

Soluzione Esame 24/01/2022

Domanda 5

$D(x)$ = "x è un docente"

$S(x)$ = "x è uno studente"

$A(x, y)$ = "x odia y"

$B(x, y)$ = "ad x piace y"

a = Laura

① $\exists x (S(x) \wedge \forall y (D(y) \rightarrow A(x, y)))$

② $\forall x (D(x) \rightarrow \exists y (S(y) \wedge B(x, y)))$

③ $D(a) \wedge \forall x (S(x) \rightarrow B(a, x))$

Domanda 6

$$\begin{array}{c}
 \frac{\frac{[P]^1 \quad (P \rightarrow Q) \wedge (Q \rightarrow P)}{\rightarrow e} \quad P \rightarrow Q}{(ii) \quad \frac{P}{P \wedge Q} \quad [P]^1}{[PVQ]} \quad \frac{[Q]^2 \quad (P \rightarrow Q) \wedge (Q \rightarrow P)}{\rightarrow e} \quad Q \rightarrow P}{(ii) \quad \frac{P}{P \wedge Q} \quad [Q]^2} \\
 \frac{P \wedge Q}{\wedge i} \quad [P]^1 \quad [Q]^2}{[PVQ]^2}
 \end{array}$$

$$\begin{array}{c}
 \frac{[PVQ] \quad \frac{[P]^2 \quad \frac{P \wedge Q}{\wedge e} \quad P}{\neg P} \quad [Q]^3 \quad \frac{P \wedge Q}{\wedge e} \quad Q}{\neg(P \vee Q)} \quad \neg(P \vee Q)}{\neg i} \quad \frac{\perp}{\neg(P \vee Q)} \quad [PVQ]^2}
 \end{array}$$

Diamonde \neg

$$P = \exists x ((\neg A(x) \rightarrow B(x)) \rightarrow (A(x) \vee B(x)))$$

$\frac{A(a) \wedge \neg A(a)}{\text{(vr) } \frac{A(a) \wedge P, A(a)}{\neg P, \neg A(a), A(a) \vee B(a)}}$	$\frac{B(a) \wedge \neg B(a)}{\text{(vr) } \frac{B(a) \wedge P, B(a)}{B(a) \wedge P, A(a) \vee B(a)}}$
$\frac{\frac{\neg P, \neg A(a), A(a) \vee B(a) \quad B(a) \wedge P, A(a) \vee B(a)}{\neg P, \neg A(a), A(a) \vee B(a)} \quad (\rightarrow S)}{\neg A(a) \rightarrow B(a) \wedge P, A(a) \vee B(a)} \quad (\rightarrow r)$	
$\frac{\neg P, (\neg A(a) \rightarrow B(a)) \rightarrow (A(a) \vee B(a))}{\neg P, (\neg A(a) \rightarrow B(a)) \rightarrow (A(a) \vee B(a))} \quad (\rightarrow r)$	
$\frac{\neg P, (\neg A(a) \rightarrow B(a)) \rightarrow (A(a) \vee B(a))}{\neg \exists x ((\neg A(x) \rightarrow B(x)) \rightarrow (A(x) \vee B(x)))} \quad (\exists r)$	

1)	$\neg \exists x ((\neg A(x) \rightarrow B(x)) \rightarrow (A(x) \vee B(x)))$	
2)	$\neg (\neg A(a) \rightarrow B(a) \rightarrow (A(a) \vee B(a)))$	(\exists da 1))
3)	$\neg A(a) \rightarrow B(a)$] (\neg da 2))
4)	$\neg (A(a) \vee B(a))$	
5)	$\neg A(a)$] ($\neg \vee$ da 4))
6)	$\neg B(a)$	
7)	$\neg A(a) \rightarrow B(a)$] (riscontro 3))
	$\neg \neg A(a)$ $A(a)$	
	$B(a)$	

Diamonds 8

- 1) $\exists x (B(x) \wedge C(x)) \wedge \forall x (B(x) \rightarrow A(x)) \wedge \exists x (C(x) \rightarrow C(f(x)))$
- 2) $\exists x (B(x) \wedge C(x)) \wedge \forall y (B(y) \rightarrow A(y)) \wedge \exists z (C(z) \rightarrow C(f(z)))$
- 3) $\exists x (B(x) \wedge C(x)) \wedge \forall y (B(y) \rightarrow A(y)) \wedge \forall z (\neg(C(z) \rightarrow C(f(z))))$
- 4) $\exists x \forall y \exists z (B(x) \wedge C(x) \wedge (B(y) \rightarrow A(y)) \wedge (\neg(C(z) \rightarrow C(f(z))))$
- 5) $\forall y \exists z (B(z) \wedge C(z) \wedge (B(y) \rightarrow A(y)) \wedge (\neg(C(z) \rightarrow C(f(z))))$
- 6) $B(z) \wedge C(z) \wedge (\neg B(y) \vee A(y)) \wedge (\neg(\neg C(z) \vee C(f(z))))$
- 7) $B(z) \wedge C(z) \wedge (\neg B(y) \vee A(y)) \wedge C(z) \wedge \neg C(f(z))$
- 8) $\{ \{B(z)\}, \{C(z)\}, \{ \neg B(y), A(y) \}, \{C(z)\}, \{ \neg C(f(z)) \} \}$

1) $\neg \exists x A(x)$

2) $\forall x \neg A(x)$

3) $\{ \neg A(x) \}$

