Domanda 5

$A(x,y)$ = "x è amico di y"

$B(x)$ = "x ama il cinema"

$C(x)$ = "x ama il teatro"

b = Barbara

d = Donatella

1) $\forall x\,(\exists y\,(A(x,y) \wedge B(y)) \rightarrow B(x))$
2) $\forall x\,(C(x) \rightarrow \exists y\,(A(x,y) \wedge C(y))$
3) $A(b,d) \wedge C(b) \wedge \neg B(b)$

Domanda 6

- $\vdash (P \rightarrow Q) \rightarrow ((P \wedge R) \rightarrow (Q \wedge R))$

$$\dfrac{\dfrac{\dfrac{\dfrac{[P \wedge R]^2}{P} \quad \dfrac{[P \rightarrow Q]^1}{Q} \quad \dfrac{[P \wedge R]^2}{R}}{Q \wedge R}\,2(\rightarrow i)}{\vdash (P \wedge R) \rightarrow (Q \wedge R)} \quad [P \wedge R]^2}{\vdash (P \rightarrow Q) \rightarrow ((P \wedge R) \rightarrow (Q \wedge R))}\,1(\rightarrow i) \quad [P \rightarrow Q]^1$$

- $\neg(P \vee Q) \vdash \neg P \wedge \neg Q$

$$\dfrac{\dfrac{(\perp i)\ \dfrac{\neg(P \vee Q) \qquad \dfrac{[P]^2}{P \vee Q}(\vee i)}{\perp}}{\neg P}\,1(\neg i) \qquad \dfrac{\dfrac{\neg(P \vee Q) \qquad \dfrac{[Q]^2}{P \vee Q}(\vee i)}{\perp}(\perp i)}{\neg Q}\,2(\neg i)}{\neg P \wedge \neg Q}\,(\wedge i)$$

Domanda 7

- $\neg\exists x(P(x) \to \forall x P(x))$

($\neg\exists$) $\neg(P(a) \to \forall x P(x))$

($\neg\to$) $P(a)$

($\neg\forall$) $\neg\forall x P(x)$

$\neg P(b)$

(riscrittura) $\neg\exists x(P(x) \to \forall x(P(x))$

($\neg\exists$) $\neg(P(b) \to \forall x P(x))$

($\neg\to$) $P(b)$

$\neg\forall x P(x)$

$\bot$

- (imbeb) $\dfrac{}{P(b) \vdash P(b)}$

($\forall r$) $\dfrac{P(b) \vdash P(b)}{P(a), P(b) \vdash \exists x(P(x) \to \forall x P(x)), P(b), \forall x P(x)}$

($\exists r$) $\dfrac{P(a), P(b) \vdash \exists x(P(x) \to \forall x P(x)), P(b), \forall x P(x)}{P(a) \vdash \exists x(P(x) \to \forall x P(x)), P(b), P(b) \to \forall x P(x)}$

($\forall r$) $\dfrac{P(a) \vdash \exists x(P(x) \to \forall x P(x)), P(b), P(b) \to \forall x P(x)}{P(a) \vdash \exists x(P(x) \to \forall x P(x)), P(b)}$

($\to r$) $\dfrac{P(a) \vdash \exists x(P(x) \to \forall x P(x)), P(b)}{P(a) \vdash \exists x(P(x) \to \forall x P(x)), \forall x P(x)}$

($\exists r$) $\dfrac{P(a) \vdash \exists x(P(x) \to \forall x P(x)), \forall x P(x)}{\vdash \exists x(P(x) \to \forall x P(x)), P(a) \to \forall x P(x)}$

$\vdash \exists x(P(x) \to \forall x P(x))$

Domanda 8

Siano

$Q_1 = \exists x (B(x) \wedge E(x))$,

$Q_2 = \exists x (B(x) \wedge \forall y (E(y) \rightarrow \neg A(x,y)))$

$P = \exists x (B(x) \wedge \neg \forall y (B(y) \rightarrow A(y,x)))$

Devo dimostrare per risoluzione che

$\{Q_1, Q_2\} \cup (\neg P)^c$ è insoddisfacibile

① Ridenominare le variabili se necessario di ogni fbf
② Forma normale prenesse
③ Forma di Skolem
④ Eliminazione di quantificatori universali
⑤ Forma normale congiunta
⑥ Forma a Clausole

$Q_1 = \exists x (B(x) \wedge E(x))$

$B(a) \wedge E(a)$ ③

$\{\{B(a)\}, \{E(a)\}\}$ ⑥

$Q_2 = \exists x (B(x) \wedge \forall y (E(y) \rightarrow \neg A(x,y)))$

$\exists x \forall y (B(x) \wedge (E(y) \rightarrow \neg A(x,y)))$ ②

$\forall y (B(b) \wedge (E(y) \rightarrow \neg A(b,y)))$ ③

$B(b) \wedge (E(y) \rightarrow \neg A(b,y))$ ④

$B(b) \wedge \neg E(y) \vee \neg A(b,y)$ ⑤

$\{\{B(b)\}, \{\neg E(y), \neg A(b,y)\}\}$ ⑥

$$\neg P = \neg \exists x (B(x) \wedge \neg \forall y (B(y) \rightarrow A(y,x)))$$

$$\forall x \neg (B(x) \wedge \neg \forall y (B(y) \rightarrow A(y,x)))$$

$$\forall x (\neg B(x) \vee \forall y (B(y) \rightarrow A(y,x)))$$

$$\forall x \forall y (\neg B(x) \vee (B(y) \rightarrow A(y,x))) \quad (2)$$

$$\neg B(x) \vee (B(y) \rightarrow A(y,x)) \quad (4)$$

$$\neg B(x) \vee \neg B(y) \vee A(y,x) \quad (5)$$

$$\{\neg B(x), \neg B(y), A(y,x)\} \quad (6)$$