INTRODUZIONE AL CORSO DI STATICA 24/2/22

RIPASSO: GRANDEZZE FISICHE

- scalari : massa [M], lunghezza [L], temperatura [T], pressione [$FL^{-2}$]
- vettoriali : forza [F], spostamento [L]

UNITÀ DI MISURA

massa : Kg

Peso = $M \cdot g$

lunghezza : m

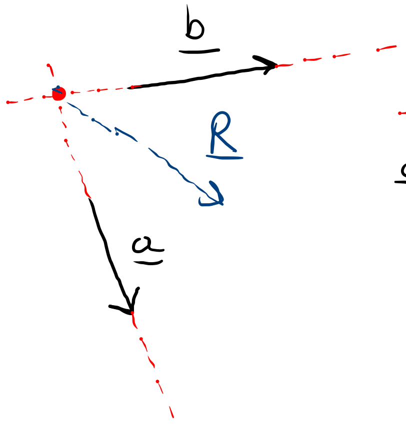
forza : N

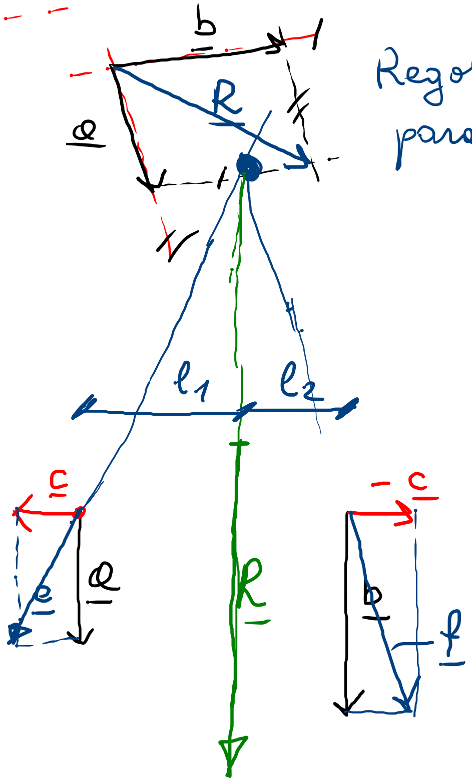
pressione : $Pa = \frac{N}{m^2}$ ⇒ grado di sollecitazione dei materiali

⇝ $MPa = N/mm^2$ ; $GPa \sim 10^9$ Pa

$10^6$ Pa

# ALCUNE CONSIDERAZ. SUI VETTORI




Regola del parallelogramma

$l_1 > l_2$

VETTORI //
CONCORDI

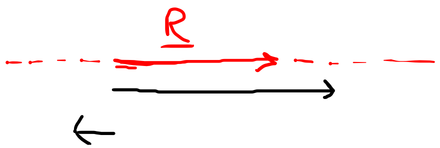
$\{\underline{a}, \underline{b}\}$

$\Downarrow$

$\{\underline{e}, \underline{f}\}$

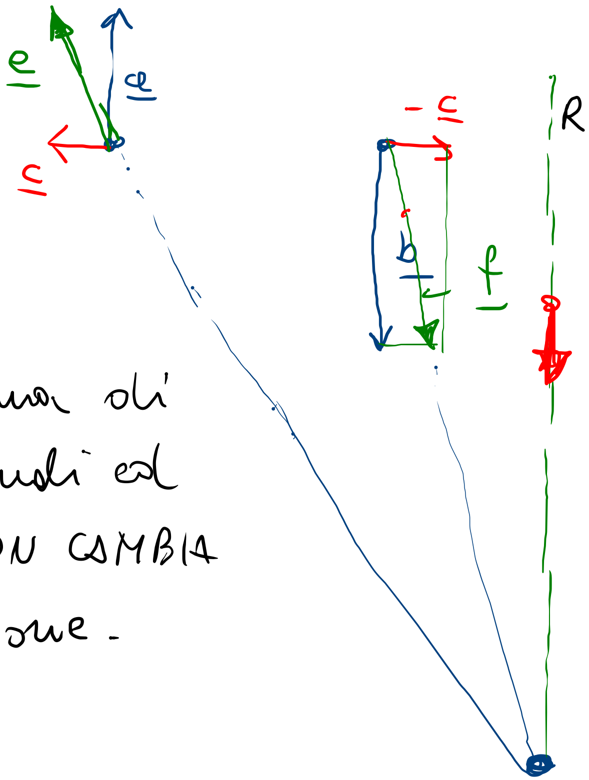
STESSA RISULTANTE

$|\underline{R}| = 5$




VETTORI //
DISCORDI

$|\underline{a}| \neq |\underline{b}|$

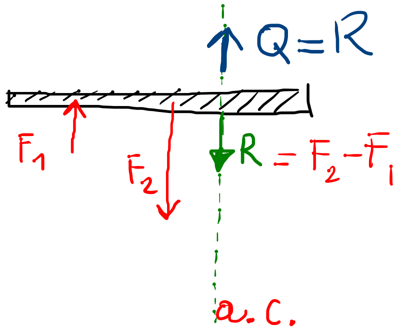


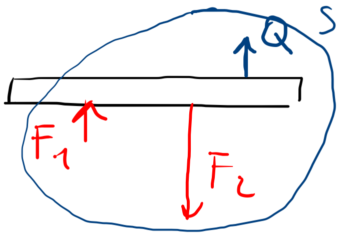

RETTA D'AZIONE DELLA RISULT.
=
<u>ASSE CENTRALE</u>

$\underline{R} \rightsquigarrow |\underline{R}| = 1$

Aggiungere ad un sistema di vettori 2 vettori uguali ed opposti $\{\underline{c}, -\underline{c}\}$ NON CAMBIA $\underline{R}$ e la sua posizione.

RAGIONIAMO:




SIST. DI FORZE EQUILIBR.

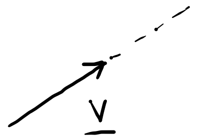
$R_1 = 0$

Problema: assegno $\{F_1, F_2\}$

come posso metterlo in equilibrio?

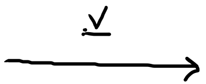
Risposta: applico una $Q$ sull'asse centrale tale che $Q = R$

# VETTORI, LORO RAPPRESENTAZIONE E OPERAZIONI




INTENSITÀ o MODULO : $|\underline{v}|$ , $v$ : LUNGHEZZA DEL SEGMENTO

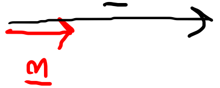
VERSORE : VETTORE UNITARIO : $|\underline{m}| = 1$

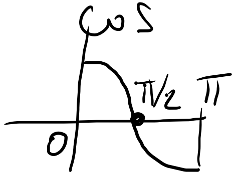



$|\underline{v}| = 3$

INTENSITÀ

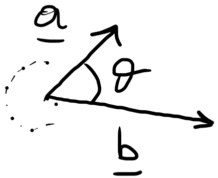
$\underline{v} = 3\,\underline{m}$



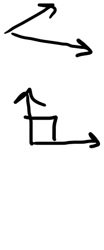



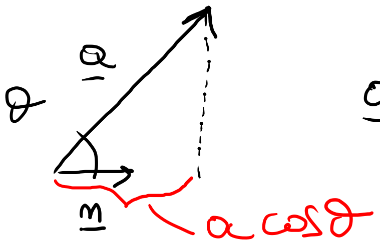

• DUE OPERAZIONI SUI VETTORI

1) PRODOTTO SCALARE $\underline{a} \cdot \underline{b} \Rightarrow$ NUMERO REALE




$$\underline{a} \cdot \underline{b} = ab\cos\vartheta \gtreqless 0$$





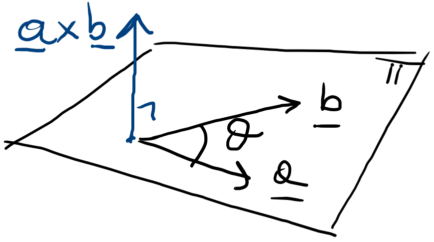

$\underline{m}$ : VERSORE

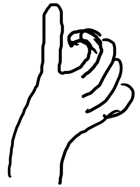
$\underline{a}\cdot\underline{m} = a\cdot 1\cos\vartheta = a\cos\vartheta$ ; PROIEZ. DEL VETTORE $\underline{a}$ NELLA DIREZ. DI $\underline{m}$

2) PRODOTTO VETTORIALE

$\underline{a}, \underline{b} \longrightarrow \underline{a}\times\underline{b}$ è un vettore

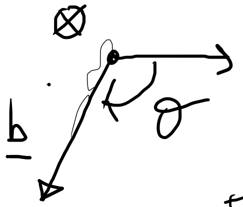
↳ $\perp \underline{a}, \perp \underline{b}$



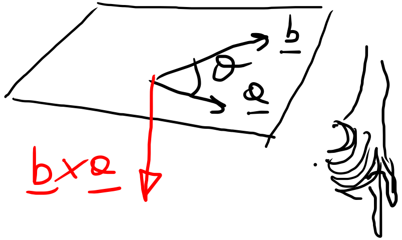
$|\underline{a}\times\underline{b}| = ab\sin\vartheta \rightarrow$ se $\vartheta=0 \Rightarrow |\underline{a}\times\underline{b}| = 0$

ES.



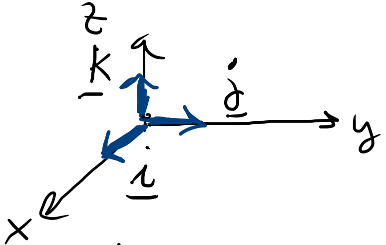

$\underline{a}\times\underline{b}$ entra o esce dal foglio?

ENTRA!




$$\underline{b}\times\underline{a} = -\,\underline{a}\times\underline{b}$$

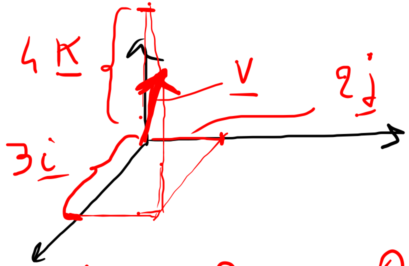
RAPPRESENTAZ. CARTESIANA DEI VETTORI




$\underline{i}, \underline{j}, \underline{k}$ : 3 versori $\perp$ tra di loro

$|\underline{i}| = |\underline{j}| = |\underline{k}| = 1$ ; $\underline{i}\cdot\underline{i} = 1$ $\left(\underline{j}\cdot\underline{j} = 1 = \underline{k}\cdot\underline{k}\right)$

$\underline{i}\times\underline{j} = \underline{k}$ ......

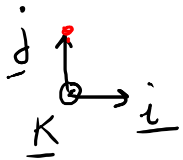


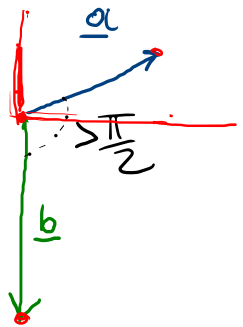

$\{\underline{i}, \underline{j}, \underline{k}\}$ : BASE DEI VETTORI NELLO SPAZIO

$\underline{V} = 3\,\underline{i} + 2\,\underline{j} + 4\,\underline{k}$ ; RAPPRES. CART. DI $\underline{V}$

$\underline{v}\cdot\underline{i}$ : COMP DI $\underline{v}$ lungo $\underline{i}$ $\Rightarrow \left(3\underline{i} + 2\underline{j} + 4\underline{k}\right)\cdot\underline{i} = 3\,\overset{1}{\underline{i}\cdot\underline{i}} + 2\,\overset{0}{\cancel{\underline{j}\cdot\underline{i}}} + 4\,\overset{0}{\cancel{\underline{k}\cdot\underline{i}}} = 3 + \cdots = 3$

ES




$\underline{a} = 2\underline{i} + \underline{j}$ ; $\underline{b} = -3\underline{j}$

RAPPR. CART. DI $\underline{a}$ e $\underline{b}$

$$\underline{a} \cdot \underline{b} \, ? = (2\underline{i} + \underline{j}) \cdot (-3\underline{j}) = 2\underline{i} \cdot (-3\underline{j}) + \underline{j} \cdot (-3\underline{j})$$

$$= -6 \underbrace{\cancel{\underline{i} \cdot \underline{j}}}_{0} - 3 \underbrace{\underline{j} \cdot \underline{j}}_{1} = -3$$

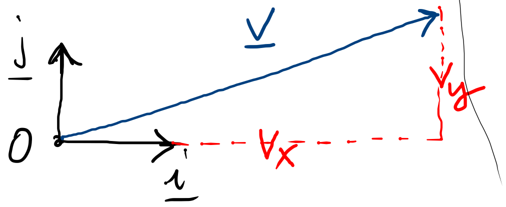
$\underline{a} = a_x \underline{i} + a_y \underline{j}$

$\underline{b} = b_x \underline{i} + b_y \underline{j}$ $\Rightarrow$ $\boxed{\underline{a} \cdot \underline{b} = a_x b_x + a_y b_y \; (+ a_z b_z)}$

SOMMA PRODOTTO COMPONENTI OMOLOGHE

ES: $\underline{a}$ e $\underline{b}$ ORTOGONALI PER COSTRUZIONE $\Rightarrow \underline{a} \cdot \underline{b} = 0$

ES: MODULO o INTENSITA' DI UN VETTORE IN RAPPR. CARTESIANA

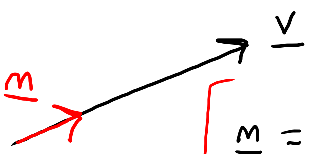



$\underline{v} = v_x \underline{i} + v_y \underline{j}$

$|\underline{v}|$ ? $\quad |\underline{v}|^2 = v_x^2 + v_y^2 \Rightarrow |\underline{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

COME POSSO SCRIVERE $v_x^2 + v_y^2$ ? $\quad v_x v_x + v_y v_y = \underline{v} \cdot \underline{v} \Rightarrow |\underline{v}| = \sqrt{\underline{v} \cdot \underline{v}}$

COME CALCOLARE IL VERSORE ASSOCIATO AD UN VETTORE ASSEGNATO




$\underline{v} \Rightarrow \underline{m}$ ? $\quad \underline{v} = |\underline{v}| \, \underline{m}$

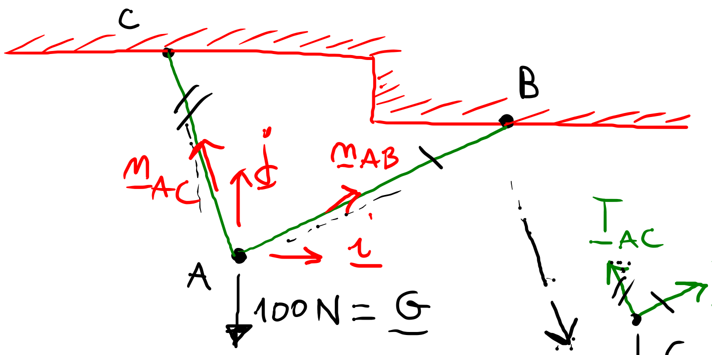
$$\boxed{\underline{m} = \frac{\underline{v}}{|\underline{v}|}}$$

ES: $\underline{v} = 3\underline{i} + 4\underline{j} \Rightarrow |\underline{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{9+16} = \sqrt{25} = 5$

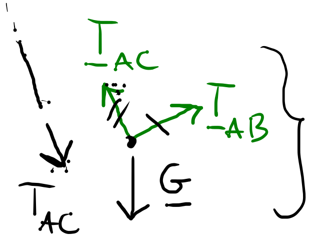
$\underline{m} = \frac{1}{5}(3\underline{i} + 4\underline{j}) = \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$ $\quad$ VERIF. $|\underline{m}| \overset{?}{=} 1$

# ES. EQUILIBRIO DI UN PUNTO MATERIALE "PESANTE"

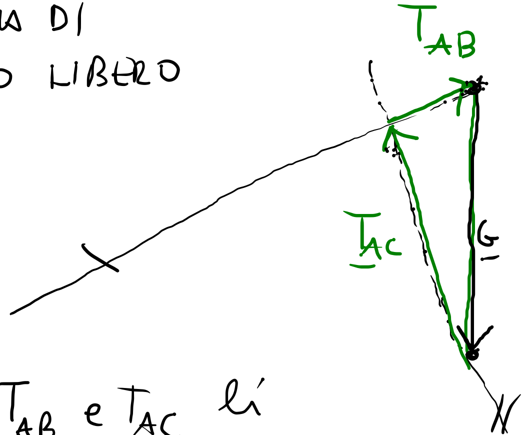
? CALCOLARE LE TENSIONI NEI FILI $\left(\sum \underline{F}_i = \underline{0}\right)$ (A)




1) METODO GRAFICO: SOMMA DI $\underline{G}$, $\underset{?}{\underline{T}_{AB}}$, $\underset{?}{\underline{T}_{AC}}$ È UGUALE A ZERO.




SCHEMA DI CORPO LIBERO




POLIGONO DELLE FORZE CHIUSO!

$\underline{R} = \underline{0}$

25 N

$T_{AB}$ e $T_{AC}$ li ottengo da una misura in scala del poligono

2) METODO ANALITICO

$$\boxed{\underline{G} + \underline{T}_{AB} + \underline{T}_{AC} = \underline{0}}$$ EQ. DI EQUILIBRIO

$$\underline{G} = -100\,\underline{j}\,N$$

$$\underline{T}_{AB} = T_{AB}\,\overbrace{(4\underline{i}+2\underline{j})\frac{1}{\sqrt{20}}}^{\underline{m}_{AB}}$$

$$\underline{T}_{AC} = T_{AC}\,\overbrace{(-\underline{i}+3\underline{j})\frac{1}{\sqrt{10}}}^{\underline{m}_{AC}}$$

SCRIVO LE COMPONENTI LUNGO $\underline{i}$ e $\underline{j}$ DELL'EQ. DI EQUILIBRIO

$$\left.\begin{aligned} \underline{i}:&\quad \frac{4}{\sqrt{20}}T_{AB} - \frac{1}{\sqrt{10}}T_{AC} = 0 \\ \underline{j}:&\quad -100 + \frac{2}{\sqrt{20}}T_{AB} + \frac{3}{\sqrt{10}}T_{AC} = 0 \end{aligned}\right\}$$

2 EQUAZ. IN 2 INCOGNITE

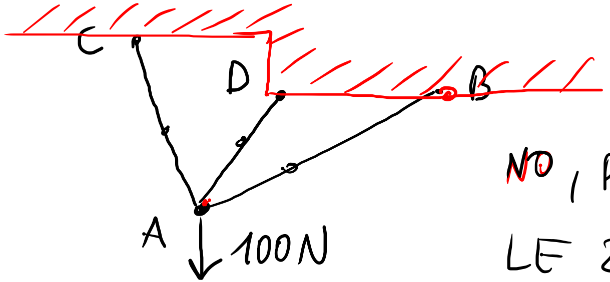
$\Rightarrow$ 1 SOLUZ. UNICA CON

$T_{AC}, T_{AB} \neq 0$

$$\left.\begin{aligned} T_{AB} &= \frac{100\sqrt{5}}{7}\ N \\ T_{AC} &= \frac{200\sqrt{10}}{7}\ N \end{aligned}\right\}$$

RAGIONIAMO:

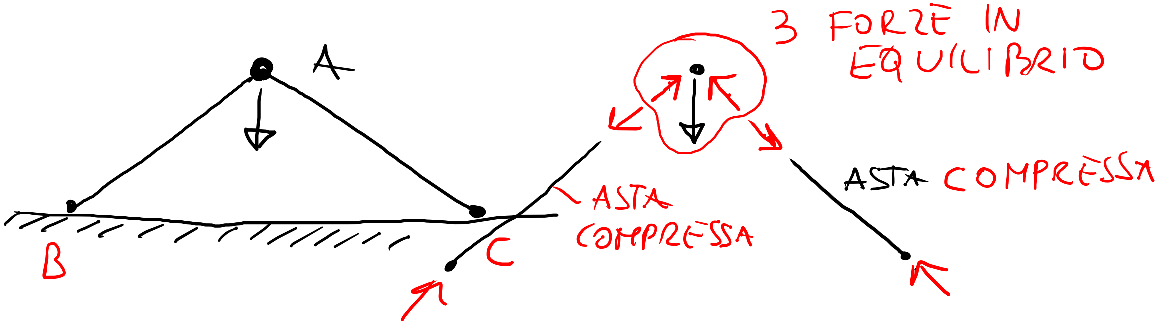
RIUSCIREMMO A CALCOLARE LE TENSIONI NEI FILI DI QUESTO PROBLEMA?




$T_{AC}, T_{AD}, T_{AB}$?

NO, PERCHÉ HO A DISPOSIZ. SEMPRE SOLO LE 2 EQUAZ. SCALARI LUNGO $\underline{i}$ e LUNGO $\underline{j}$.

LE SOLUZ. SONO $\infty^1 = \infty^{3-2}$

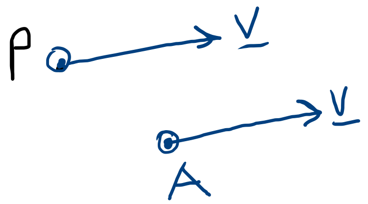


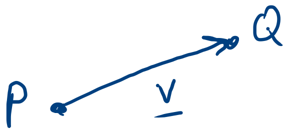

POSSO RISOLVERE IL PROBLEMA PERCHÉ HO 2 INCOGNITE CON 2 EQ. SCALARI

---

## CENNI ALLA TEORIA DEI VETTORI APPLICATI

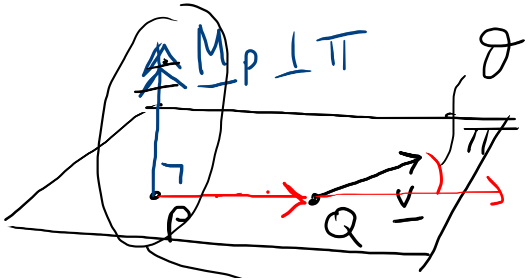
VETTORE APPLICATO: UNA COPPIA $(P, \underline{v})$ dove $P$ è un pto nello spazio




$$(P, \underline{v}) \doteq PQ \quad , \quad |\underline{v}| = \overline{PQ} = |PQ|$$

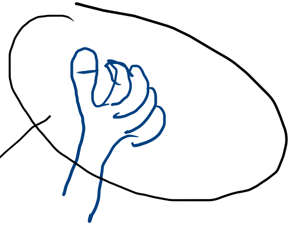
# DEFINIZIONE DI MOMENTO DI UN VETTORE APPLICATO




CHIAMO MOMENTO DEL VETTORE $(Q, \underline{v})$

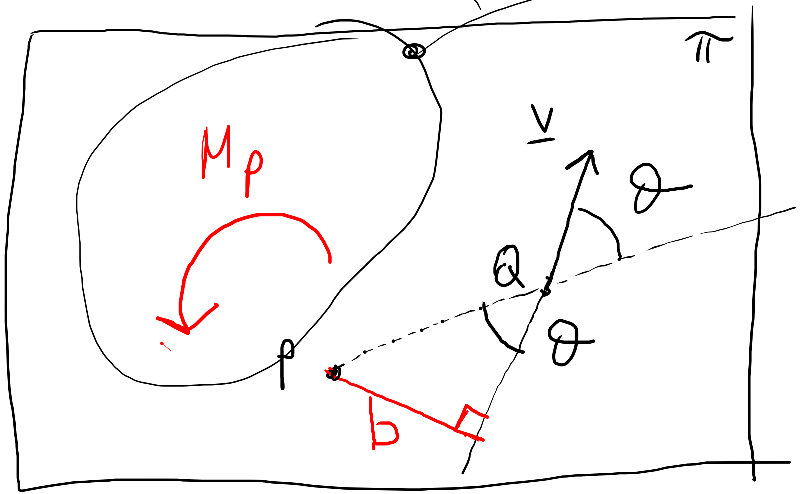
RISPETTO A P:   P: POLO o PUNTO DI RIDUZIONE

$$\underline{M}_{(P)} = PQ \times \underline{v}$$



$$|\underline{M}_P| = |PQ|\,|\underline{v}|\,\sin\vartheta \; - b = |\underline{v}| \cdot b$$

↳ braccio




DISEGNO $\underline{M}_P$ NEL PIANO CON UNA FRECCIA RICURVA

ES.

$M_A$ ? $\Rightarrow$ $\circlearrowright +$ $M_A = +3 \cdot 2 = 6\ \mathrm{Nm}$