us from

$$f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

to

$$\rho \frac{\partial^2 u_i}{\partial t^2} = f_i + \frac{\partial \sigma_{ij}}{\partial x_j}$$

the equations of motion for dynamic elasticity

$$\rho\partial_t^2\mathbf{u} = \mathbf{f} + (\lambda + 2\mu)\nabla\nabla \bullet \mathbf{u} - \mu\nabla \times \nabla \times \mathbf{u}$$

Let us apply the **div** operator to this equation, we obtain

$$\rho\partial_t^2\theta = (\lambda + 2\mu)\Delta\theta$$

where

$$\theta = \nabla \bullet \mathbf{u}$$

Acoustic wave equation

$$\frac{1}{\alpha^2}\partial_t^2\theta = \Delta\theta$$

or

P-wave velocity

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$\rho\partial_t^2\mathbf{u} = \mathbf{f} + (\lambda + 2\mu)\nabla\nabla\bullet\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

Let us apply the **curl** operator to this equation, we obtain

$$\rho\partial_t^2\nabla\times\mathbf{u} = (\lambda+\mu)\nabla\times\nabla\theta + \mu\Delta(\nabla\times\mathbf{u})$$

we now make use of $\nabla\times\nabla\theta = 0$ and define

$$\boldsymbol{\varphi} = \nabla\times\mathbf{u}$$ to obtain

Shear wave equation

$$\frac{1}{\beta^2}\partial_t^2\varphi = \Delta\varphi$$

S-wave velocity

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

Elastodynamic Potentials

Any vector may be separated into scalar and vector potentials

$$\mathbf{u} = \nabla\Phi + \nabla \times \Psi$$

where Φ is the potential for P waves and Ψ the potential for shear waves

$$\Rightarrow \theta = \Delta\Phi \qquad \Rightarrow \boldsymbol{\varphi} = \nabla \times \mathbf{u} = \nabla \times \nabla \times \boldsymbol{\Psi} = -\Delta\boldsymbol{\Psi}$$

P-waves have no rotation

Shear waves have no change in volume

$$\frac{1}{\alpha^2}\partial_t^2\theta = \Delta\theta \qquad \frac{1}{\beta^2}\partial_t^2\varphi = \Delta\varphi$$

... what can we say about the direction of displacement, the **polarization** of seismic waves?

$$\mathbf{u} = \nabla\Phi + \nabla \times \Psi \qquad \Rightarrow \mathbf{u} = \mathbf{P} + \mathbf{S}$$

$$\mathbf{P} = \nabla\Phi \qquad \mathbf{S} = \nabla \times \Psi$$

... we now assume that the potentials have the well known form of plane harmonic waves

$$\Phi = A\exp i(\mathbf{k} \bullet \mathbf{x} - \omega t) \qquad \Psi = \mathbf{B}\exp i(\mathbf{k} \bullet \mathbf{x} - \omega t)$$

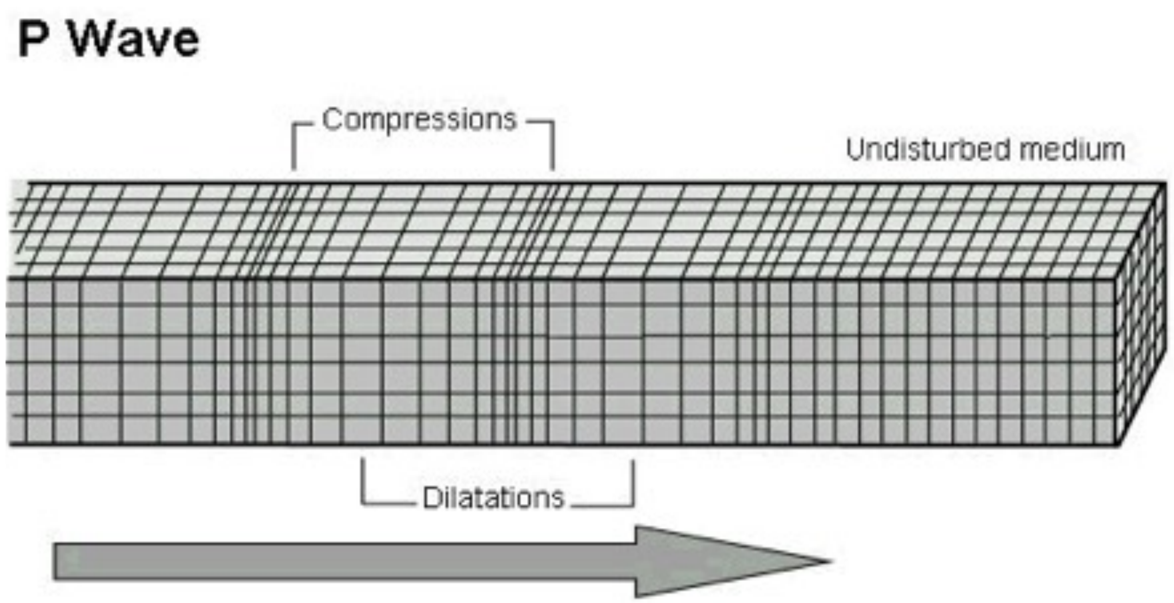
$$\downarrow \qquad \downarrow$$

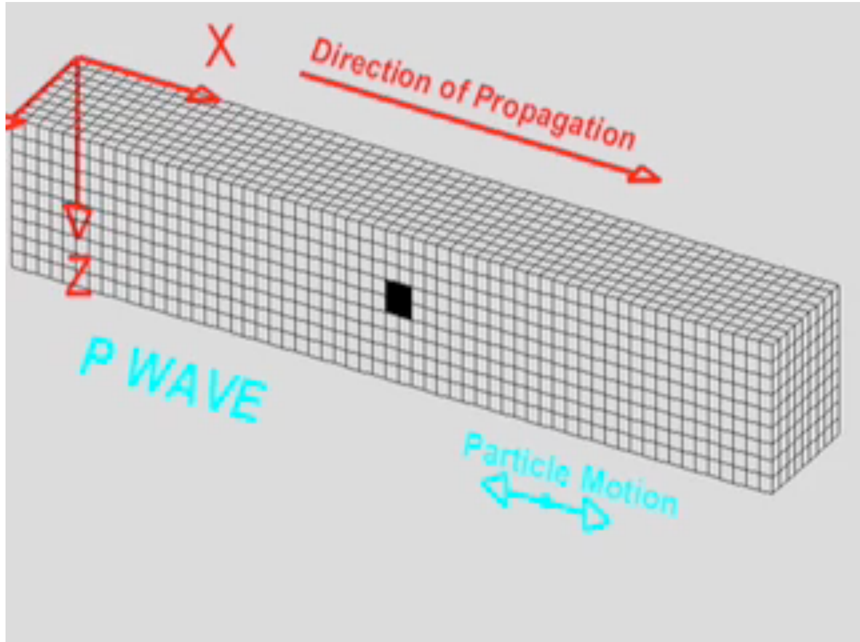
$$\mathbf{P} = A\mathbf{k}\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)] \qquad \mathbf{S} = \mathbf{k} \times \mathbf{B}\exp[i(\mathbf{k} \bullet \mathbf{x} - \omega t)]$$

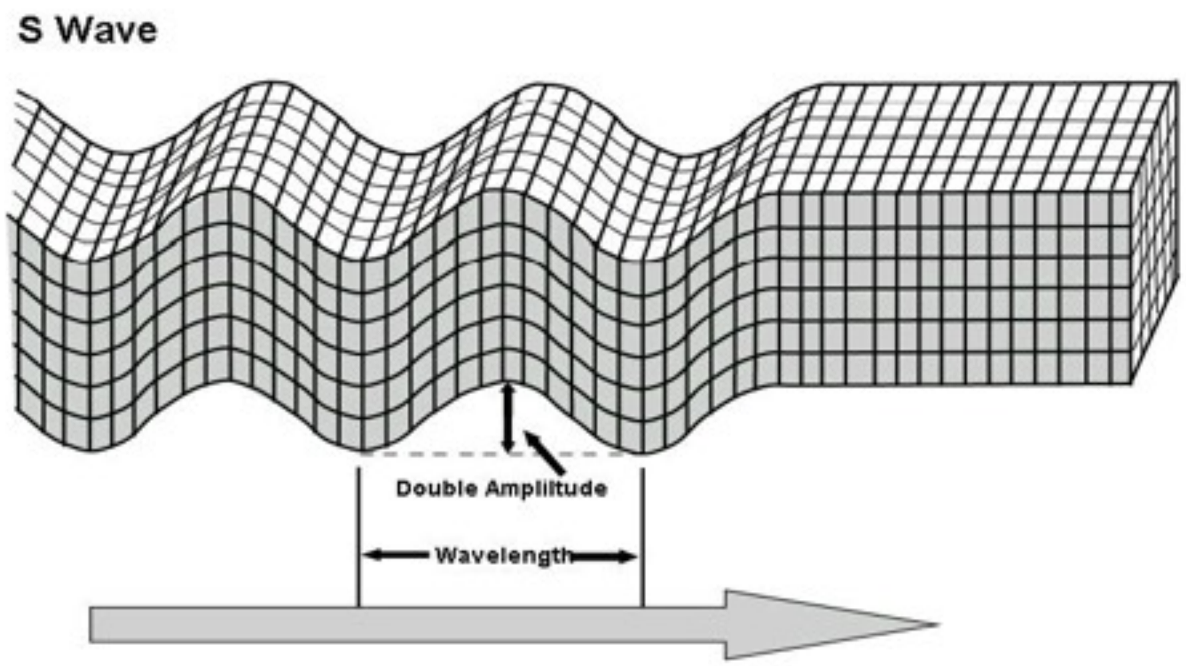
P waves are **longitudinal** as P is parallel to k

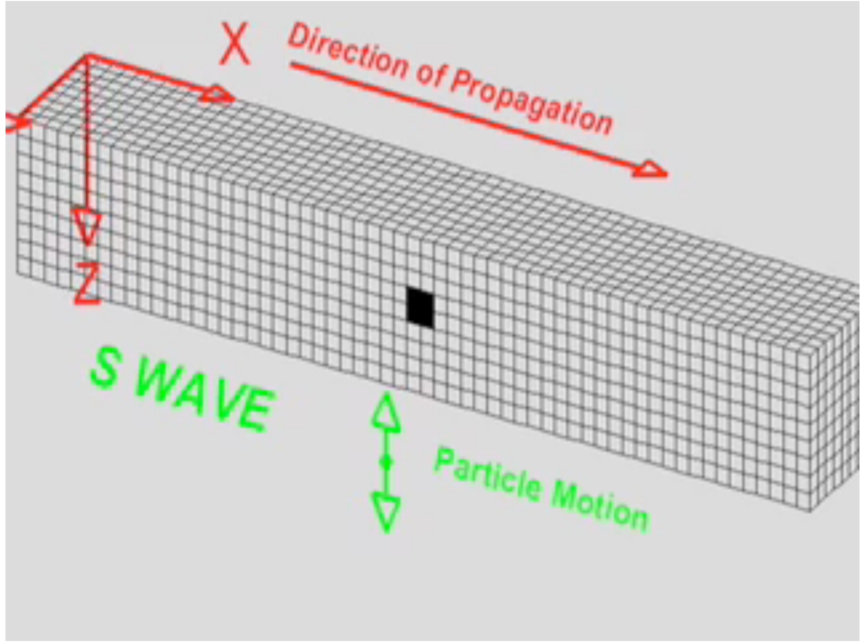
S waves are **transverse** because S is normal to the wave vector k

Wavefields visualization









https://www.iris.edu/hq/inclass/animation/seismic_wave_motions4_waves_animated