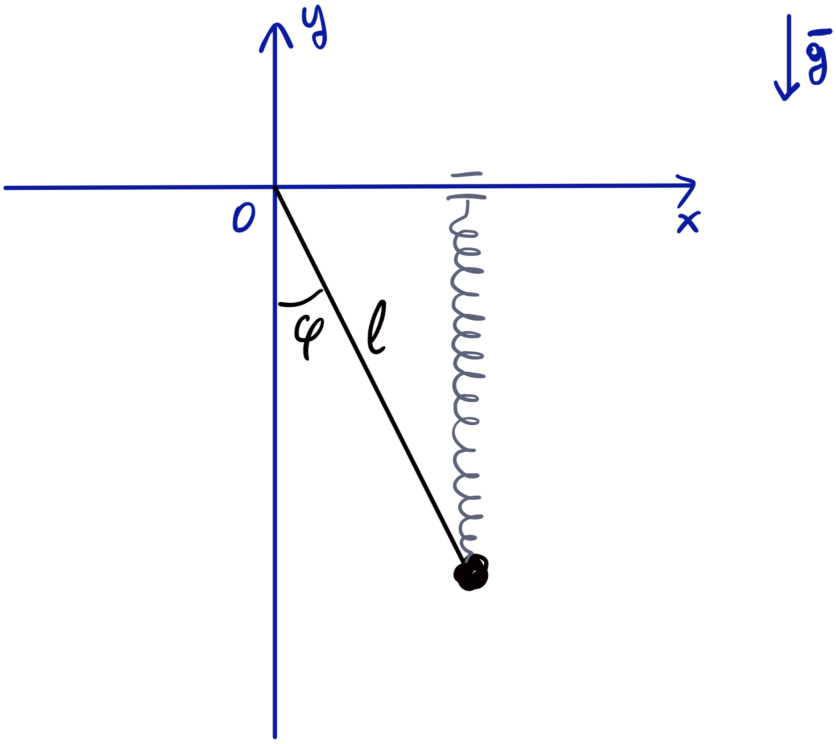
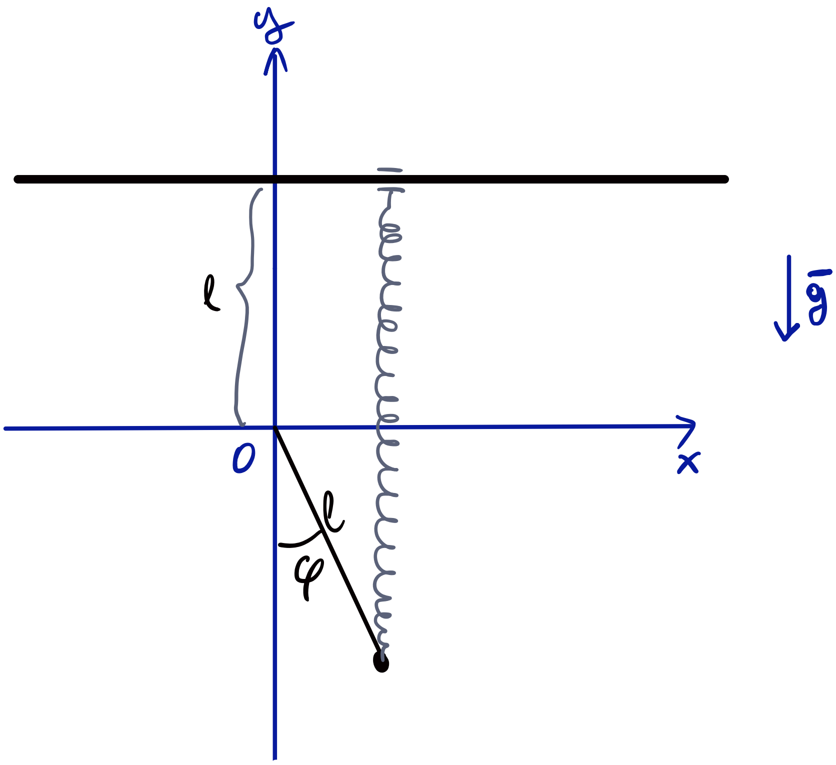


$$x = \ell \operatorname{sen}\varphi$$
$$y = -\ell \cos\varphi$$

$$V = -mg\ell\cos\varphi + \frac{k}{2}\ell^2\cos^2\varphi$$

$$V' = mg\ell \operatorname{sen}\varphi - k\ell^2\cos\varphi \operatorname{sen}\varphi$$



$$x = \ell \operatorname{sen}\varphi$$
$$y = -\ell \cos\varphi$$
$$T = \frac{m\ell^2}{2}\dot{\varphi}^2$$
$$V = -mg\ell\cos\varphi + \frac{k}{2}\ell^2(1+\cos\varphi)^2$$

1) $L = \frac{m\ell^2}{2}\dot{\varphi}^2 + mg\ell\cos\varphi - \frac{k\ell^2}{2}(1+\cos\varphi)^2$

2) $\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} = m\ell^2\ddot{\varphi}$ $\qquad \frac{\partial L}{\partial\varphi} = -mg\ell \operatorname{sen}\varphi + k\ell^2(1+\cos\varphi)\operatorname{sen}\varphi$

$$\ddot{\varphi} = -\frac{g}{l}\operatorname{sen}\varphi + \frac{k}{m}(1+\cos\varphi)\operatorname{sen}\varphi$$

→ Linearizzazione attorno $\varphi = 0$

$$\ddot{\varphi} = f(\varphi) \rightarrow \ddot{\varphi} = f(0) + f'(0)\varphi + \cancel{\dots}$$

$$\stackrel{!}{=} 0 + \left(-\frac{g}{l} + 2\frac{k}{m}\right)\varphi$$

3) $V(\varphi) = -mgl\cos\varphi + \frac{kl^2}{2}(1+\cos\varphi)^2$

$$V'(\varphi) = mgl\operatorname{sen}\varphi - kl^2(1+\cos\varphi)\operatorname{sen}\varphi = kl^2\operatorname{sen}\varphi\left(\frac{mg}{kl} - 1 - \cos\varphi\right)$$

$$V''(\varphi) = mgl\cos\varphi - kl^2(1+\cos\varphi)\cos\varphi + kl^2\operatorname{sen}^2\varphi$$

$V'(\varphi) = 0 \rightarrow \operatorname{sen}\varphi = 0 \qquad \varphi = 0, \pi$

$\cos\varphi = \frac{mg}{kl} - 1$ ⇝ soluzioni quando $-1 \leq \frac{mg}{kl} - 1 \leq 1$, cioè

$$\varphi^*_\pm = \pm\arccos\left(\frac{mg}{kl} - 1\right) \qquad 0 \leq \boxed{\frac{mg}{kl} \leq 2} \qquad mg \leq 2kl$$

$V''(0) = (mg - 2kl)l \rightarrow$ stab. se $mg > 2kl$

$V''(\pi) = -mgl < 0 \rightarrow$ INSTAB

$$V''(\varphi^*_\pm) = mgl\left(\frac{mg}{kl} - 1\right) - kl^2\Bigg[\underbrace{\frac{mg}{kl}\left(\frac{mg}{kl} - 1\right) + \left(\frac{mg}{kl} - 1\right)^2}_{\left(\frac{mg}{kl} - 1\right)\left(\frac{mg}{kl} + \frac{mg}{kl} - 1\right)} - 1\Bigg]$$

$$= \left(\frac{mg}{kl} - 1\right) l \left(mg - kl\left(2\frac{mg}{kl} - 1\right)\right) + kl^2 =$$

$$= -\left(\frac{mg}{kl} - 1\right) kl^2 \left(\frac{mg}{kl} - 1 \right) + kl^2 =$$

$$= kl^2 \left(1 - \underbrace{\left(\frac{mg}{kl} - 1\right)^2}_{= \cos\varphi_* \leq 1} \right) \geq 0$$

$\rightarrow$ STAB (quando esiste)

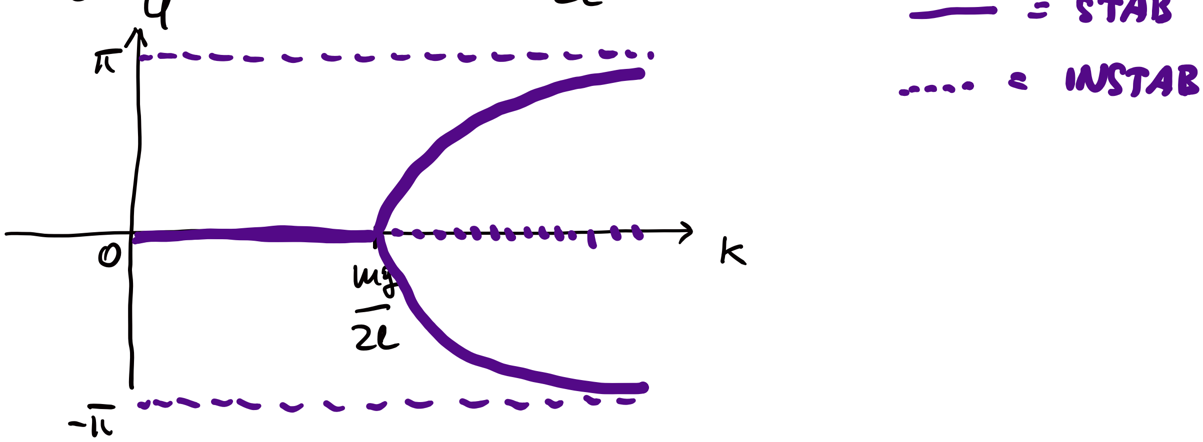
$$V'' = mgl\cos\varphi - kl^2(1+\cos\varphi)\cos\varphi + kl^2\sin^2\varphi =$$

$$\cos\varphi\,(mgl - kl^2) + kl^2(\sin^2\varphi - \cos^2\varphi) =$$

$$= kl^2 \left\{ \cos\varphi \underbrace{\left(\frac{mg}{kl} - 1 \right)}_{\cos\varphi_*} + \sin^2\varphi - \cos^2\varphi \right\} \Big|_{\varphi_*} =$$

$$= kl^2 \left(\cos^2\varphi_* + \sin^2\varphi_* - \cos^2\varphi_* \right) = kl^2 \sin^2\varphi_* \geq 0$$

4) $\frac{mg}{kl} \gtreqless 2 \quad \sim \quad k \lesseqgtr \frac{mg}{2l}$



5) $L = \frac{ml^2}{2}\dot{\varphi}^2 + mgl\cos\varphi - \frac{kl^2}{2}(1+\cos\varphi)^2$

$$L_{lin} = \frac{ml^2}{2}\dot{\varphi}^2 + \cancel{mgl} - \frac{mgl}{2}\varphi^2 - \frac{kl^2}{2}\underbrace{\left(1+1-\frac{\varphi^2}{2}+\dots\right)^2}_{\cancel{4}-2\varphi^2+\dots}$$

$$= \frac{ml^2}{2}\dot{\varphi}^2 - \frac{1}{2}\varphi^2(mgl-2kl^2) = \frac{ml^2}{2}\dot{\varphi}^2 - \frac{ml^2}{2}\varphi^2\left(\frac{g}{l}-\frac{2k}{m}\right)$$

$$\cancel{ml^2}\ddot{\varphi} = -\cancel{ml^2}\left(\frac{g}{l}-\frac{2k}{m}\right)\varphi^2$$

$A = ml^2 \qquad B = V''(0) = (mg-2kl)l$

$\Rightarrow L_{lin} = \frac{1}{2}ml^2\varphi^2 - \frac{1}{2}(mg-2kl)l\,\varphi^2$

6) $\omega^2 = \frac{g}{l} - \frac{2k}{m}$ da L_{lin} o eq. lin.

$B-\lambda A = 0 \to (mg-2kl)l - \lambda ml^2 = 0 \to \lambda = \frac{g}{l} - \frac{2k}{m}$

$\varphi(t) = A\cos\left(\left(\frac{g}{l}-\frac{2k}{m}\right)t + \varphi_0\right)$

7) vedi risoluz. al pto 2.