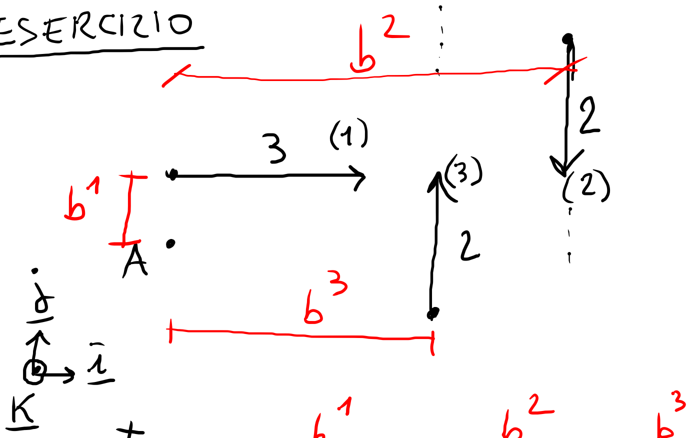
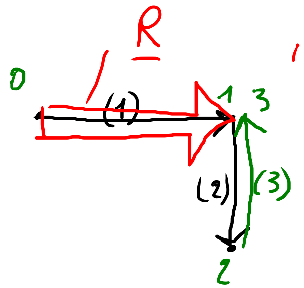
ESERCIZIO

SIST. PIANO DI VETTORI — 3/03/22



$\underline{R}$? M_A?

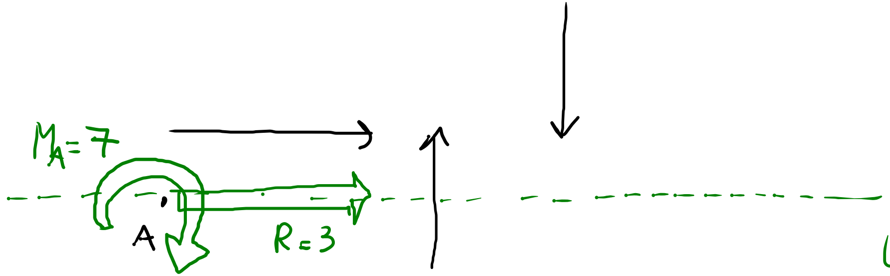
$\underline{R}$: metodo grafico poligono dei vettori.



, $|\underline{R}| = 3$

$$M_A: -3\cdot 1 - 2\cdot 6 + 2\cdot 4 = -7$$

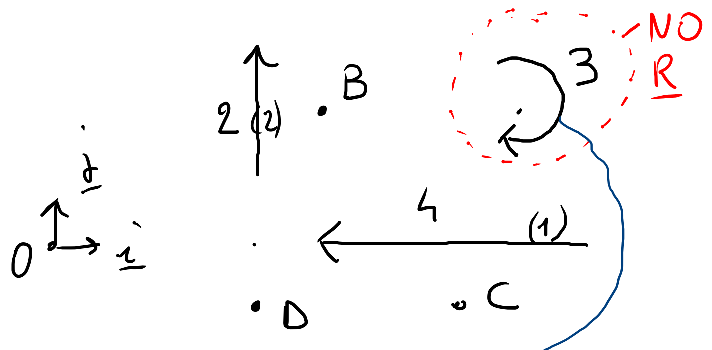
Se i vettori sono FORZE (N), i MOMENTI → Nm



NON è L'ASSE CENTRALE

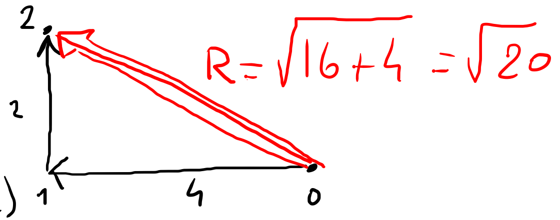
SISTEMA PIANO ⟹ $\underline{R}$, M_A

ES: SIST. PIANO DI VETTORI CON MOMENTO CONCENTRATO



$\underline{R}$? M_B

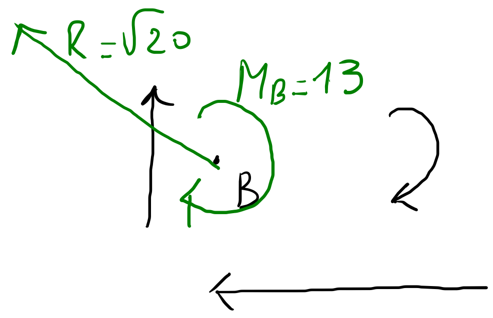
$\underline{R}$:



(1) (2)

$\underline{R} = -4\underline{i} + 2\underline{j}$; $|\underline{R}| = \sqrt{16+4}$

M_B (+): $-4 \cdot 2 - 2 \cdot 1 - 3 = -13$



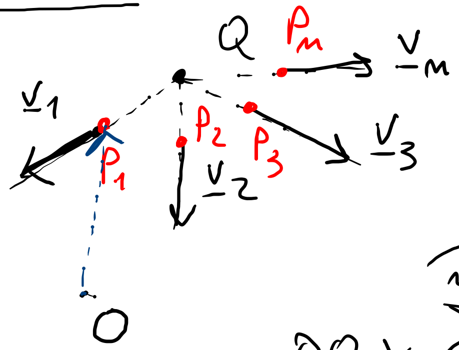
Cosa succede se cambio il polo?

$\underline{R}$ non cambia!

M_B cambia in funzione del polo stesso! ($M_B \neq M_C \neq M_D$)

TEOREMA DI VARIGNON

Nello spazio:



$\underline{M}_O$?

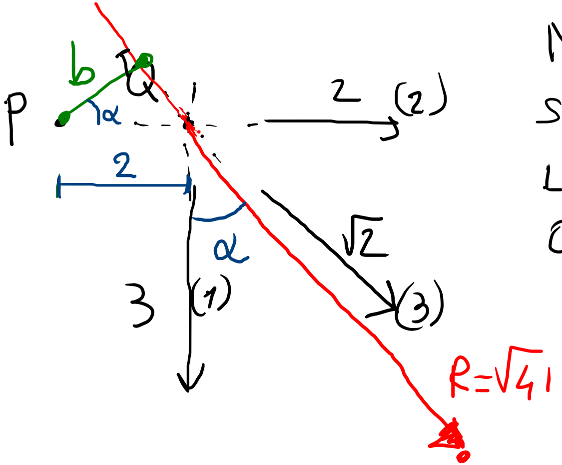
$$\boxed{\underline{M}_O = \sum_{i=1}^{n} OP_i \times \underline{v}_i} = OQ \times \overbrace{\sum_{i=1}^{n} \underline{v}_i}^{\underline{R}} = \boxed{OQ \times \underline{R}}$$

Osservo che $OP_i \times \underline{v}_i = OQ \times \underline{v}_i$ perché il singolo momento non cambia se traslo $\underline{v}_i$ sulla propria retta d'azione.

ENUNCIATO

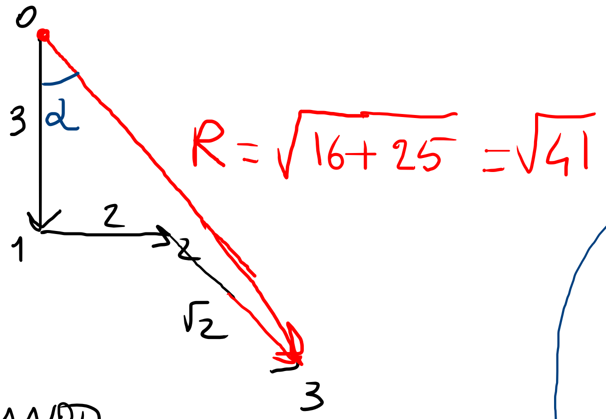
Il Momento del sistema rispetto al punto Q è uguale al Momento della Risultante applicata al punto Q stesso.

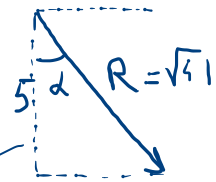
ES.



M_P ?

SFRUTTO IL TH. DI VARIGNON, CALCOLO LA RISULTANTE E LA APPLICO IN Q.





$$\overset{+}{\curvearrowright} \quad M_P = R \cdot b = \sqrt{41} \cdot b \quad (*)$$

ES DA FARE: CALCOLARE M_P (**) CONSIDERANDO I SINGOLI VETTORI; DETERMINARE GEOMETRIC. b E VERIFICARE CHE IL MOM. (*) SIA UGUALE A (**)

AGG. 8/3: $\cos\alpha = 5/\sqrt{41}$

$b = 2\cos\alpha = 10/\sqrt{41}$

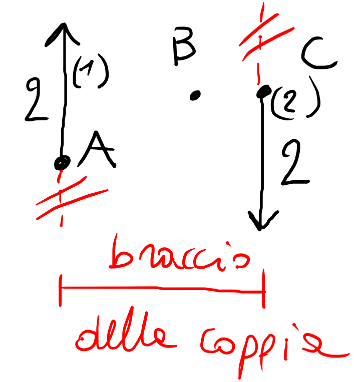
(*) $M_P = \sqrt{41}\,\frac{10}{\sqrt{41}} = 10$

(**) $\overset{+}{\curvearrowright}\; M_P = 3\cdot 2 + 2\cdot 2 = 10$

↑(1) ↑(componente verticale del vett. (3))

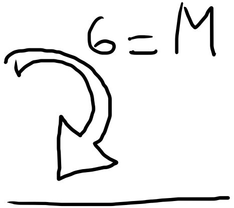
il vettore (2) passa per P e non ha braccio.

UN SISTEMA (PIANO) PARTICOLARE: LA COPPIA



$\underline{R}$? ↕ $\underline{R}$ SEMPRE $\underline{0}$

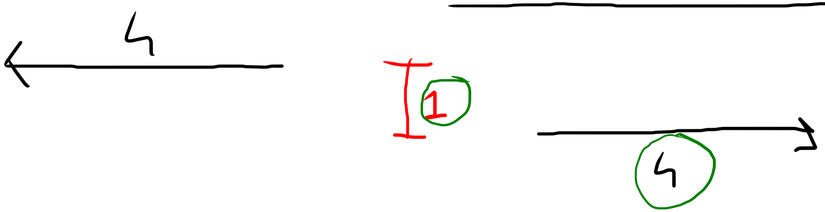
$$M_A = M_B = M_C = M_P \ (\forall P) \Rightarrow \text{COSTANTE} = -6$$



$$\overset{\curvearrowleft +}{M_A}: \ 2 \cdot 0 - 2 \cdot 3 = -6$$

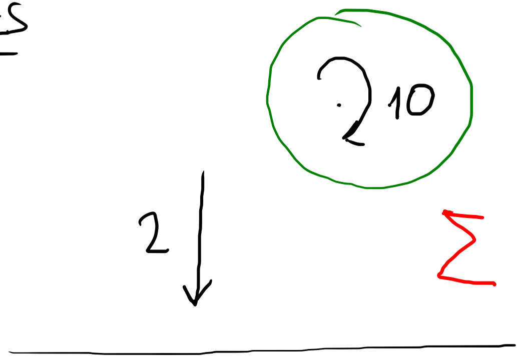
$$\overset{\curvearrowleft +}{M_B}: \ -2 \cdot 2 - 2 \cdot 1 = -6$$

IL MOMENTO DELLA COPPIA È 6 ED È ORARIO $= 2 \cdot \text{braccio}$



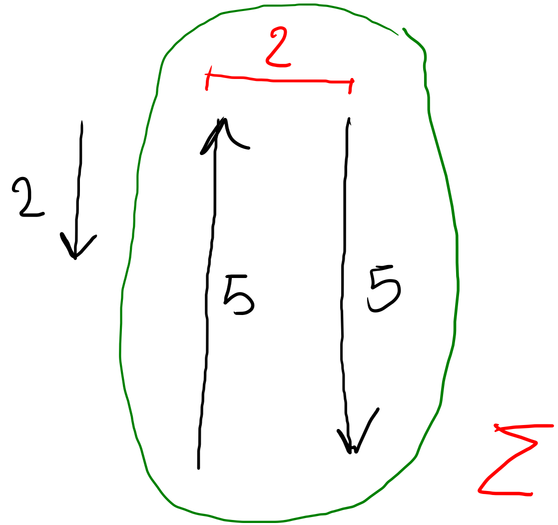
M? ↺ 4

ES

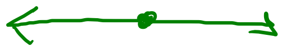


Per il calcolo di $\underline{R}$ e M_p del sistema assegnato (Σ) posso sostituire il Momento concentrato 10 una COPPIA EQUIVALENTE

Σ e Σ' HANNO STESSA $\underline{R}$ e STESSO $M_p, \forall p$



Se il braccio della coppia è NULLO ....



COPPIA DI BRACCIO NULLO

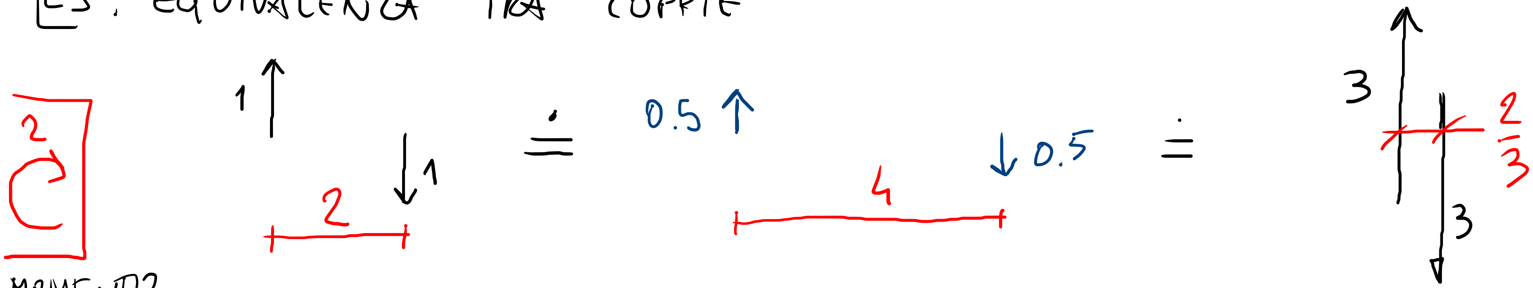
$(\underline{R}=\underline{0})$ $(\underline{M}=\underline{0})$

EQUIVALENZA DI SISTEMI DI VETTORI

DUE SISTEMI DI VETTORI Σ e Σ' SONO **EQUIVALENTI** SE E SOLO SE CONDIVIDONO LA STESSA **RISULTANTE** E LO STESSO **MOMENTO** RISPETTO AD UN POLO GENERICO:

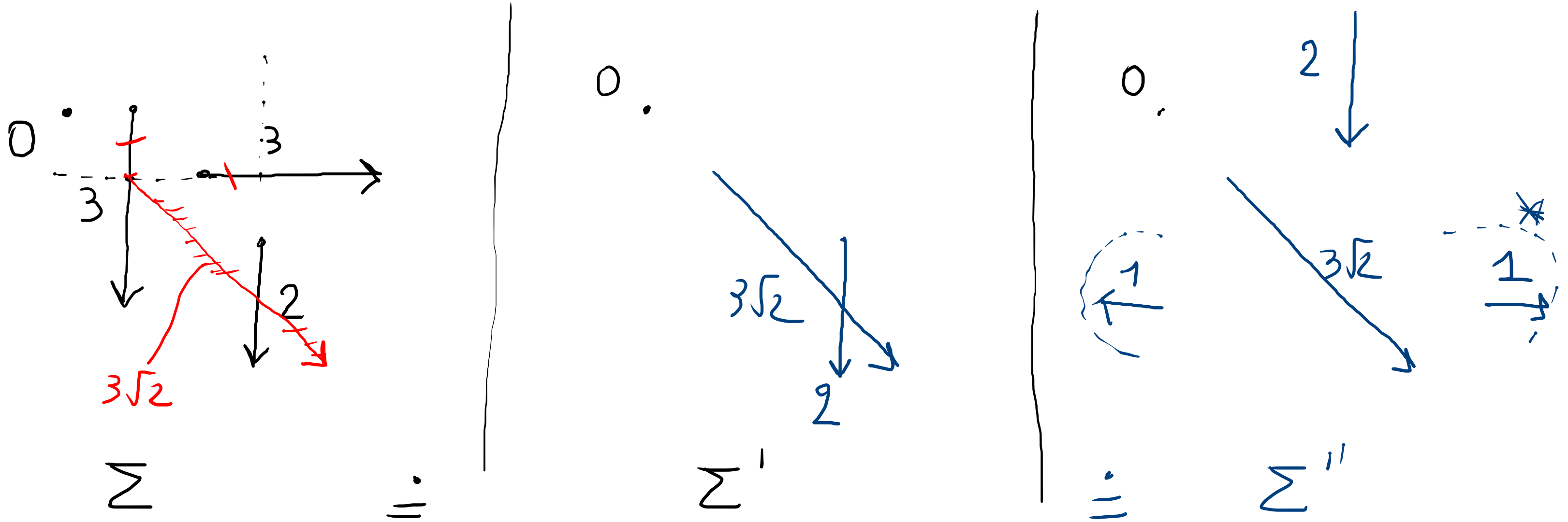
$$\Sigma \doteq \Sigma' \iff \underline{R} = \underline{R}' \quad \text{e} \quad \underline{M}_P = \underline{M}'_P \,, \forall P$$

ES: EQUIVALENZA TRA COPPIE



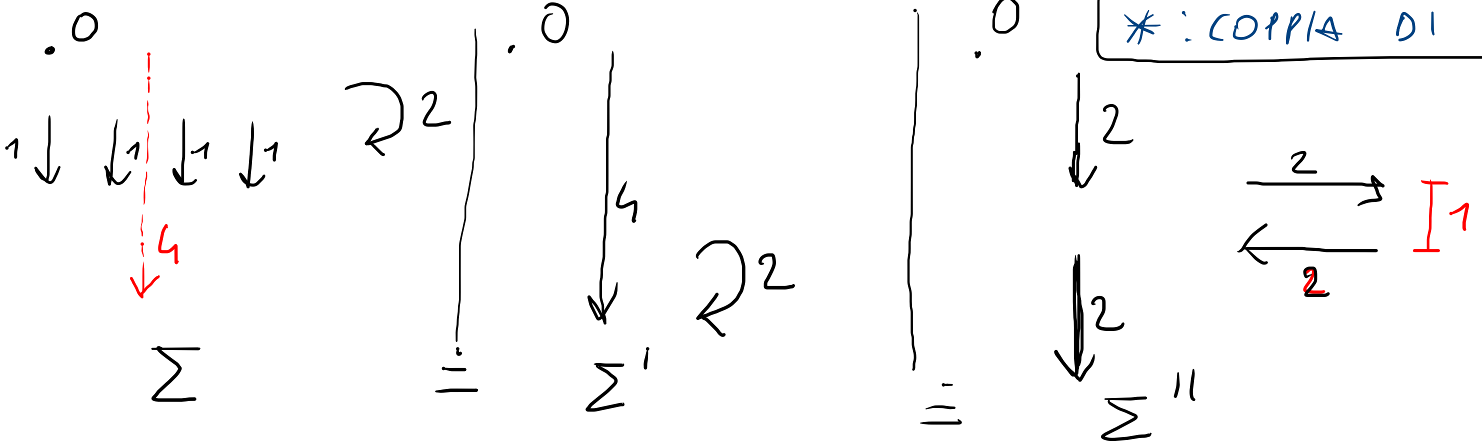
MOMENTO DELLA COPPIA

ES

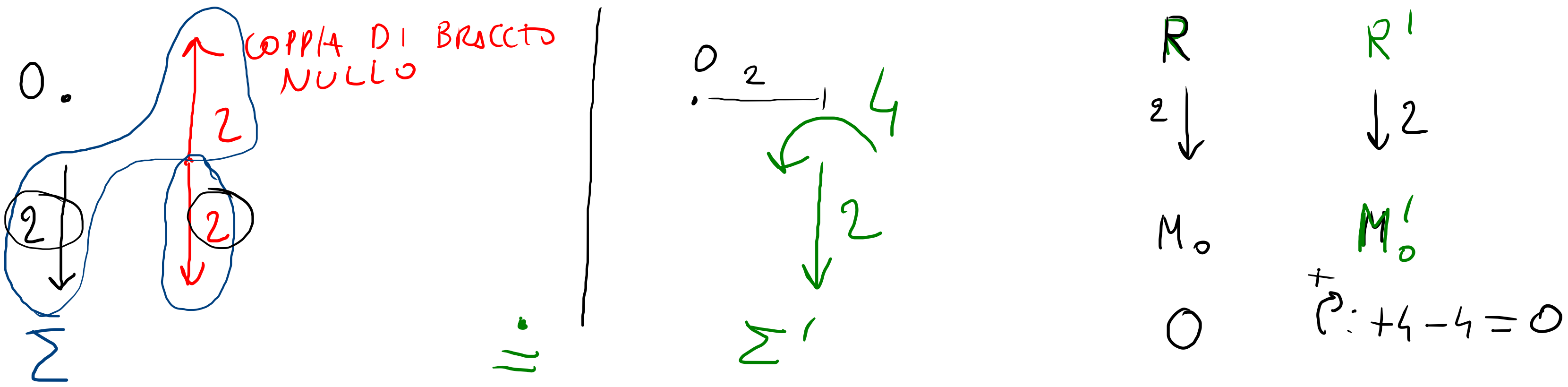


ES

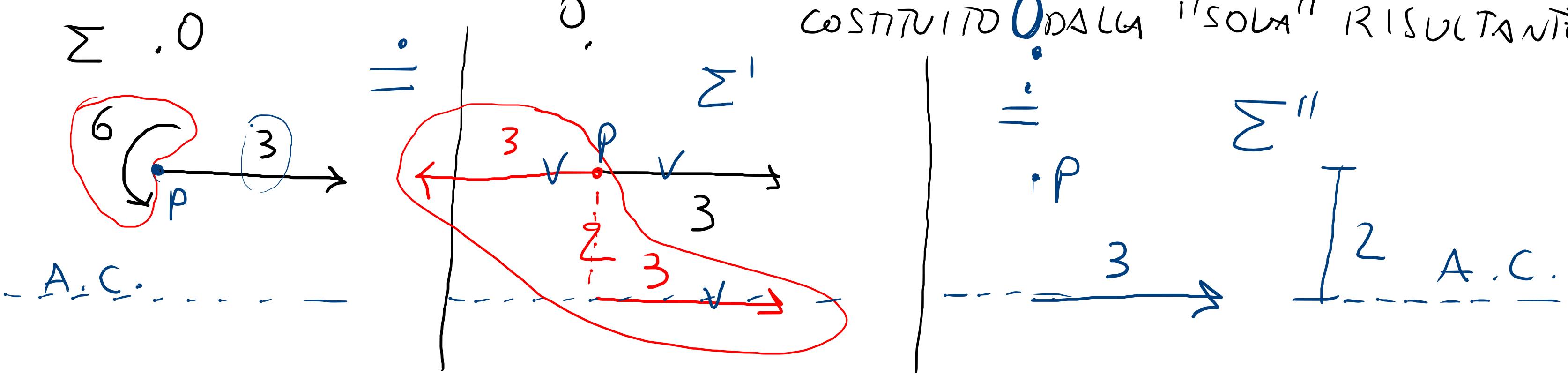
* : COPPIA DI BRACCIO



ES: TRASLAZIONE TRASVERSALE DI UN VETTORE

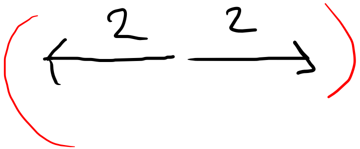


ES: DUALE A QUELLO PRECEDENTE (OTTENERE UN SIST. EQUIVALENTE A Σ COSTITUITO DALLA "SOLA" RISULTANTE)

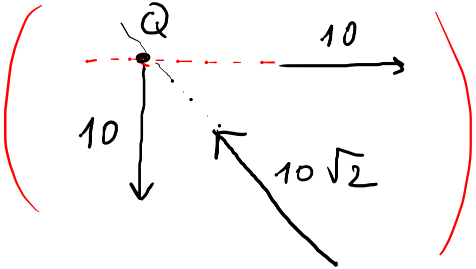


SISTEMA NULLO O EQUILIBRATO

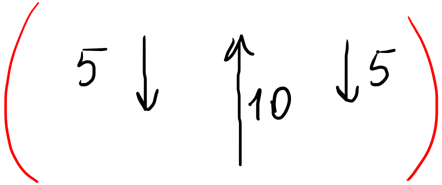
Σ È UN SIST. NULLO O EQUIL. SE $\boxed{\underline{R} = \underline{0}\ ,\ \underline{M}_P = \underline{0}}\ \forall P$



COPPIA DI BRACCIO NULLO $(\underline{R} = \underline{0},\ \underline{M}_P = \underline{0},\ \forall P)$

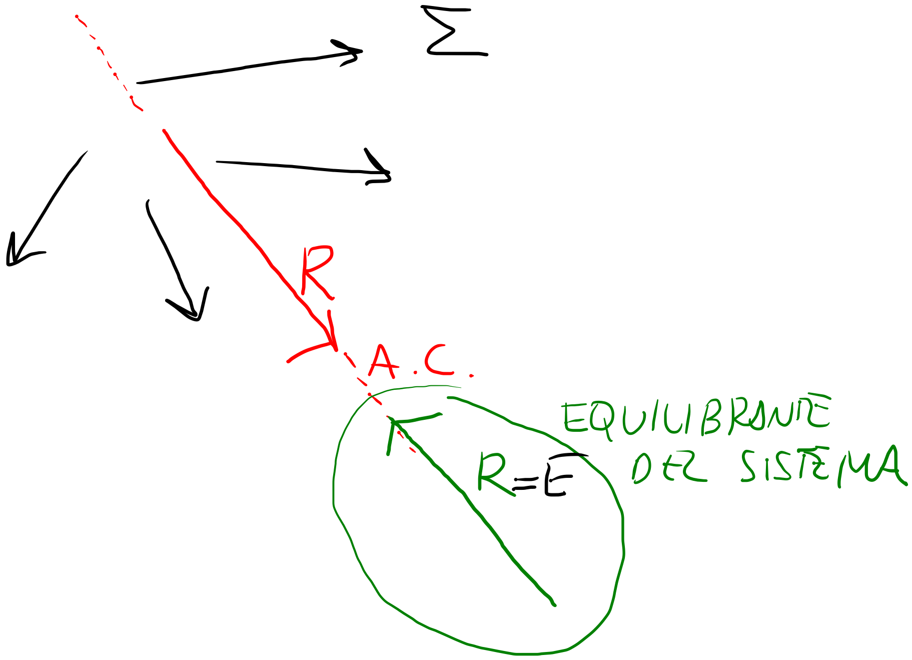


$M_Q = 0,\ R = 0$



SIST. NULLI

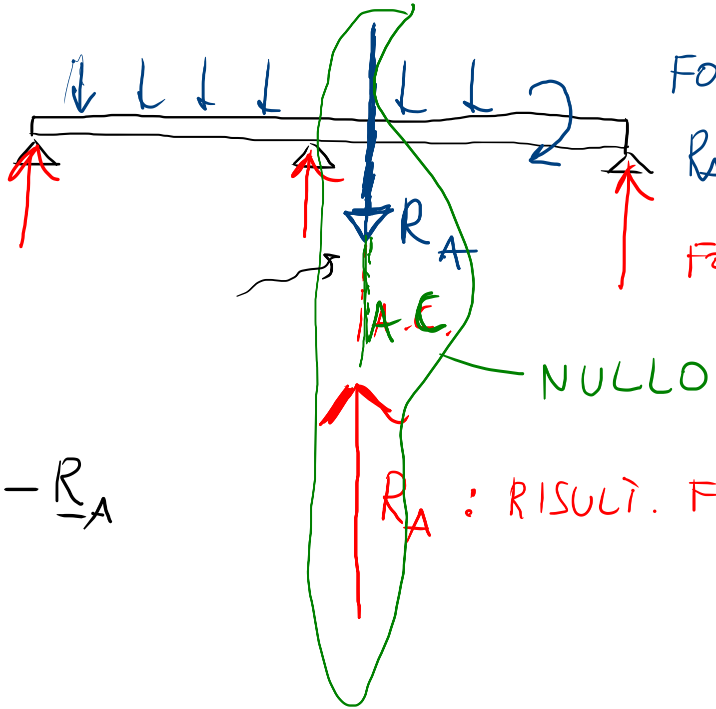
EQUILIBRANTE DI UN SISTEMA



L'EQUILIBR. DI UN SISTEMA È QUEL VETTORE CHE EQUILIBRA O ANNULLA LA RISULTANTE

$$\underline{R} + \underline{E} = \underline{0}$$

OSSERVAZ. -



FORZE ATTIVE

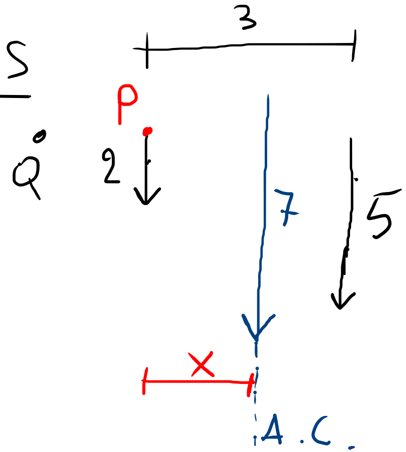
R_A : RISULT. FORZE ATTIVE

FORZE REATTIVE

R_A : RISULT. FORZE REATTIVE = EQUILIBRANTE DELLE FORZE ATTIVE

$\underline{R}_A$, l'equilibr è $-\underline{R}_A$

ES



$\Sigma = \{2, 5\}$

1) TROVA L'ASSE CENTRALE
2) $\underline{R}$? = 7
3) ANNULLA IL SISTEMA

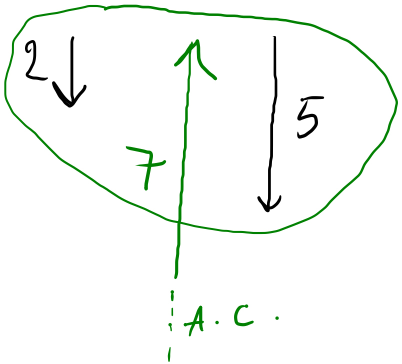
$\Sigma' = \{7\}$ → APPL. ALL'ASSE CENTRALE

CALCOLO x IMPONENDO $M_P^{\Sigma} = M_P^{\Sigma'}$ (+ ↺)

$$2 \cdot 0 - 5 \cdot 3 = -7 \cdot x \Rightarrow +15 = +7x \Rightarrow x = \frac{15}{7} < \frac{21}{7}$$

$\simeq 2.1$

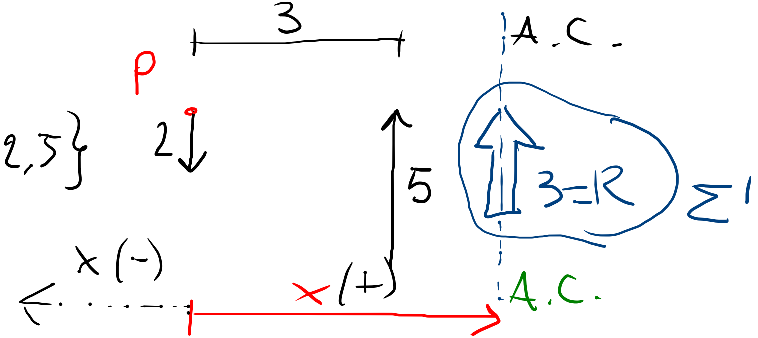
3)



Σ'' NON È EQUIVALENTE A Σ e Σ', Σ'' È NULLO

ES

$\Sigma = \{2, 5\}$

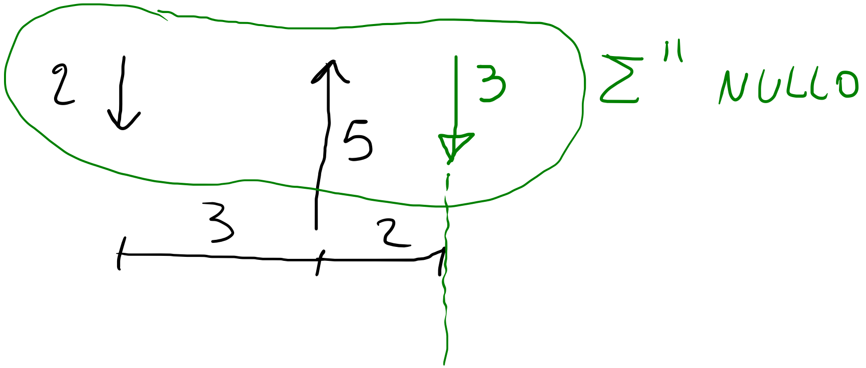


$$M_P^{\Sigma} = M_P^{\Sigma'} \quad (+\circlearrowleft)$$

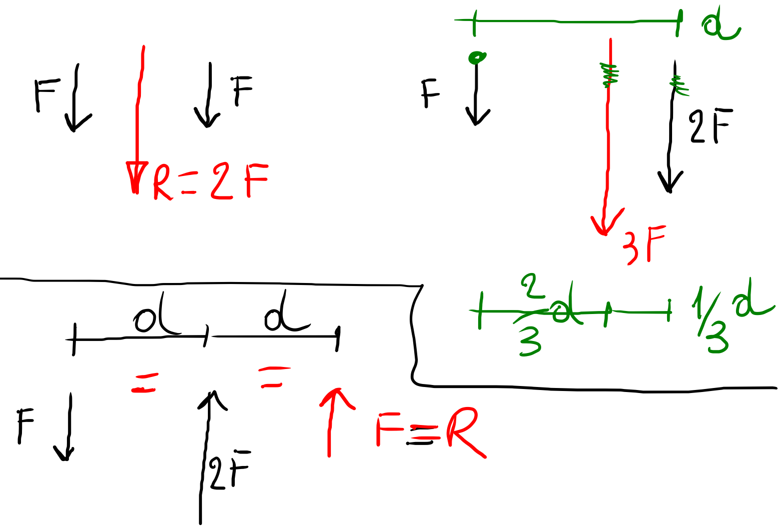
$$\cancel{2 \cdot 0} + 5 \cdot 3 = 3 \cdot X$$

$$\boxed{X = 5} > 3$$

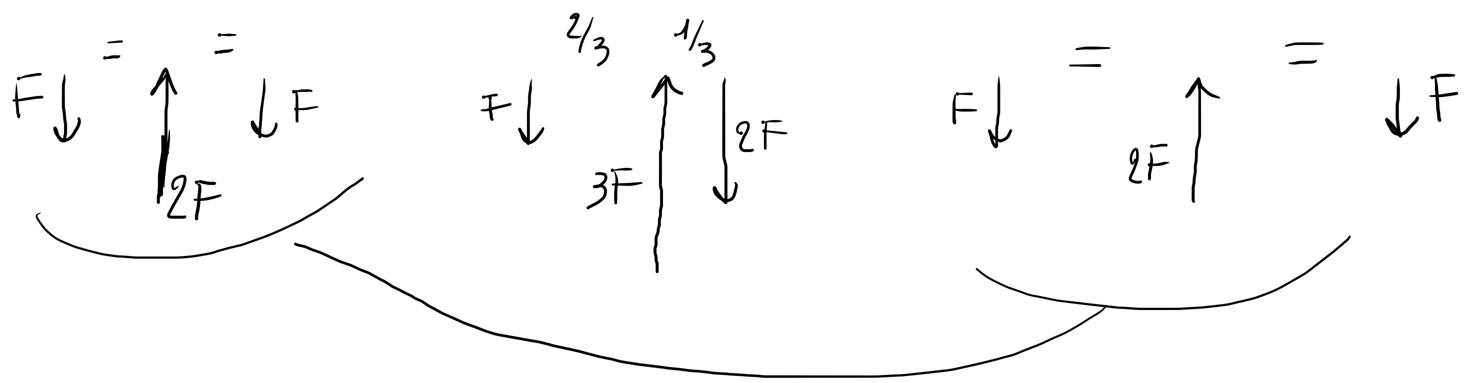
COME POSSO EQUILIBRARE Σ ?



POSIZ. RISULT. SISTEMI NOTEVOLI

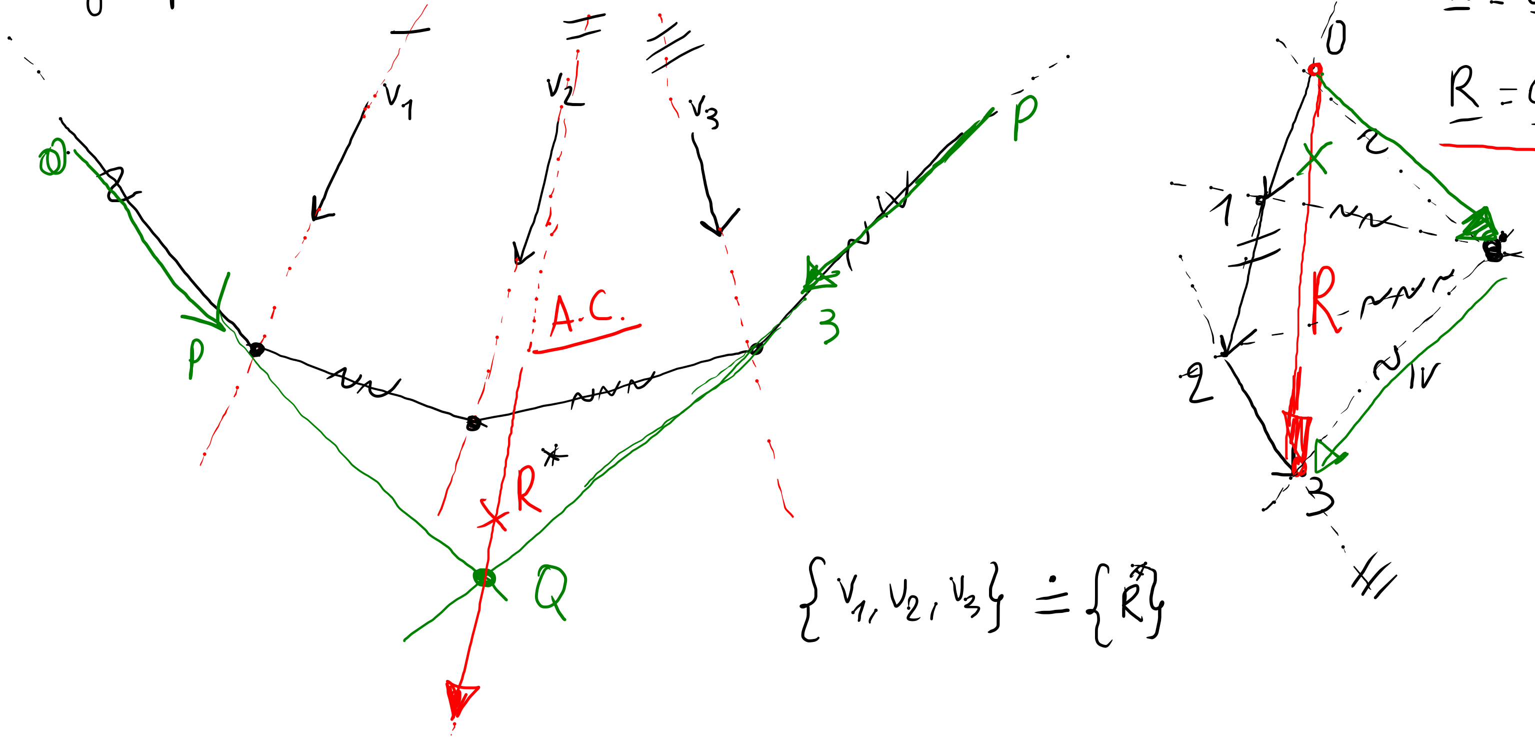


SIST. EQUILIBRATI NOTEVOLI $\left(\underline{R} = \underline{0},\ M_P = 0\ \forall P \right)$



POLIGONO FUNICOLARE (UNA COSTRUZIONE GRAFICA NOTEVOLE)

Assegnato un sist piano di vettori, il P.F. permette di determinare graficamente l'A.C.

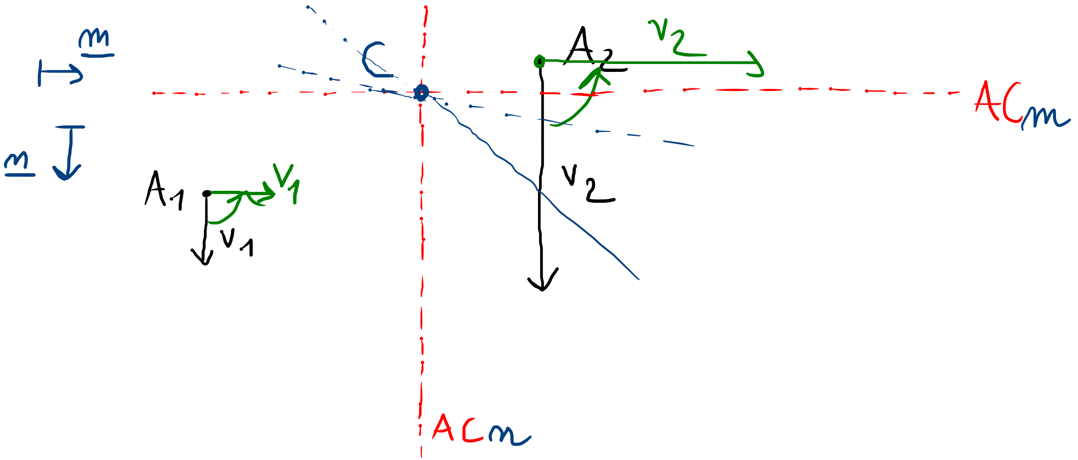


$R = \underline{01} + \underline{12} + \underline{23}$ (v_1, v_2, v_3)

$R = \underline{0P} + \underline{P3}$

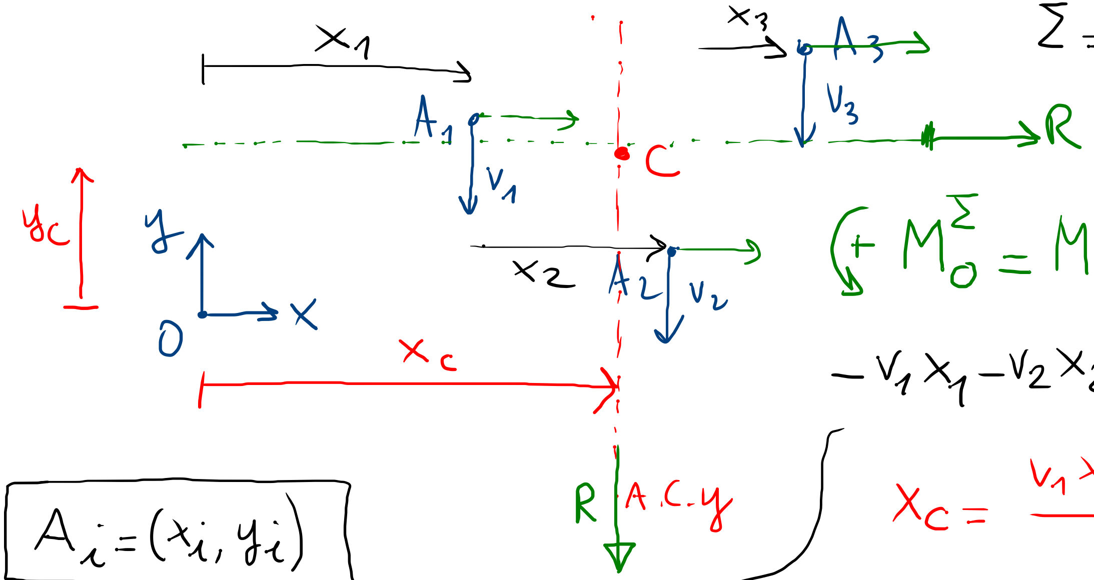
$\{v_1, v_2, v_3\} \doteq \{R^*\}$

CENTRO DI UN SISTEMA DI VETTORI PARALLELI



C: "CENTRO" DEL SISTEMA ASSEGNATO $\{v_1, v_2\}$

Nelle pag successive determiniamo le coord. di C rispetto ad un sist. cartesiano.



$$\Sigma = \{V_1, V_2, V_3\} \; ; \; \Sigma' = \{R\}$$

$$R = V_1 + V_2 + V_3$$

$$(+ \; M_O^{\Sigma} = M_O^{\Sigma'}$$

INCOGNITA

$$-V_1 x_1 - V_2 x_2 - V_3 x_3 = -R x_c$$

$$x_c = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3}{R} = \frac{\sum_{i=1}^{n} V_i x_i}{\sum_{i=1}^{n} V_i}$$

$$y_c = \frac{\sum_{i=1}^{n} V_i y_i}{\sum_{i=1}^{n} V_i}$$

$$\boxed{A_i = (x_i, y_i)}$$

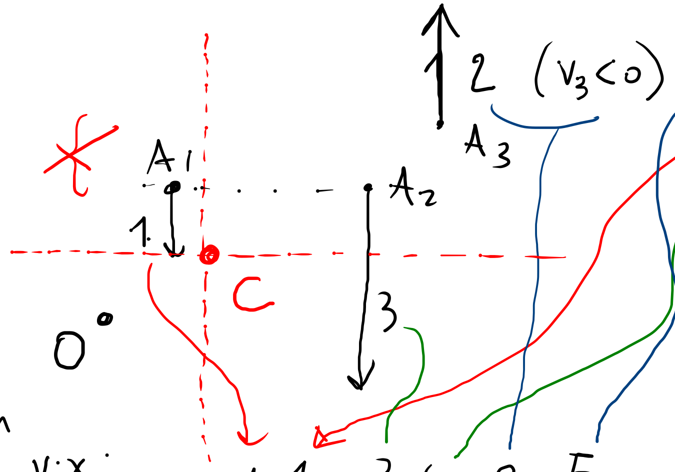
$i = 1, 2, 3$

Ripeto l'esempio con y_c:

COORDINATE DI C

$V_i \gtrless 0$

$V_1 > 0 \downarrow \quad V_2 > 0 \downarrow \quad \uparrow V_3 < 0$



$A_1=(1,2)$; $A_2=(4,2)$; $A_3=(5,3)$

$$R = v_1+v_2+v_3 = +1+3-2 = 2$$

$$x_c = \frac{\sum_{i=1}^{n} v_i x_i}{\sum_{i=1}^{n} v_i} = \frac{1\cdot 1+3\cdot 4-2\cdot 5}{2} = \frac{3}{2}$$

$$y_c = \frac{\cdots\, y_i}{\cdots\cdots} = \frac{1\cdot 2+3\cdot 2-2\cdot 3}{2} = 1$$