Exercises 3

key words: Absolutely continuous functions. Properties of AC functions, differentiability of AC functions. Characterisation of an AC function as the integral function of a L^1 function. Fundamental theorem of integral calculus.

1) Prove that Cantor's function is not an AC function

- with a direct computation;
- using the fundamental theorem of integral calculus.

2) Let $f \in C([0,1])$ and suppose that $f \in AC([\varepsilon,1])$ for all $\varepsilon > 0$.

- prove that if $f \in BV([0,1])$ then $f \in AC([0,1])$;
- find an example for $f \notin AC([0,1])$.

3) Let

$$f_\alpha(x) = \begin{cases} 0 & \text{if } x = 0, \\ x^\alpha \cos(\frac{1}{x}) & \text{if } x \in]0,1]. \end{cases}$$

- For what α we have that $f_\alpha \in BV$?
- For what α we have that $f_\alpha \in AC$?
- For what α, f_α has a bounded derivative in $]0,1[$ and a finite right derivative in 0?