SEISMOLOGY

Master Degree Programme in Physics - UNITS
Physics of the Earth and of the Environment

GF FOR 1D HALSFPACE

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Elastodynamic equations

Considering an elastic body of volume V and surface S, the application of body forces, as well as the application of tractions, will generate a displacement field that is constrained to satisfy the equations of motion:

$$\rho \ddot{u}_i = f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = f_i + \sigma_{ij,j}$$

The equation for elastic displacement can be written also using the vector differential operator, $(L(\mathbf{u}))_i = \rho \ddot{u}_i - \left(c_{ijkl} u_{k,l}\right)_{,j} = \rho \ddot{u}_i - \sigma_{ij,j}$ as:

$L(\mathbf{u}) = 0$ homogeneous

$L(\mathbf{u}) = \mathbf{f}$ inhomogeneous

Isotropic medium

And for an isotropic medium, in absence of body forces, the equations of motion become:

$$(\mathbf{L}(\mathbf{u}))_i = \rho\ddot{\mathbf{u}}_i - \frac{\partial}{\partial_j}\left(\lambda\partial_k \mathbf{u}_k \delta_{ij} + \mu(\partial_i \mathbf{u}_j + \partial_j \mathbf{u}_i)\right) = 0$$

i.e. a linear system of three differential equations with three unknowns: the components of the displacement vector, whose coefficients depend upon the elastic parameters of the material. It is not possible to find the analytic solution for this system of equations, therefore it is necessary to add further approximations, chosen according to the adopted resolving method. Two ways can be followed:

a) an **exact definition of the medium is given**, and a **direct numerical integration technique** is used to solve the set of differential equations;

b) **exact analytical techniques** are applied to an **approximated model** of the medium that may have the elastic parameters varying along one or more directions of heterogeneity.

1D heterogeneity

- Let us consider a halfspace in a system of Cartesian coordinates with the vertical z axis positive downward and the **free surface**, where vertical stresses (σ_{xz}, σ_{yz}, σ_{zz}) are null, is defined by the plane z=0.

- Let us assume that ρ, λ and μ are piecewise continuous functions of z, that displacement and stress components are continuous along z, and that body wave velocities, α and β, assume their largest value, α_H and β_H, when z=H, remaining constant for greater depths.

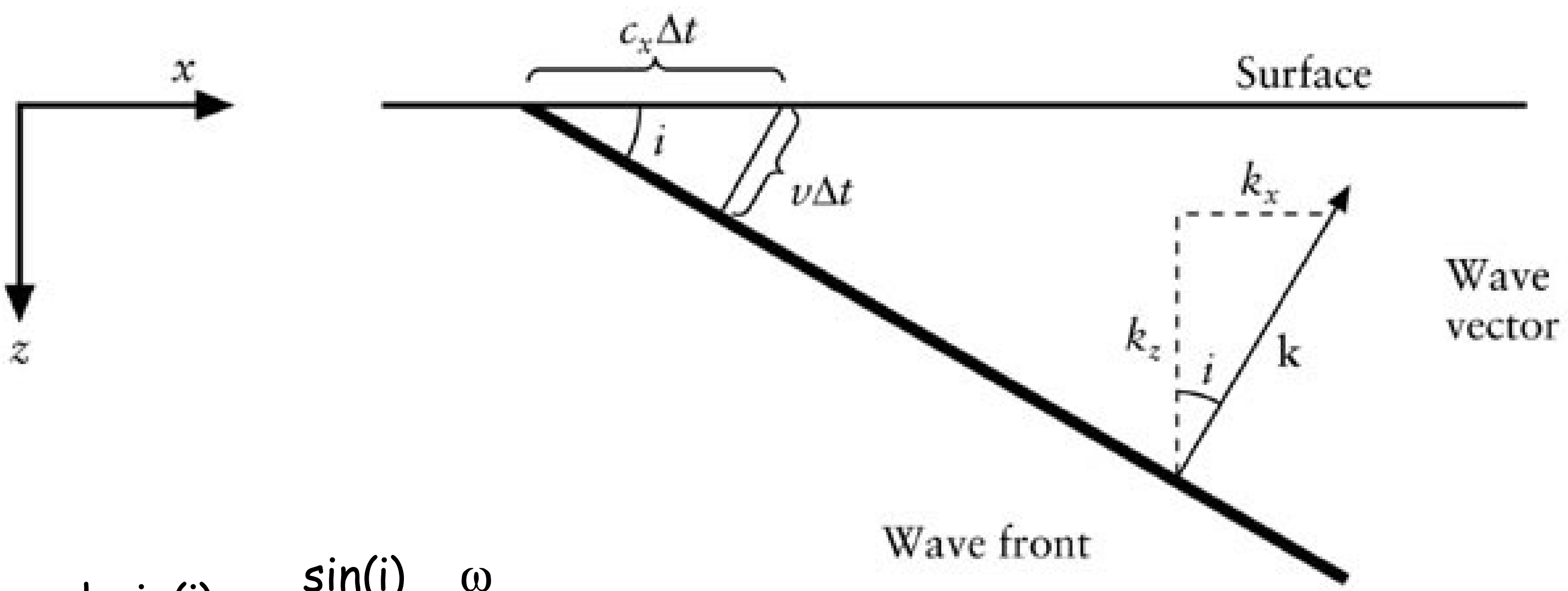
If the parameters depend only upon the vertical coordinate, the equations become:

$$\rho\ddot{\mathbf{u}} = (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} + \frac{\partial\lambda}{\partial z}(\hat{\mathbf{z}}\nabla\cdot\mathbf{u}) + \frac{\partial\mu}{\partial z}\left[(\nabla\cdot\hat{\mathbf{z}})\mathbf{u} + \nabla(\hat{\mathbf{z}}\cdot\mathbf{u})\right]$$

we can consider solutions of having the form of plane harmonic waves propagating along the positive x axis:

$$\mathbf{u}(\mathbf{x},t) = \mathbf{F}(z)e^{i(\omega t - kx)}$$

Apparent horizontal (phase) velocity



$$k_x = k\sin(i) = \omega\frac{\sin(i)}{\alpha} = \frac{\omega}{c}$$

$$k_z = k\cos(i) = \sqrt{k^2 - k_x^2} = \omega\sqrt{\left(\frac{1}{\alpha}\right)^2 - \left(\frac{1}{c}\right)^2} = \frac{\omega}{c}\sqrt{\left(\frac{c}{\alpha}\right)^2 - 1} = k_x r_\alpha$$

$$k_x = k\sin(i) = \omega\frac{\sin(i)}{\beta} = \frac{\omega}{c}$$

$$k_z = k\cos(i) = \sqrt{k^2 - k_x^2} = \omega\sqrt{\left(\frac{1}{\beta}\right)^2 - \left(\frac{1}{c}\right)^2} = \frac{\omega}{c}\sqrt{\left(\frac{c}{\beta}\right)^2 - 1} = k_x r_\beta$$

Remember: when c is less then the body wave velocity k_z is imaginary and represent **inhomogeneous** waves, i.e. waves exponentially **decaying** or increasing with depth; examples are Rayleigh waves in a homogenous halfspace, or Love waves in low velocity layer over a homogeneous halfspace

In current terminology, k_x is k

P-SV problem

We have to solve two independent eigenvalue problems for the three components of the vector **F**=(F_x,F_y,F_z). The first one describes the motion in the plane **(x,z)**, i.e., **P-SV** waves and it has the form:

$$\frac{\partial}{\partial z}\left[\mu\frac{\partial F_x}{\partial z} - ik\mu F_z\right] - ik\lambda\frac{\partial F_z}{\partial z} + \left[\omega^2\rho - k^2(\lambda+2\mu)\right]F_x = 0$$

$$\frac{\partial}{\partial z}\left[(\lambda+2\mu)\frac{\partial F_z}{\partial z} - ik\lambda F_x\right] - ik\mu\frac{\partial F_x}{\partial z} + \left[\omega^2\rho - k^2\mu\right]F_z = 0$$

and must be solved with the free surface boundary condition at z = 0

$$\sigma_{zz} = \left[(\lambda+2\mu)\frac{\partial F_z}{\partial z} - ik\lambda F_x\right]_{z=0} = 0$$

$$\sigma_{xz} = \left[\mu\frac{\partial F_z}{\partial z} - ik\mu F_z\right]_{z=0} = 0$$

SH problem

The second eigenvalue problem describes the case when the particle motion is limited to the **y-axis**, and determines phase velocity and amplitude of **SH waves**. It has the (Sturm-Liouville) form:

$$\frac{\partial}{\partial z}\left(\mu \frac{\partial F_y}{\partial z}\right) + \left(\omega^2 \rho - k^2 \mu\right) F_y = 0$$

and must be solved with the free surface boundary condition at z = 0

$$\left[\mu \frac{\partial F_y}{\partial z}\right]_{z=0} = 0$$

Layered halfspace

Let us now assume that the vertical heterogeneity in the halfspace is modelled with **a series of N-1 homogeneous flat layers**, parallel to the free surface, overlying a homogeneous halfspace.

Let ρ_m, α_m, β_m, and d_m, respectively be the density, P-wave and S-wave velocities, and the thickness of the m-th layer.

Furthermore, let us define:

$$
r_{\alpha m} = \begin{cases} \sqrt{\left(\dfrac{c}{\alpha_m}\right)^2 - 1} & \text{if } c > \alpha_m \\ -i\sqrt{1 - \left(\dfrac{c}{\alpha_m}\right)^2} & \text{if } c < \alpha_m \end{cases}
\qquad
r_{\beta m} = \begin{cases} \sqrt{\left(\dfrac{c}{\beta_m}\right)^2 - 1} & \text{if } c > \beta_m \\ -i\sqrt{1 - \left(\dfrac{c}{\beta_m}\right)^2} & \text{if } c < \beta_m \end{cases}
$$

Love (SH) problem

The SH solutions (displacement and stress) for the m-th layer are:

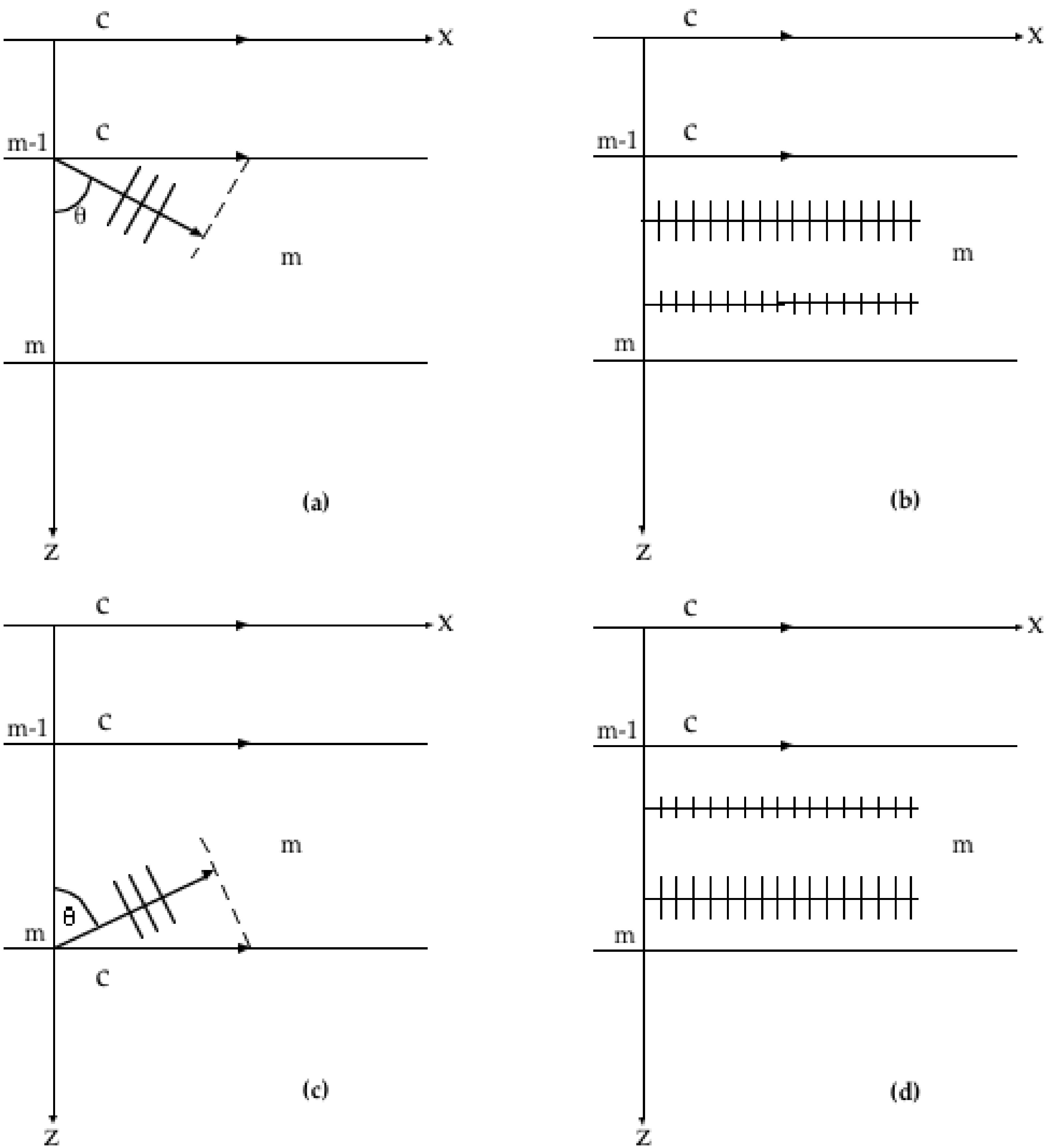
$u_x = u_z = 0$

$$u_y = \left(v'_m e^{-ikr_{\beta m}z} + v''_m e^{+ikr_{\beta m}z}\right) e^{i(\omega t - kx)}$$

$$\sigma_{zy} = \mu \frac{\partial u_y}{\partial z} = ik\mu r_{\beta m}\left(-v'_m e^{-ikr_{\beta m}z} + v''_m e^{+ikr_{\beta m}z}\right) e^{i(wt - kx)}$$

where v_m' and v_m'' are constants.
Given the sign conventions adopted, the term in v' represents a plane wave whose direction of propagation makes an angle $\cot^{-1} r_{\beta m}$ with the +z direction when $r_{\beta m}$ is real, and a wave propagating in the +x direction with amplitude diminishing exponentially in the +z direction when $r_{\beta m}$ is imaginary. Similarly the term in v'' represents a plane wave making the same angle with the direction -z when $r_{\beta m}$ is real and a wave propagating in the +x direction with amplitude increasing in the +z direction when $r_{\beta m}$ is imaginary.

Love (SH) problem



the term in v' represents a plane wave whose direction of propagation makes an angle $\cot^{-1} r_{\beta m}$ with the +z direction when $r_{\beta m}$ is real (a),
and a wave propagating in the +x direction with amplitude diminishing exponentially in the +z direction when $r_{\beta m}$ is imaginary (b).

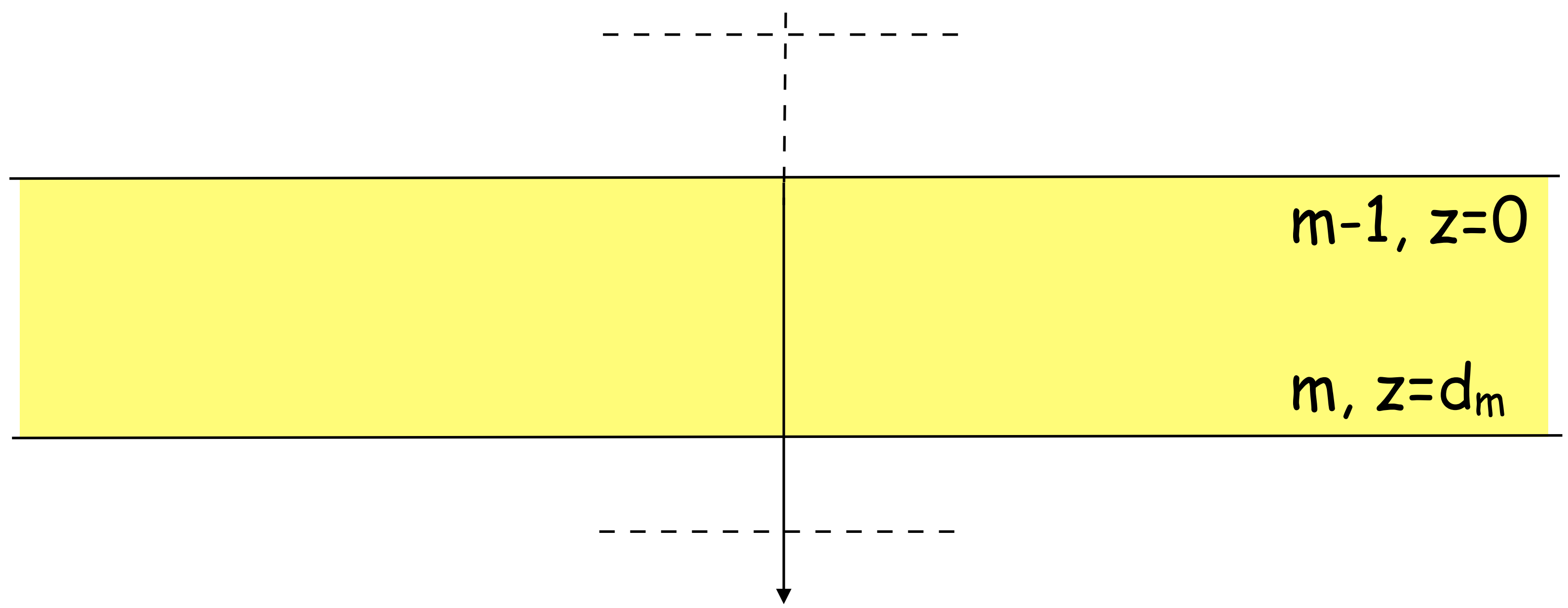
Similarly the term in v'' represents a plane wave making the same angle with the direction -z when $r_{\beta m}$ is real (c)
and a wave propagating in the +x direction with amplitude increasing in the +z direction
when $r_{\beta m}$ is imaginary d).

Love (SH) problem

Consider the m-th layer and the (m-1) interface, set temporarily as the origin of the coordinate system. It is convenient to use $[(du_y/dt)/c]=iku_y$ instead of displacement, to deal with adimensional quantities.

$$\left(\frac{\dot{u}_y}{c}\right)_{m-1} = ik(v'_m + v''_m)$$

$$\left(\sigma_{zy}\right)_{m-1} = ik\mu_m r_{\beta_m}(v''_m - v'_m)$$



$$\left(\frac{\dot{u}_y}{c}\right)_{m} = ik(v'_m + v''_m)\cos Q_m - k(v''_m - v'_m)\sin Q_m$$

$Q_m = k r_{\beta m} d_m$

$$\left(\sigma_{zy}\right)_{m} = -k\mu_m r_{\beta_m}(v''_m + v'_m)\sin Q_m + ik\mu_m r_{\beta_m}(v''_m - v'_m)\cos Q_m$$

Love layer matrix

$$\left(\frac{\dot{u}_y}{c}\right)_m = \left(\frac{\dot{u}_y}{c}\right)_{m-1} \cos Q_m + i\left(\sigma_{zy}\right)_{m-1}\left(\mu_m r_{\beta_m}\right)^{-1} \sin Q_m$$

$$\left(\sigma_{zy}\right)_m = \left(\frac{\dot{u}_y}{c}\right)_{m-1} i\,\mu_m r_{\beta_m} \sin Q_m + \left(\sigma_{zy}\right)_{m-1} \cos Q_m$$

$$a_m = \begin{bmatrix} \cos Q_m & \dfrac{i \sin Q_m}{\mu_m r_{\beta_m}} \\ i\,\mu_m r_{\beta_m} \sin Q_m & \cos Q_m \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{\dot{u}_y}{c}\right)_m \\ \left(\sigma_{zy}\right)_m \end{bmatrix} = a_m \begin{bmatrix} \left(\frac{\dot{u}_y}{c}\right)_{m-1} \\ \left(\sigma_{zy}\right)_{m-1} \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{\dot{u}_y}{c}\right)_{N-1} \\ \left(\sigma_{zy}\right)_{N-1} \end{bmatrix} = A \begin{bmatrix} \left(\frac{\dot{u}_y}{c}\right)_0 \\ \left(\sigma_{zy}\right)_0 \end{bmatrix}$$

$$A = a_{N-1} a_{N-2} \ldots a_2 a_1$$

Love dispersion equation

remembering that the boundary conditions of a) surface waves and b) the free surface implies that $v_N''=0$ and $\sigma_{zy}(z=0)=0$, we have that:

$$A_{21} + \mu_N r_{\beta_N} A_{11} = 0$$

The left-hand side is the **dispersion function** for Love modes (SH waves), where A_{21} and A_{11} are elements of the matrix A.
The couples (ω,c) for which the dispersion function is equal to zero are its roots and represent the **eigenvalues** of the problem.

Eigenvalues, according to the number of zeroes of the corresponding **eigenfunctions**, $u_y(z,\omega,c)$ and $\sigma_{zy}(z,\omega,c)$,
can be subdivided in the **dispersion curve** of the fundamental mode (which has no nodal planes), of the first higher mode (having one nodal plane), of the second higher mode and so on.

Once the phase velocity c is determined, we can compute analytically the **group velocity** using the implicit functions theory, and the eigenfunctions.

Rayleigh (P-SV) problem

The P-SV solutions (displacement and stress) for the m-th layer can be found combining dilatational and rotational potentials:

$$\Delta_m = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = \left(\Delta_m^{'} e^{-ikr_{\alpha m}z} + \Delta_m^{''} e^{+ikr_{\alpha m}z}\right) e^{i(\omega t - kx)}$$

$$\delta_m = \frac{1}{2}\left[\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right] = \left(\delta_m^{'} e^{-ikr_{\beta m}z} + \delta_m^{''} e^{+ikr_{\beta m}z}\right) e^{i(\omega t - kx)}$$

where Δ_m', Δ_m'', δ_m' and δ_m'' are constants.
Given the sign conventions adopted, the term in Δ_m' represents a plane wave whose direction of propagation makes an angle $\cot^{-1} r_{am}$ with the +z direction when r_{am} is real, and a wave propagating in the +x direction with amplitude diminishing exponentially in the +z direction when r_{am} is imaginary. Similarly the term in Δ_m'' represents a plane wave making the same angle with the direction -z when r_{am} is real and a wave propagating in the +x direction with amplitude increasing in the +z direction when r_{am} is imaginary.
The same considerations can be applied to the terms in δ_m' and δ_m'', substituting r_{am} with $r_{\beta m}$.

Rayleigh (P-SV) problem

The P-SV solutions (displacement and stress) components can be written as:

$$u_x = -\frac{\alpha_m^2}{\omega^2}\left(\frac{\partial \Delta_m}{\partial x}\right) - 2\frac{\beta_m^2}{\omega^2}\left(\frac{\partial \delta_m}{\partial z}\right)$$

$$u_z = -\frac{\alpha_m^2}{\omega^2}\left(\frac{\partial \Delta_m}{\partial z}\right) + 2\frac{\beta_m^2}{\omega^2}\left(\frac{\partial \delta_m}{\partial x}\right)$$

$$\sigma_{zz} = \rho_m\left\{\alpha_m^2\Delta_m + 2\beta_m^2\left[\frac{\alpha_m^2}{\omega^2}\left(\frac{\partial^2 \Delta_m}{\partial x^2}\right) + 2\frac{\beta_m^2}{\omega^2}\left(\frac{\partial^2 \delta_m}{\partial z^2}\right)\right]\right\}$$

$$\sigma_{zx} = 2\beta_m^2\rho_m\left\{-\frac{\alpha_m^2}{\omega^2}\left(\frac{\partial^2 \Delta_m}{\partial x \partial z}\right) + \frac{\beta_m^2}{\omega^2}\left[\left(\frac{\partial^2 \delta_m}{\partial x^2}\right) - \left(\frac{\partial^2 \delta_m}{\partial z^2}\right)\right]\right\}$$

Starting with the free surface condition ($\sigma_{zz}(z=0)=\sigma_{zx}(z=0)=0$), iterating the continuity boundary conditions at every interface, and applying the condition of no radiation in the final halfspace, one can build up the **dispersion function whose roots are the eigenvalues associated with the Rayleigh modes**.

GF for heterogeneous halfspace

The GF, at a large distance, will consist entirely of **surface waves** propagating outward from the source:

$$G_{ik}^{L,R} = \sum_{m=1}^{\infty} G_{ik}^{L,Rm}(\mathbf{x},\mathbf{x}_0;t)$$

for **Love** (L) modes (m):

$$G_{ik}^{mL}(\omega) = \frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}} \frac{e^{-ik_m x}}{\sqrt{x}} \frac{\left(RP_{ik}^{L}(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}} \frac{\left(RC_{ik}^{L}(z,\omega)\right)}{\sqrt{v_m I_m}}$$

propagation source receiver

and **Rayleigh** (R) modes (m):

$$G_{ik}^{mR}(\omega) = \frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}} \frac{e^{-ik_m x}}{\sqrt{x}} \frac{\left(RP_{ik}^{R}(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}} \frac{\left(RC_{ik}^{R}(z,\omega)\right)}{\sqrt{v_m I_m}}$$

RP for heterogeneous halfspace

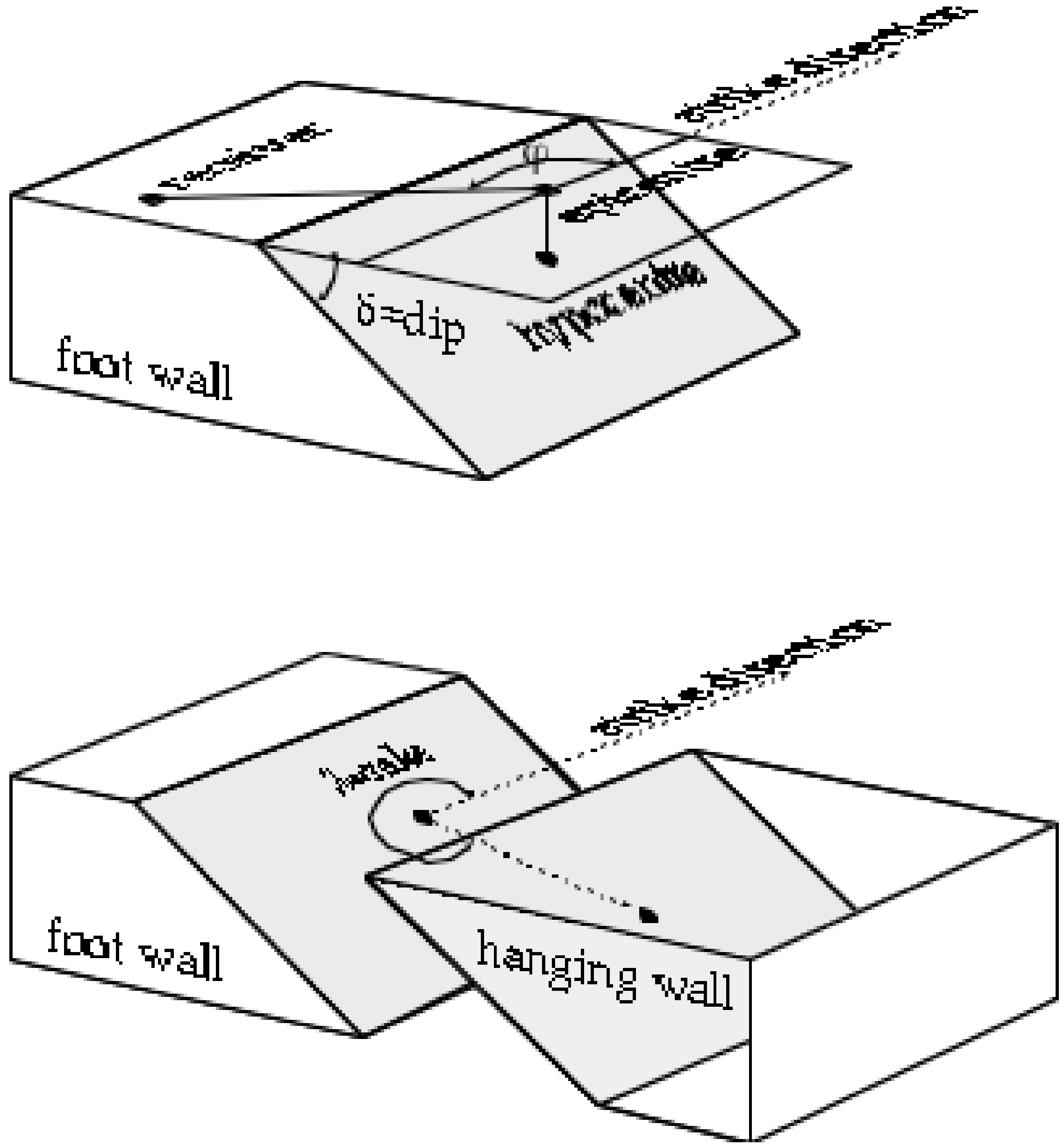
where **x**, is the source-receiver distance; **c** is the phase velocity, **v** is the group velocity (c, v are calculated for the m-th Love or Rayleigh mode, at frequency ω, and thus are the "**eigenvalues**"); **I** is the energy integral, **RP** is the radiation pattern and **RC** is the receiver factor (calculated for the m-th Love or Rayleigh mode, at frequency ω, and thus are connected to the "**eigenvectors**" (F_x,F_y.F_z)):

$$RP_{ik}^{mL}RC_{ik}^{mL} = F_y^m(h_s,\omega)\begin{pmatrix} \sin^2\phi & -\sin\phi\cos\phi & 0 \\ -\sin\phi\cos\phi & \cos^2\phi & 0 \\ 0 & 0 & 0 \end{pmatrix} F_y^m(z,\omega)$$

$$RP_{ik}^{mR}RC_{ik}^{mR} = \begin{pmatrix} F_x^m(h_s,\omega)F_x^m(z,\omega)\cos^2\phi & F_x^m(h_s,\omega)F_x^m(z,\omega)\sin\phi\cos\phi & -iF_z^m(h_s,\omega)F_x^m(z,\omega)\cos\phi \\ F_x^m(h_s,\omega)F_x^m(z,\omega)\sin\phi\cos\phi & F_x^m(h_s,\omega)F_x^m(z,\omega)\sin^2\phi & -iF_z^m(h_s,\omega)F_x^m(z,\omega)\sin\phi \\ iF_x^m(h_s,\omega)F_z^m(z,\omega)\cos\phi & iF_x^m(h_s,\omega)F_z^m(z,\omega)\sin\phi & F_z^m(h_s,\omega)F_z^m(z,\omega) \end{pmatrix}$$

Seismic source in a layered halfspace

The source is introduced in the medium by representing the (planar) fault



as a discontinuity in the displacement field (shear dislocation), and thus it is equivalent to a double-couple.

GF for DC in heterogeneous halfspace

If the **surface waves** are excited by a double-couple, and we are in the far-field:

for **Love** (L) modes (m):

$$u_y^L(x,z,\omega) = \sum_{m=1}^{\infty} \frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}} \frac{e^{-ik_m x}}{\sqrt{x}} \frac{\left(\chi_m^L(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}} \frac{\left(F_y(z,\omega)\right)}{\sqrt{v_m I_m}}$$

and **Rayleigh** (R) modes (m):

$$u_x^R(x,z,\omega) = \sum_{m=1}^{\infty} \frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}} \frac{e^{-ik_m x}}{\sqrt{x}} \frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}} \frac{\left(F_x(z,\omega)\right)}{\sqrt{v_m I_m}}$$

$$u_z^R(x,z,\omega) = \sum_{m=1}^{\infty} \frac{e^{-i\pi/4}}{\sqrt{8\pi\omega}} \frac{e^{-ik_m x}}{\sqrt{x}} \frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}} \frac{\left(F_z(z,\omega)\right)}{\sqrt{v_m I_m}}$$

RP for DC in heterogeneous halfspace

where, χ, the radiation pattern represents the azimuthal dependence of the excitation factor:

$$\chi_L = i(d_{1L}\sin\varphi + d_{2L}\cos\varphi) + d_{3L}\sin 2\varphi + d_{4L}\cos 2\varphi$$
$$\chi_R = d_0 + i(d_{1R}\sin\varphi + d_{2R}\cos\varphi) + d_{3R}\sin 2\varphi + d_{4R}\cos 2\varphi$$

$$d_{1L} = G(h_s)\cos\lambda\sin\delta$$
$$d_{2L} = -G(h_s)\sin\lambda\cos 2\delta$$
$$d_{3L} = \frac{1}{2}V(h_s)\sin\lambda\sin 2\delta$$
$$d_{4L} = V(h_s)\cos\lambda\sin\delta$$

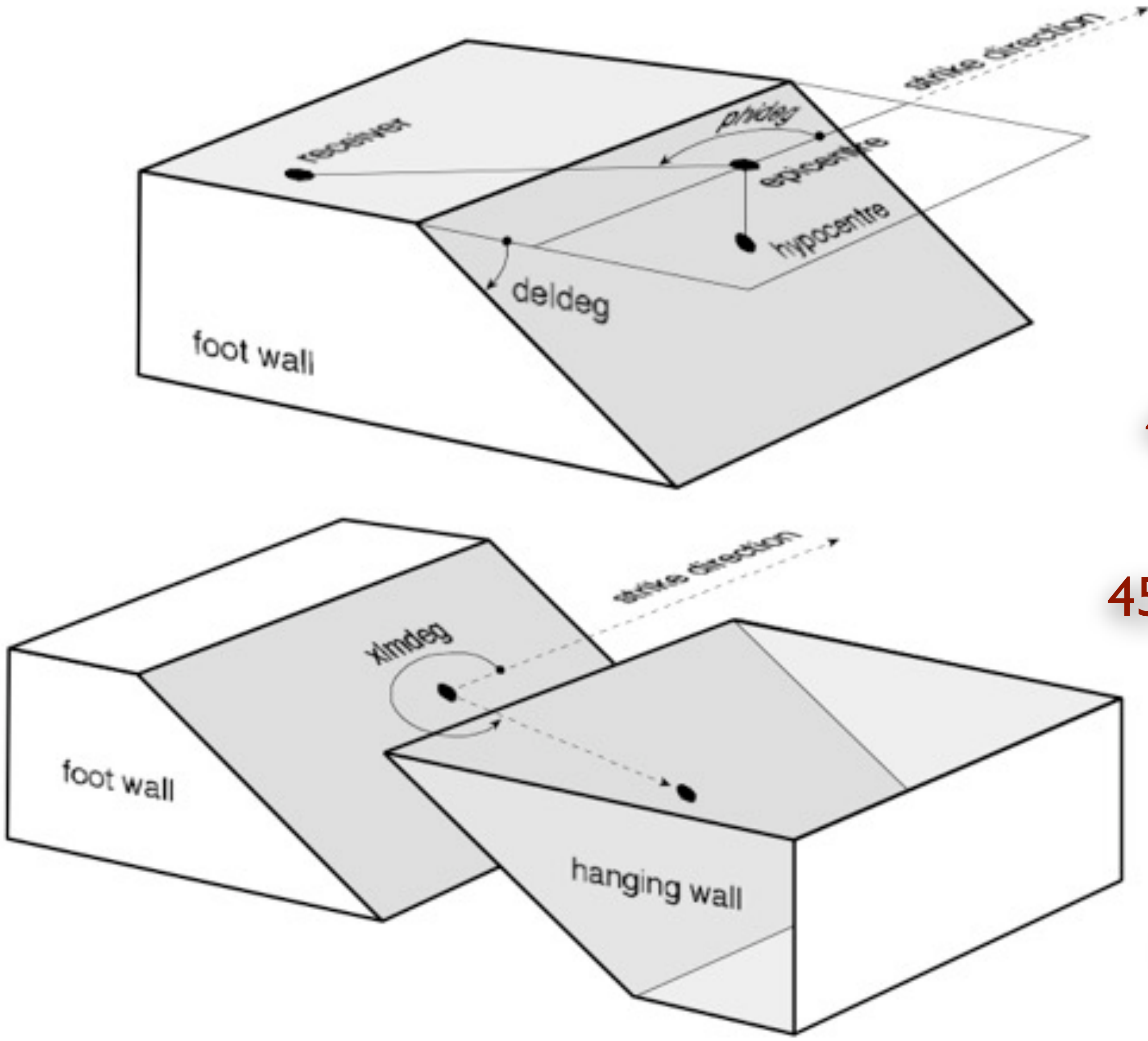
where φ is the angle between the strike of the fault and the direction obtained connecting the epicenter with the station, measured anticlockwise, δ is the dip angle and λ is the rake angle, and

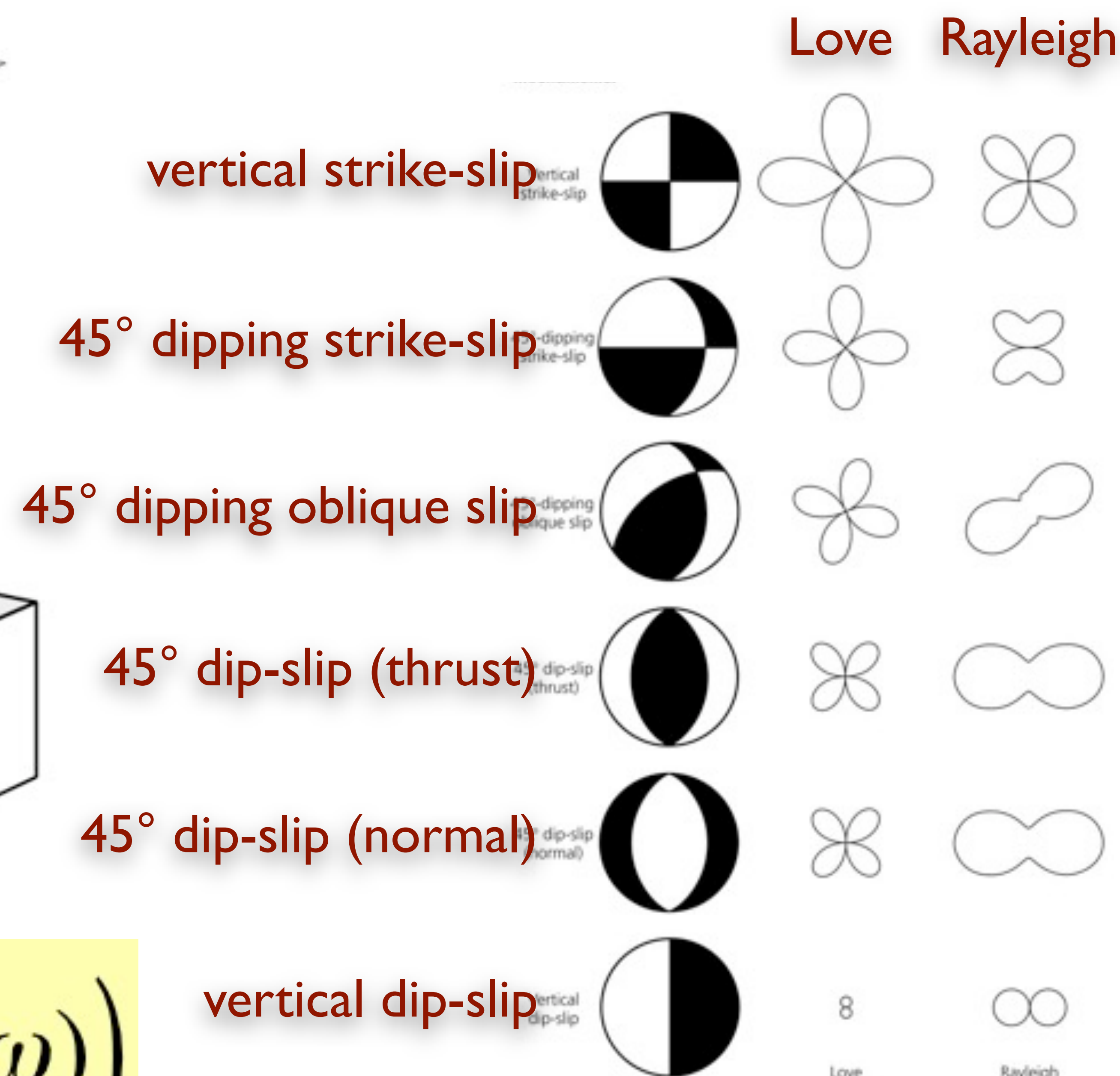
$$d_0 = \frac{1}{2}B(h_s)\sin\lambda\sin 2\delta$$
$$d_{1R} = -C(h_s)\sin\lambda\cos 2\delta$$
$$d_{2R} = -C(h_s)\cos\lambda\cos\delta$$
$$d_{3R} = A(h_s)\cos\lambda\sin\delta$$
$$d_{4R} = -\frac{1}{2}A(h_s)\sin\lambda\sin 2\delta$$

$$A(h_s) = -\frac{F_x^*(h_s)}{F_z(0)}$$
$$B(h_s) = -\left(3 - 4\frac{\beta^2(h_s)}{\alpha^2(h_s)}\right)\frac{F_x^*(h_s)}{F_z(0)} - \frac{2}{\rho(h_s)\,\alpha^2(h_s)}\frac{\sigma_{zz}^*(h_s)}{F_z(0)/c}$$
$$C(h_s) = -\frac{1}{\mu(h_s)}\frac{\sigma_{zx}(h_s)}{F_z(0)/c}$$
$$G(h_s) = -\frac{1}{\mu(h_s)}\frac{\sigma_{zy}^*(h_s)}{F_y(0)/c}$$
$$V(h_s) = \frac{\dot{F}_y(h_s)}{F_y(0)/c} = \frac{F_y(h_s)}{F_y(0)/c}$$

$$\left(\chi_m^L(h_s,\omega)\right) \qquad \left(\chi_m^R(h_s,\omega)\right)$$

Double couple RP & surface waves



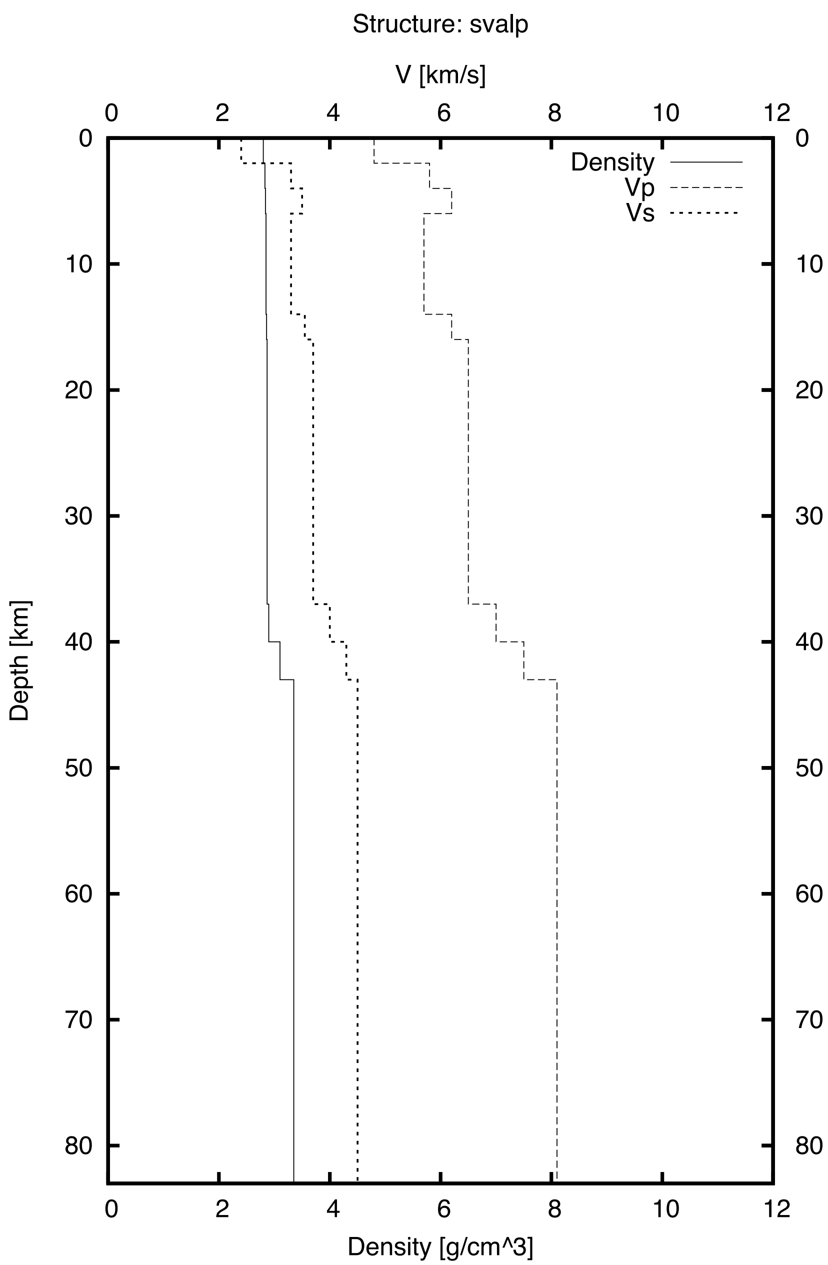


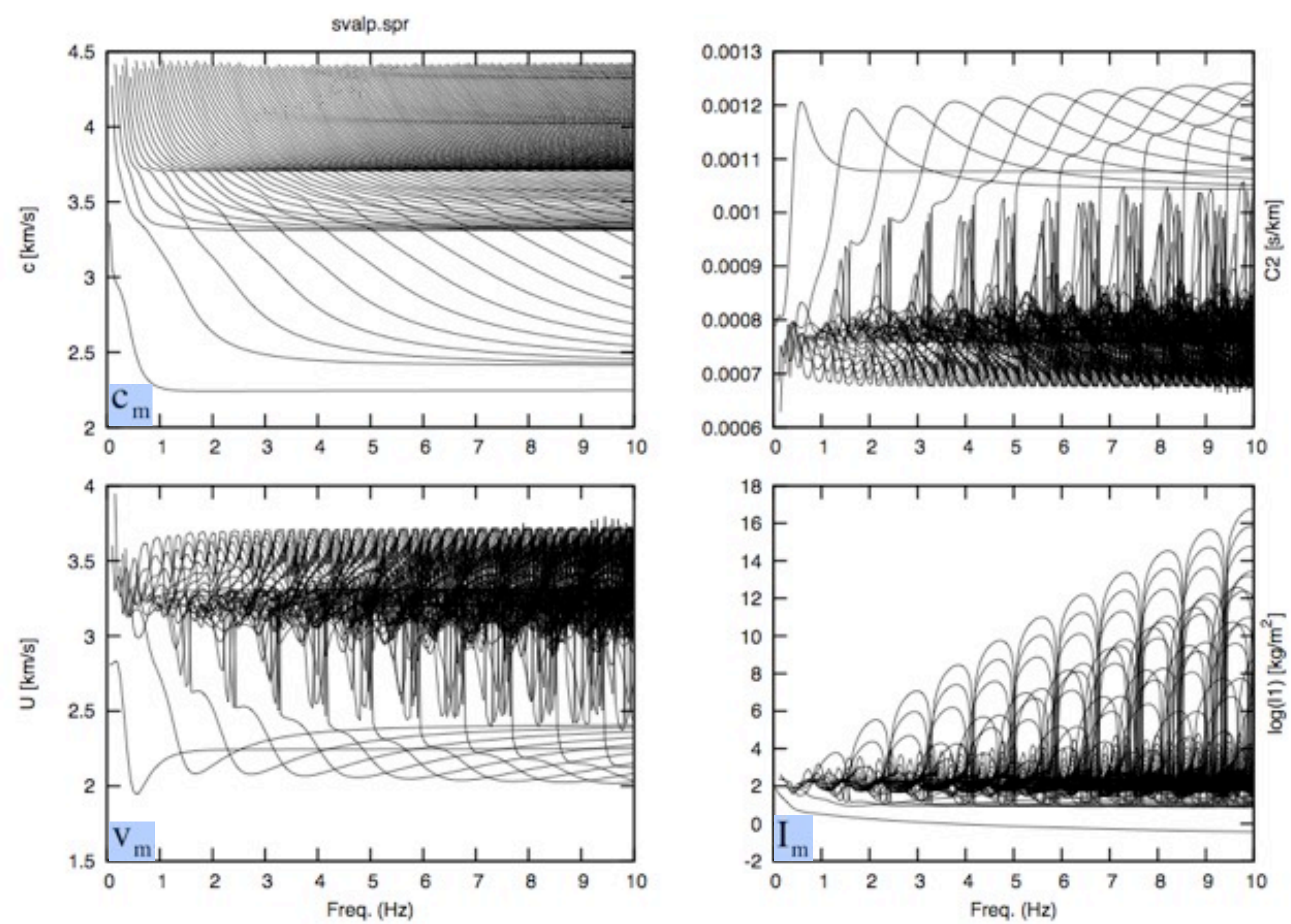
$$\left(\chi_m^L(h_s,\omega)\right) \qquad \left(\chi_m^R(h_s,\omega)\right)$$

Methodology - Modal Summation Technique

- Example of quantities associated with a structure

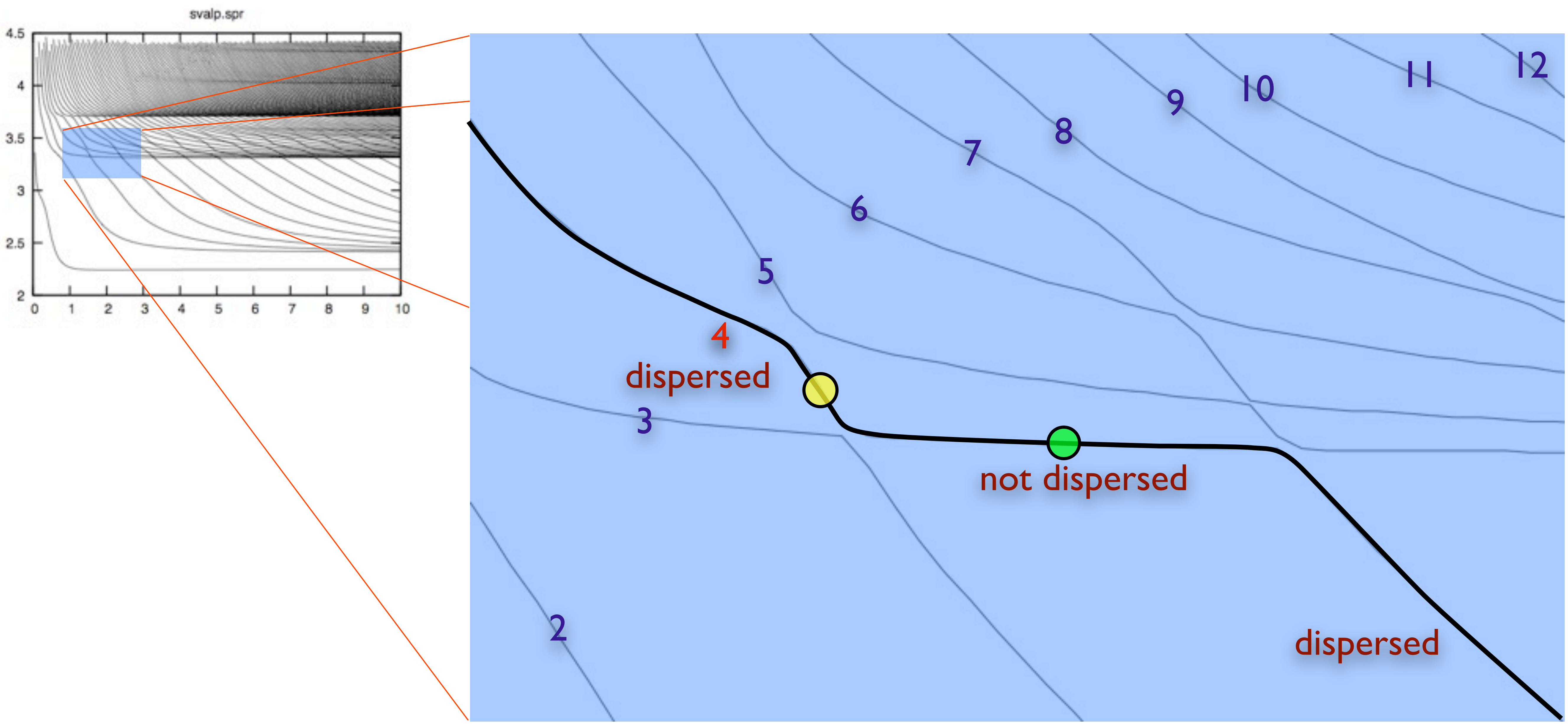
$$\sqrt{c_m v_m I_m} \qquad \sqrt{v_m I_m}$$





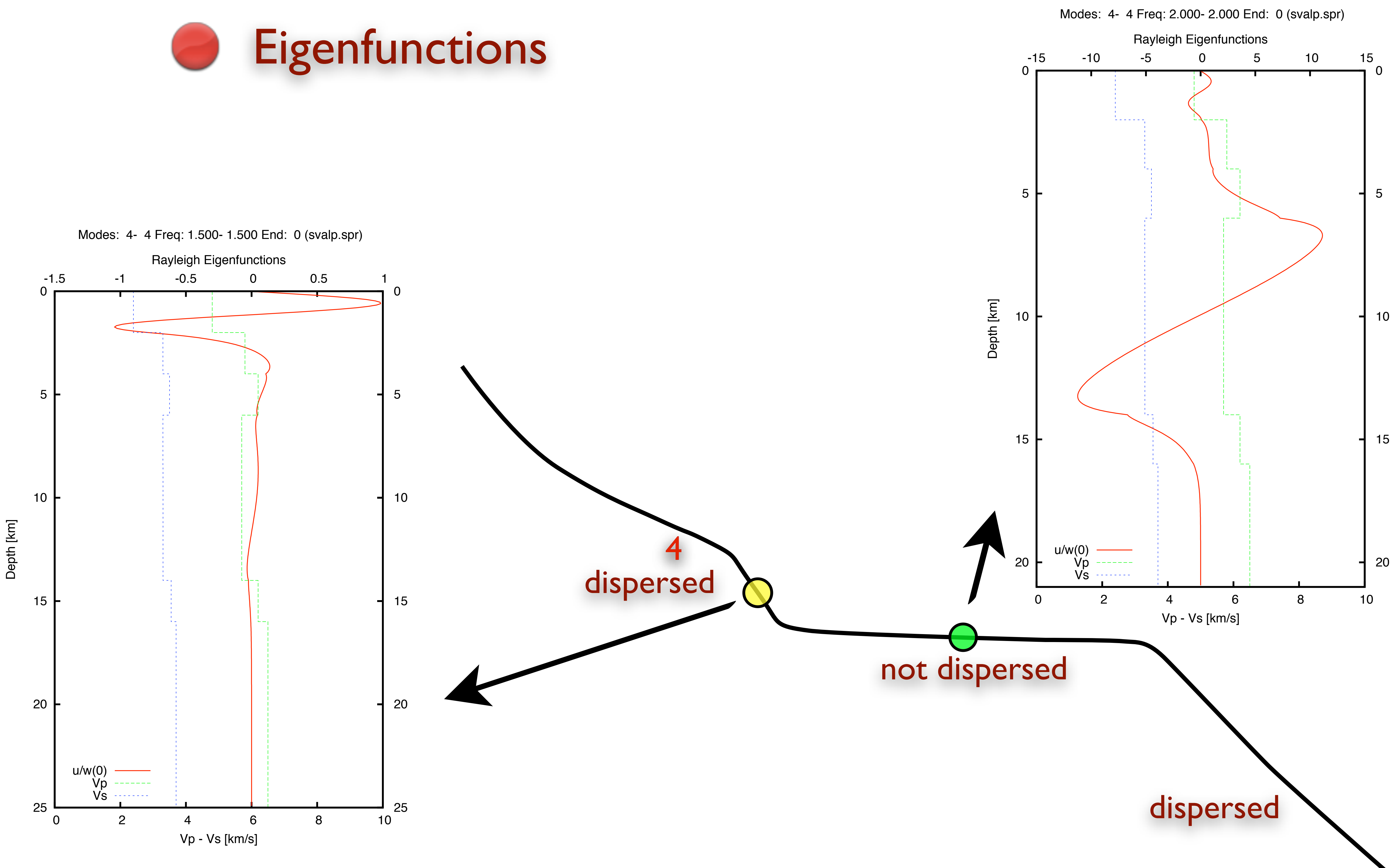
Methodology - Modal Summation Technique

- Phase velocity dispersion curve



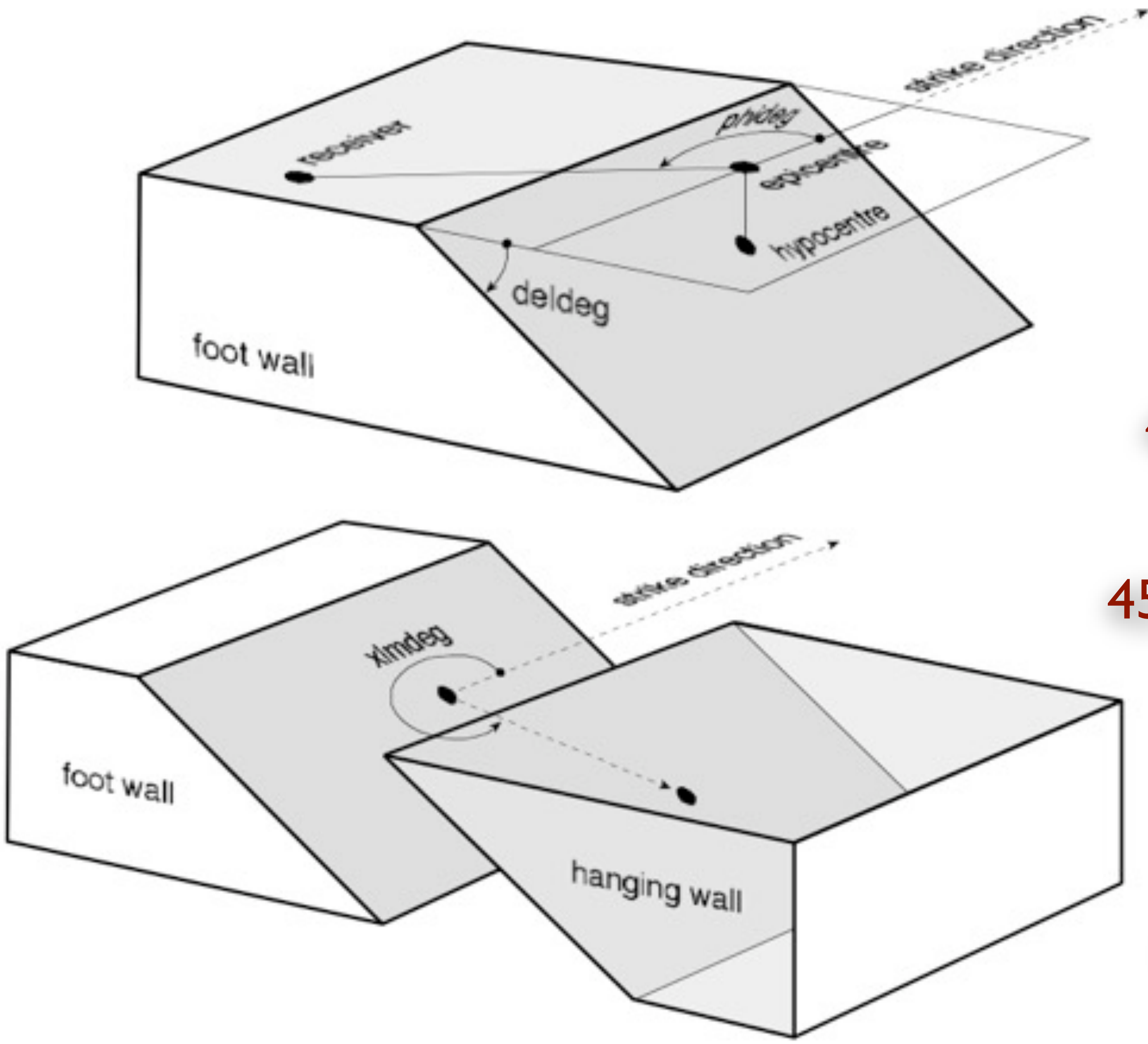
Methodology - Modal Summation Technique

- Eigenfunctions



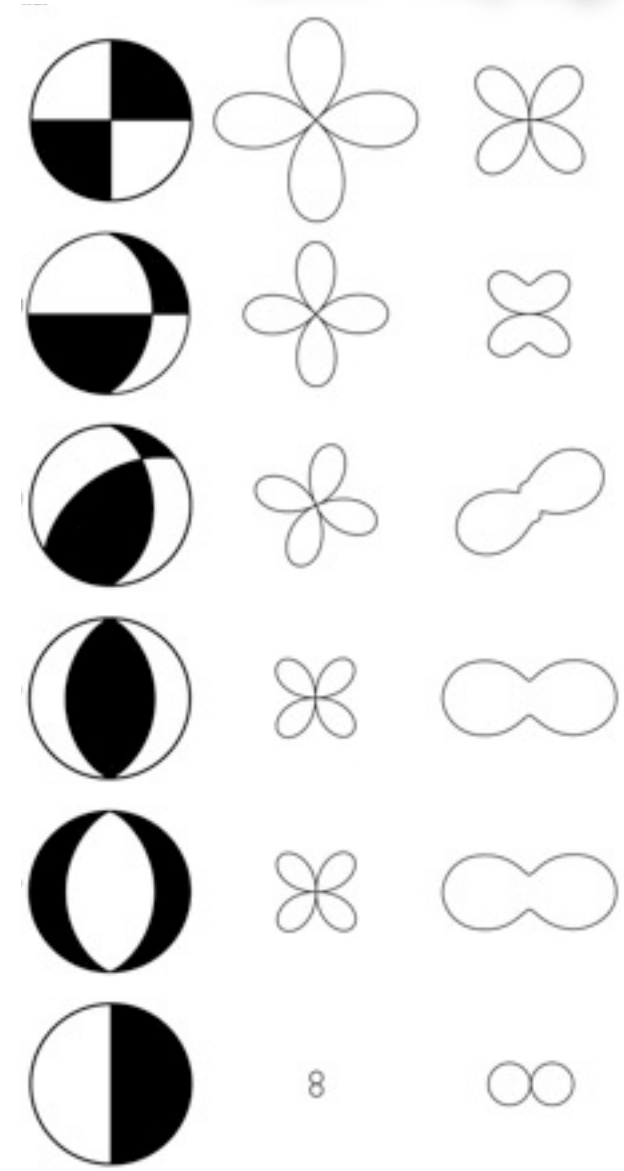
Methodology - Modal summation

- Source definition and radiation pattern



$$\left(\chi_m^L(h_s,\omega)\right) \quad \left(\chi_m^R(h_s,\omega)\right)$$

	Love	Rayleigh
vertical strike-slip		
45° dipping strike-slip		
45° dipping oblique slip		
45° dip-slip (thrust)		
45° dip-slip (normal)		
vertical dip-slip		



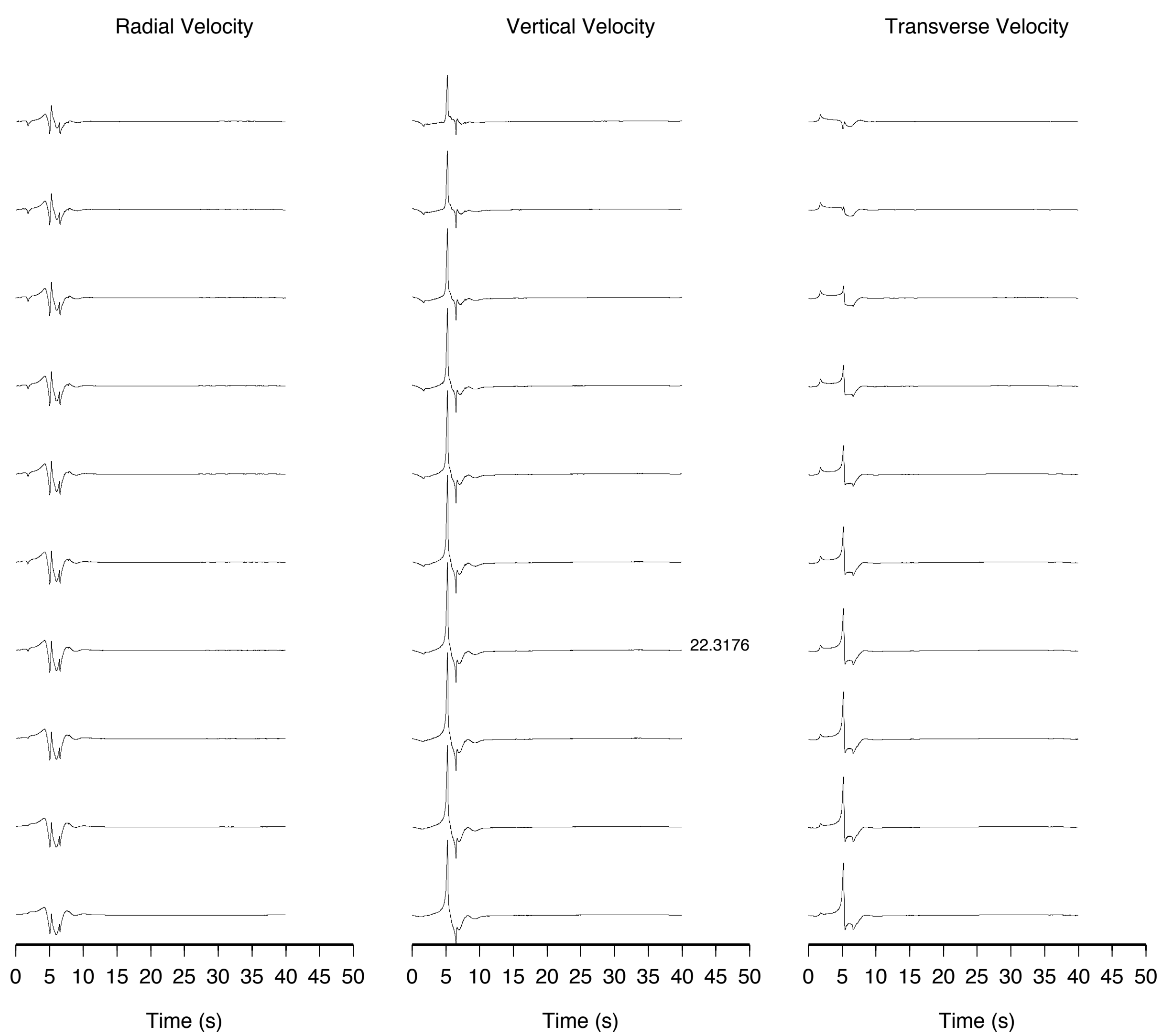
Methodology - Modal Summation Technique

- Synthetic seismograms

$$u_y^L(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^L(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_y(z,\omega)\right)}{\sqrt{v_m I_m}}$$

$$u_x^R(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_x(z,\omega)\right)}{\sqrt{v_m I_m}}$$

$$u_z^R(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_z(z,\omega)\right)}{\sqrt{v_m I_m}}$$



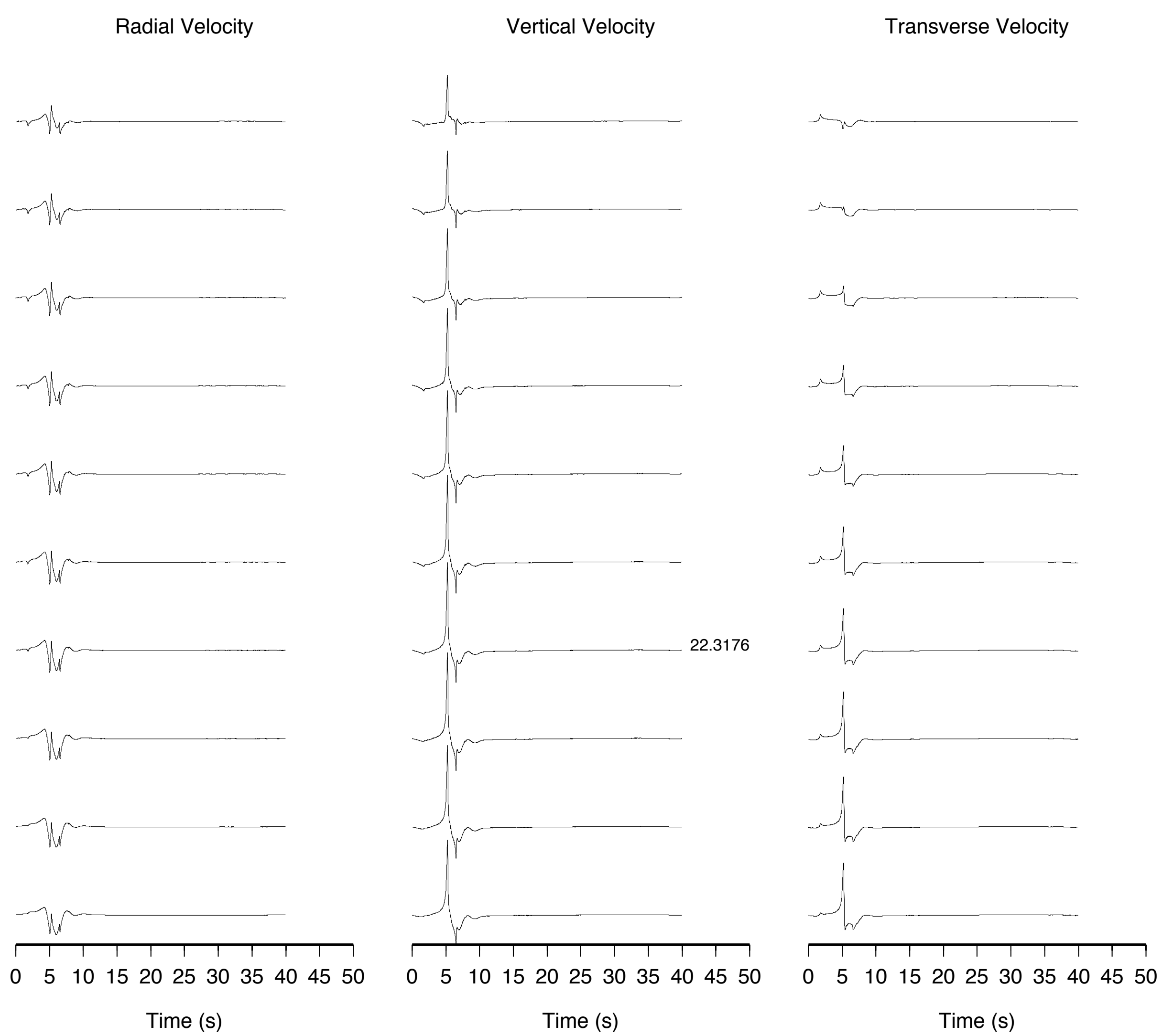
Methodology - Modal Summation Technique

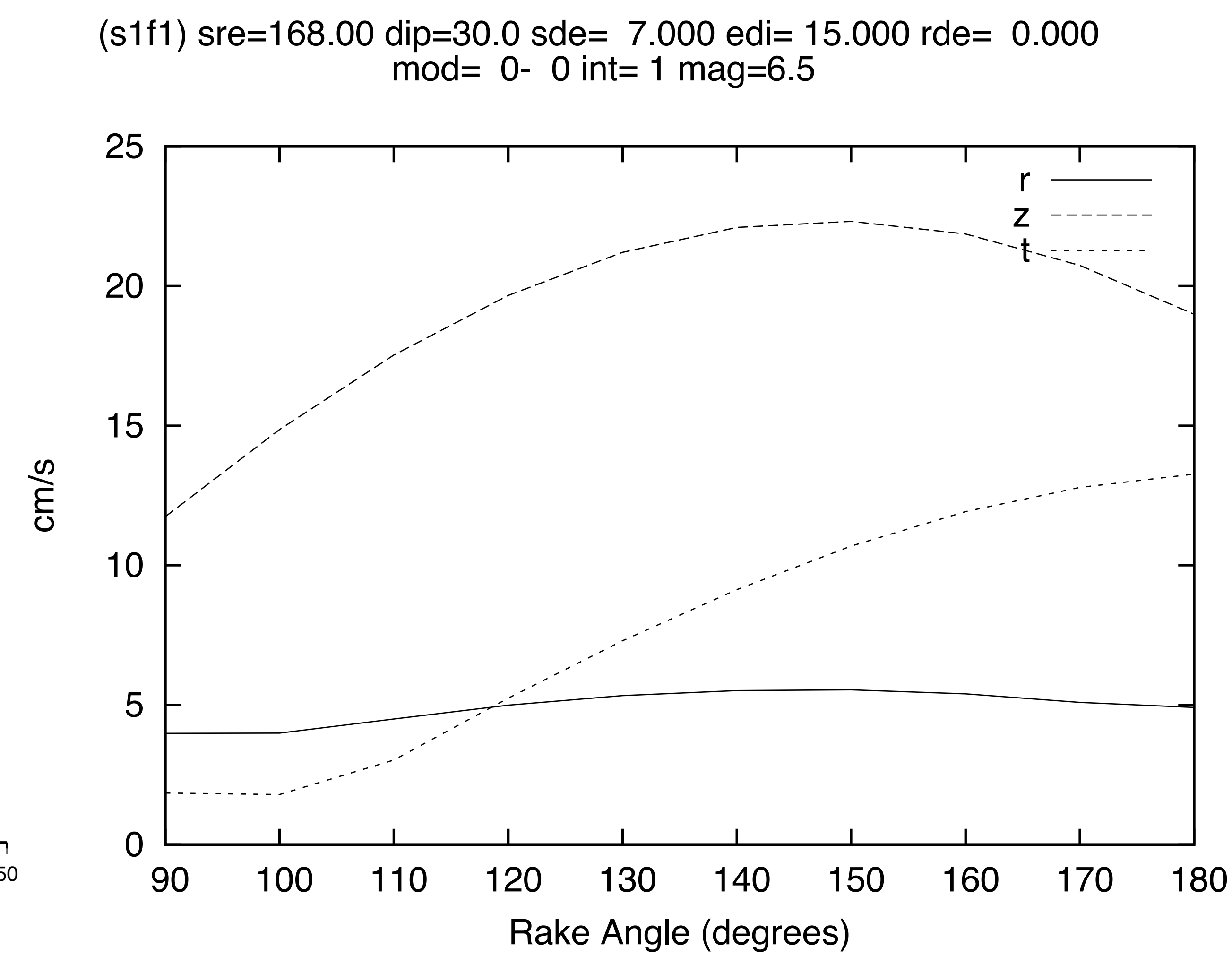
- Synthetic seismograms
- Parametric tests

$$u_y^L(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^L(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_y(z,\omega)\right)}{\sqrt{v_m I_m}}$$

$$u_x^R(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i3\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_x(z,\omega)\right)}{\sqrt{v_m I_m}}$$

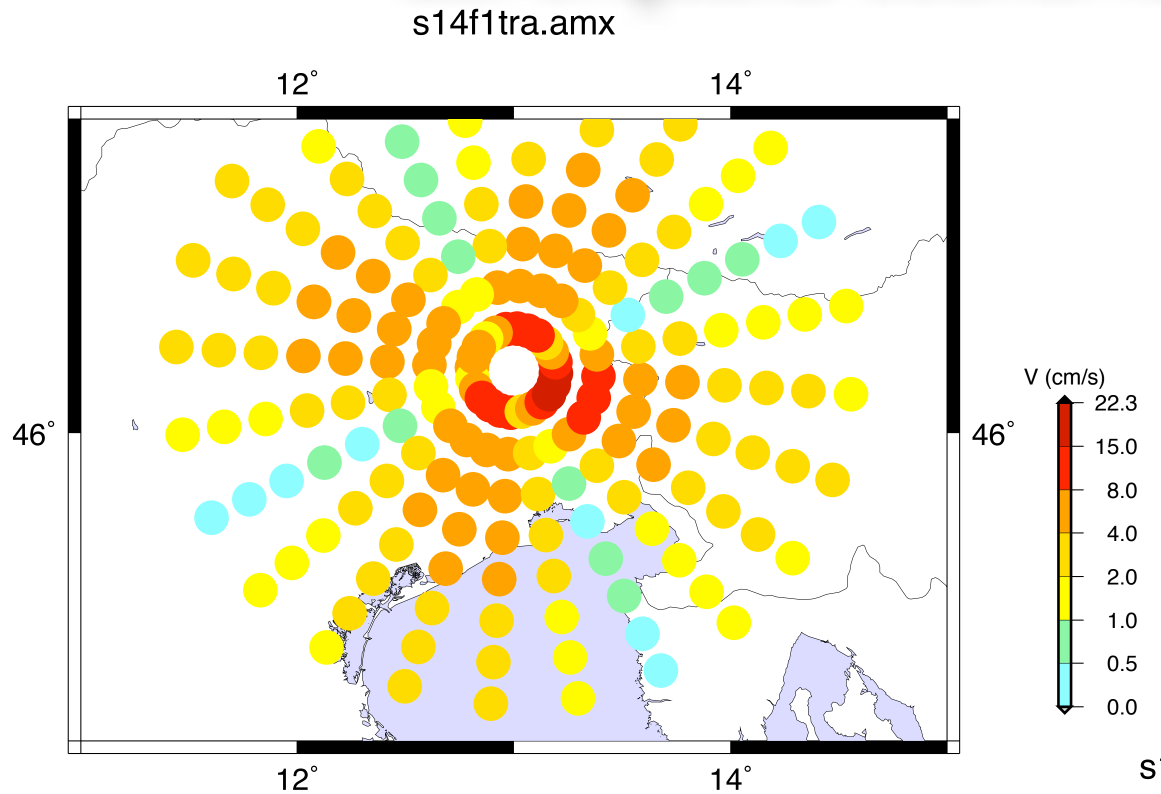
$$u_z^R(x,z,\omega)=\sum_{m=1}^{\infty}\frac{e^{-i\pi/4}}{\sqrt{8\pi\omega}}\frac{e^{-ik_m x}}{\sqrt{x}}\frac{\left(\chi_m^R(h_s,\omega)\right)}{\sqrt{c_m v_m I_m}}\frac{\left(F_z(z,\omega)\right)}{\sqrt{v_m I_m}}$$

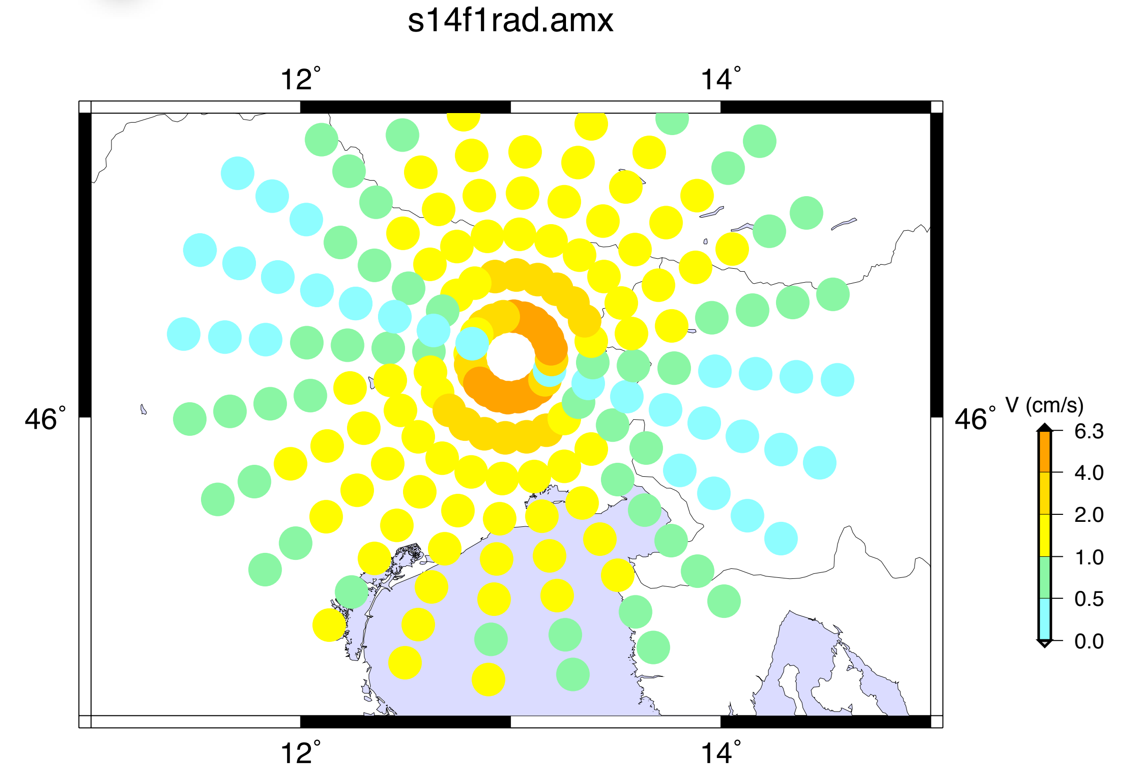


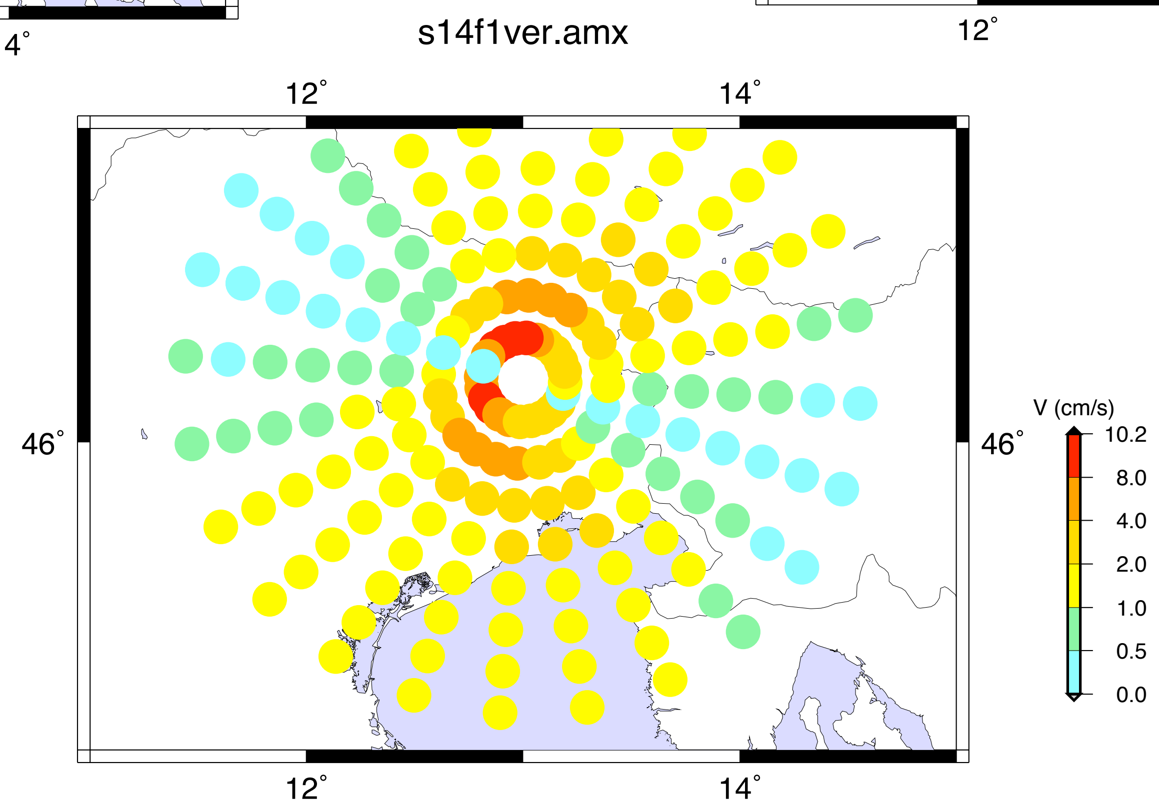


Regional Scale - Modal Summation Technique

- Earthquake scenarios for single events

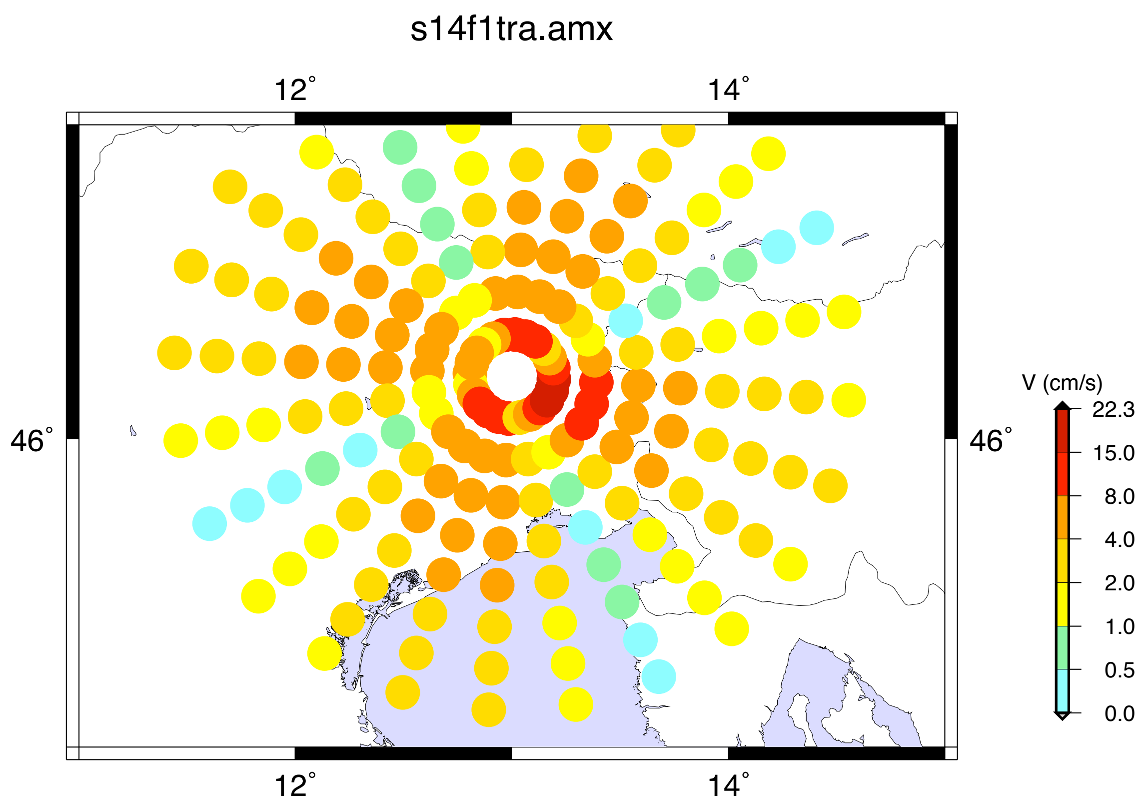


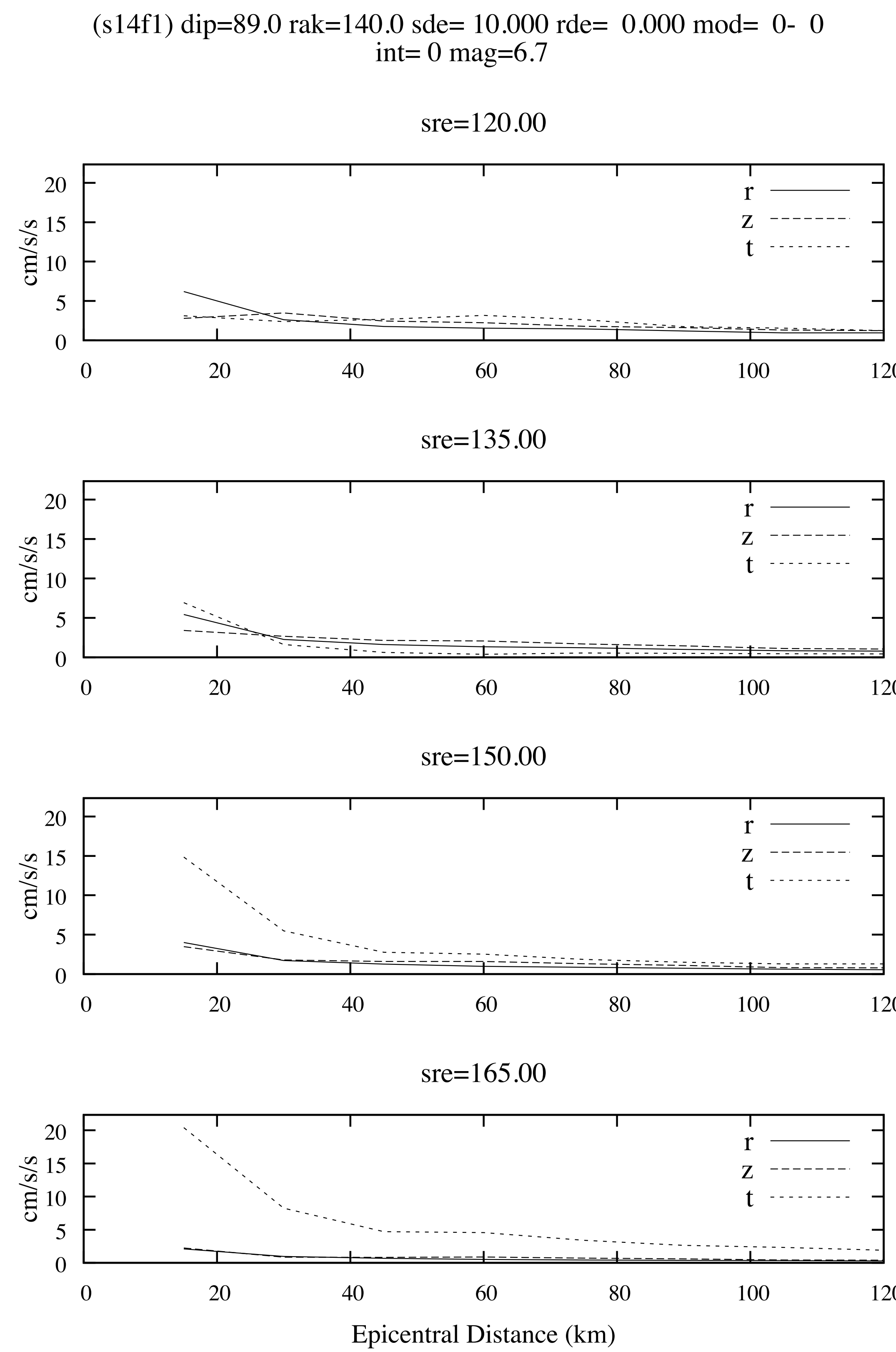




Regional Scale - Modal Summation Technique

- Earthquake scenarios

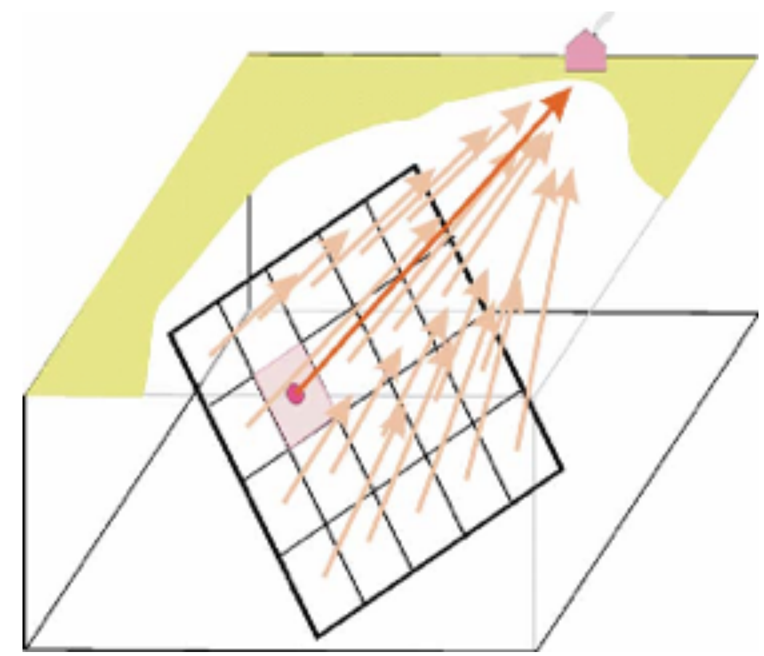




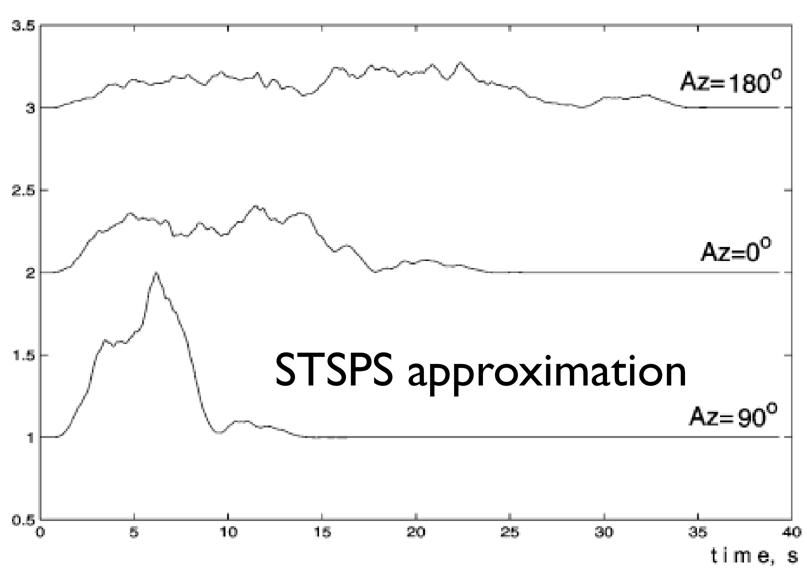
Source - Models

Point source approximation

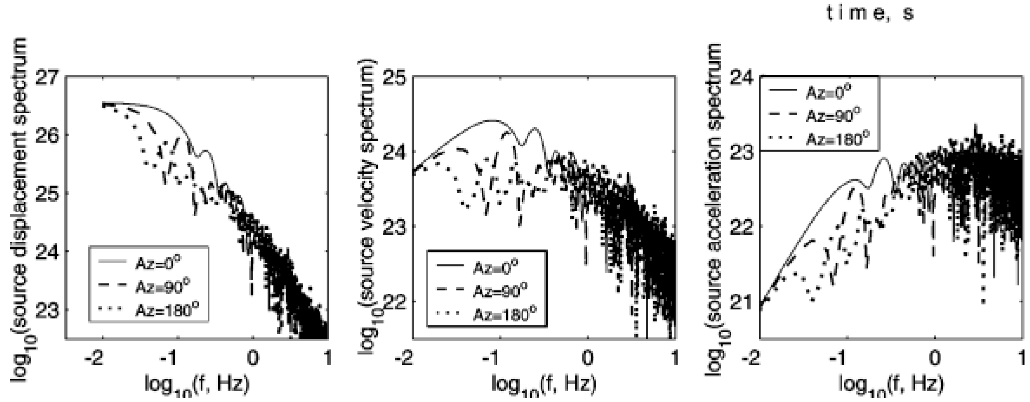
Focal mechanism and radiation pattern

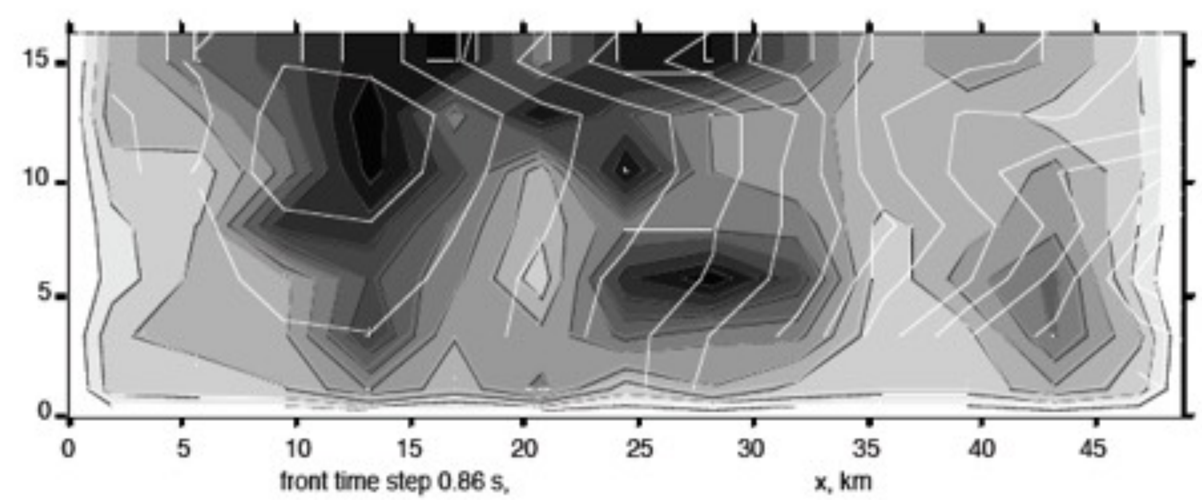


Extendend source kinematic model



STSPS approximation





2-dimensional final slip distribution over a source rectangle