PARTE I (ANALISI DIMENSIONALE)

Es 1

$|v| = 3\ m/s$

$|v| = ?\ \frac{cm}{min}$

$|v| = ?\ \frac{Km}{yr}$

$$|v| = 3\ \frac{m}{s} = 3 \cdot \frac{10^2\ cm}{\frac{1}{60}\ min} = 180 \cdot 10^2\ \frac{cm}{min}$$

$$= 1,8 \cdot 10^4\ \frac{cm}{min}$$

$$|v| = 3\ \frac{m}{s} = 3 \cdot \frac{10^{-3}}{\frac{1}{365 \cdot 24 \cdot 60 \cdot 60}} \left(\frac{Km}{yr}\right) =$$

$$= 9,46 \cdot 10^7 \cdot 10^{-3}\ \frac{Km}{yr} = 9,5\ 10^4\ \frac{Km}{yr}$$

Eg 2

$$x(t) = a t^2 + b t + c$$

$$[a] = L\, T^{-2}$$

$$[b] = L\, T^{-1}$$

$$[c] = L$$

Es 3

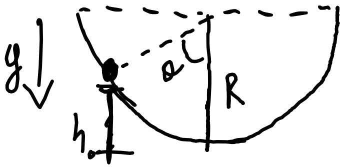
$$F = A \sin(\omega t + \varphi)$$

$$[A] = M L T^{-2}$$

$$[\omega] = T^{-1}$$

$[\varphi] =$ adimensionale

Es 4



$$\theta(t) = \left(\frac{S}{R}\right) \cos(\omega t)$$

$$[\omega] = T^{-1} \quad ; \quad S = L$$

- Significato geometrico di S?

$$S \cos(\omega t) = \theta(t)\, R \Rightarrow \text{a } t = 0 \Rightarrow S = \underline{\underline{\theta(t_0)\, R}}$$

- |v| nel punto più basso? $h_0 = R - R\cos(\theta t_0) = R(1 - \cos\theta(t_0))$

$$[v] \equiv L T^{-1} = [g^\alpha h_0^\beta m^\gamma] = L^\alpha T^{-2\alpha} L^\beta M^\gamma = L^{\alpha+\beta} T^{-2\alpha} M^\gamma$$

$$\left.\begin{array}{l} \alpha+\beta = 1 \Rightarrow \beta = 1/2 \\ -2\alpha = -1 \Rightarrow \alpha = 1/2 \\ \gamma = 0 \end{array}\right\} \Rightarrow g^{\alpha}\, h_0^{\beta}\, m^{\gamma} = g^{1/2}\, h^{1/2}$$

$$|v| \propto \sqrt{h g}$$

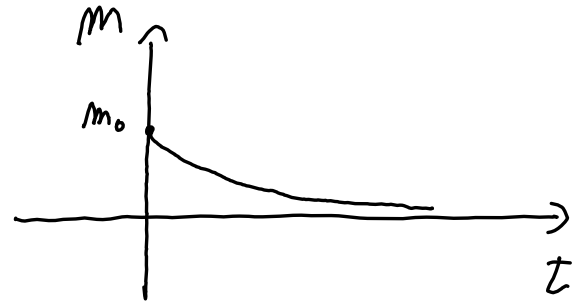
N.B la v non dipende dalla massa!

Es 5

$$m(t) = m_0 e^{-t/\tau}$$

$$[m_0] = M$$

$$[\tau] = T$$



m_0 è la massa al tempo $t=0$

Es 6

$$[F] = [m a] = M L T^{-2}$$

$$[L] = [F \cdot \Delta x] = M L^{2} T^{-2}$$

$$[E_K] = [m v^2] = M L^{2} T^{-2}$$

$$[P] = [F S^{-1}] = M L^{-1} T^{-2}$$

$$F = mg + P \cdot S \Rightarrow M L T^{-2} = M L T^{-2} + M L^{-1} T^{-2} \cdot L^{2} \qquad \text{OK}$$

$$P V = \frac{1}{2} m v^2 + L \Rightarrow M L^{2} T^{-2} = M L^{2} T^{-2} + M L^{2} T^{-2} \qquad \text{OK}$$

$$\frac{L}{v} = M \frac{V P}{g} \Rightarrow M L T^{-1} = M L \qquad \text{NO!}$$

Es 7

$[a] = L T^{-2}$

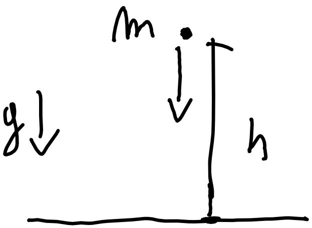
$[q] = M L T^{-1}$

$[F] = M L T^{-2}$

$[t] = T$

$\Rightarrow \left[\frac{F \cdot t}{q}\right] \Rightarrow$ dipende dal tempo!

Es 8



$t = ?$

$v_{max} = ?$

$$[m^\alpha g^\beta h^\gamma] \equiv T$$

$$M^\alpha L^\beta T^{-2\beta} L^\gamma = T$$

$$\Downarrow$$

$$\alpha = 0$$

$$\beta + \gamma = 0 \Rightarrow \gamma = \frac{1}{2}$$

$$-2\beta = 1 \Rightarrow \beta = -\frac{1}{2}$$

$$\Downarrow$$

$$t \propto g^{-1/2} h^{1/2} = \sqrt{\frac{h}{g}}$$

$$[m^\alpha g^\beta h^\gamma] \equiv L T^{-1}$$

$$M^\alpha L^{\beta+\gamma} T^{-2\beta} = L T^{-1}$$

$\alpha = 0$

$\beta + \gamma = 1 \Rightarrow \gamma = \frac{1}{2}$

$-2\beta = -1 \Rightarrow \beta = \frac{1}{2}$

$\Downarrow$

$$v_{max} \propto g^{1/2} h^{1/2} = \sqrt{hg}$$

$$v_{max} \cdot \tau_{max} = \alpha \cdot \beta \sqrt{h\cancel{g}} \cdot \sqrt{\frac{h}{\cancel{g}}} = \alpha \cdot \beta\, h = h$$

$\alpha \cdot \beta = 1 \Rightarrow \alpha = \frac{1}{\beta}$

Es 4

$$\left[v_0^{\alpha} \, g^{\beta} \, m^{\gamma} \right] = [h_{max}]$$

$$L^{\alpha} T^{-\alpha} L^{-\beta} T^{-2\beta} M^{\gamma} = L$$

$$\begin{cases} \alpha + \beta = 1 \\ -\alpha - 2\beta = 0 \\ \gamma = 0 \end{cases}$$

$$\begin{cases} \alpha = -2\beta \\ \beta = -1 \end{cases} \Rightarrow \begin{cases} \alpha = 2 \\ \beta = -1 \end{cases}$$

$$h_{max} \propto \frac{v_0^2}{g}$$

Ej 10

$h \quad \bullet\, m$

$g \downarrow \qquad \uparrow F$

$$[\gamma] = \frac{M \cancel{L} T^{-2}}{\cancel{L}\, \cancel{T^{-1}}} = M T^{-1}$$

$$[\gamma^a h^b g^c m^d] = T$$

$$M^{a+d}\, T^{-a-2c}\, L^{b+c} = T$$

$$\begin{cases} a + d = 0 \\ -a - 2c = 1 \\ b + c = 0 \end{cases}$$

$$\begin{cases} b = -c \\ a = -2c - 1 \\ d = 2c + 1 \end{cases}$$

$$\tau \propto \gamma^{-2c-1}\, h^{-c}\, g^{c}\, m^{2c+1}$$

$$\tau \propto \frac{m}{\gamma} \left(\frac{g\, m^2}{h\, \gamma^2} \right)^c$$

Punto 2

$$M^{a+d}\, T^{-a-2c}\, L^{b+c} = L T^{-1}$$

$$\begin{cases} a + d = 0 \\ -a - 2c = -1 \\ b + c = 1 \end{cases} \Rightarrow \begin{cases} d = 2c - 1 \\ a = 1 - 2c \\ b = 1 - c \end{cases}$$

$$[\gamma^a h^b g^c m^d] = [v_{lim}]$$

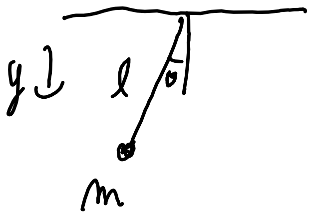
$$v_{lim} \propto \gamma^{1-2c} \, h^{1-c} \, g^c \, m^{2c-1}$$

$$v_{lim} \propto \frac{\gamma h}{m}\left(\frac{g\, m^2}{h\,\gamma^2}\right)^c$$

la v_{lim} non deve dipendere da h! $\Rightarrow c = 1$

$$\Rightarrow v_{lim} \propto \frac{\cancel{\gamma}\,\cancel{h}}{\cancel{m}} \, \frac{g\, m^{\cancel{2}}}{\cancel{h}\,\gamma^{\cancel{2}}} = \frac{m g}{\gamma}$$

Es 11



→ determina il periodo

$$[g^{\alpha} l^{\beta} m^{\gamma}] = T$$

$$L^{\alpha} T^{-2\alpha} L^{\beta} M^{\gamma} = T$$

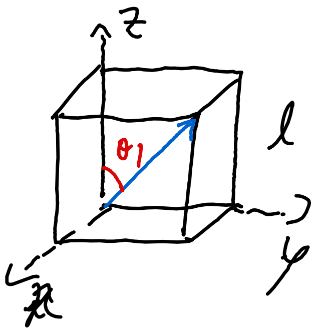
$$\begin{cases} \alpha + \beta = 0 \\ -2\alpha = 1 \\ \gamma = 0 \end{cases} \Rightarrow \begin{cases} \beta = 1/2 \\ \alpha = -1/2 \\ \gamma = 0 \end{cases} \Rightarrow \quad \tau \propto g^{-1/2} l^{1/2}$$

$$\Downarrow$$

$$\tau \propto \sqrt{\frac{l}{g}}$$

Parte II

Es 1



$$\bar{d} = l\,\hat{i} + l\,\hat{j} + l\,\hat{k} = l(\hat{i} + \hat{j} + \hat{k})$$

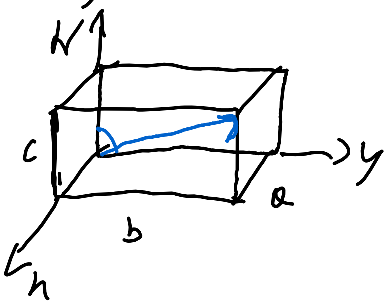
$$|d| = l\sqrt{3}$$

$$\bar{d} \cdot \hat{i} = l\,\hat{i} \Rightarrow l\sqrt{3}\cos\theta_1 = l$$

$$\Rightarrow \cos\theta_1 = \frac{1}{\sqrt{3}} \Rightarrow \theta_1 = \arccos\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta_1 = \theta_2 = \theta_3$$

b)



$$d = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

$$|d| = \sqrt{a^2+b^2+c^2}$$

$$\bar{d}\cdot\hat{\imath} = a\cdot\hat{\imath} \Rightarrow \sqrt{a^2+b^2+c^2}\,\cos\theta_1 = a$$

$$\theta_1 = \arccos\frac{a}{\sqrt{a^2+b^2+c^2}} = \arccos\left(\frac{1}{\sqrt{14}}\right)$$

$$\theta_2 = \arccos\frac{b}{\sqrt{\ldots}} = \arccos\left(\frac{2}{\sqrt{14}}\right)$$

$$\theta_3 = \arccos\frac{c}{\sqrt{}} = \arccos\left(\frac{3}{\sqrt{14}}\right)$$

Es 2

a) $c = \alpha a + \beta b$ $\qquad$ a e b non //

$$\begin{cases} \bar{c}\cdot\bar{a} = \alpha a^2 + \beta(\bar{b}\cdot\bar{a}) \\ \bar{c}\cdot\bar{b} = \alpha\, \bar{a}\cdot\bar{b} + \beta b^2 \end{cases}$$

$$\Delta = a^2 b^2 - (\bar{a}\cdot\bar{b})^2$$

$$\Delta_\alpha = \bar{c}\cdot\bar{a}\, b^2 - (\bar{c}\cdot\bar{b})(\bar{b}\cdot\bar{a})$$

$$\Delta_\beta = a^2\, \bar{c}\cdot\bar{b} - (\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{a})$$

$$\beta = \frac{\Delta_\beta}{\Delta} = \frac{a^2(\bar{c}\cdot\bar{b}) - (\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{a})}{a^2b^2 - (\bar{a}\cdot\bar{b})^2}$$

$$\alpha = \frac{\Delta_\alpha}{\Delta} = \frac{b^2\, \bar{c}\cdot\bar{a} - (\bar{c}\cdot\bar{b})(\bar{b}\cdot\bar{a})}{a^2b^2 - (\bar{a}\cdot\bar{b})^2}$$

b) $\bar{a}\,;\bar{b}$ $\qquad \bar{c} \perp \bar{b}$

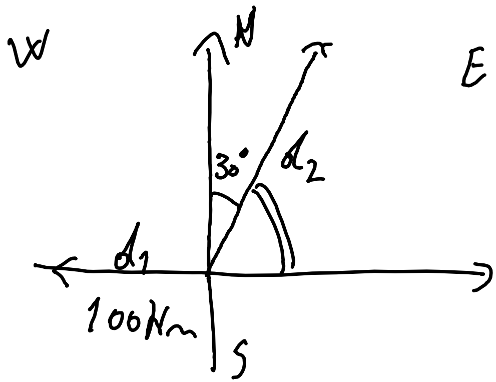
$$\bar{a} = \beta \bar{b} + c$$

$$\bar{a} \cdot \bar{b} = \beta b^2 + 0 \Rightarrow \beta = \frac{\bar{a} \cdot \bar{b}}{b^2}$$

$$c = \bar{a} - \frac{\bar{a} \cdot \bar{b}}{b^2} \bar{b}$$

$$\bar{a} = \left(\frac{\bar{a} \cdot \bar{b}}{b^2}\right) \bar{b} + \left(\bar{a} - \frac{\bar{a} \cdot \bar{b}}{b^2} \bar{b}\right)$$

Es 3



$$d_1 = -a\,\hat{\imath}$$

$$d = |d|\cos(60)\,\hat{\imath} + |d|\sin(60)\,\hat{\jmath}$$

$$= \frac{c}{2}\,\hat{\imath} + \frac{\sqrt{3}}{2}c\,\hat{\jmath}$$

$$\bar{d}_1 + \bar{d}_2 = \underbrace{\left(\frac{|d|}{2} - a\right)}_{=0}\hat{\imath} + \left(\frac{\sqrt{3}}{2}c\right)\hat{\jmath}$$

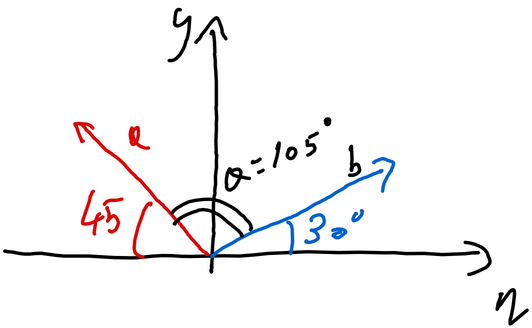
$$a = \frac{|d|}{2} \Rightarrow |d| = 2a = 200\ Km$$

$$|\overline{\Delta d}| = 100\sqrt{3}\ Km$$

Es 4

$|a| = 2\sqrt{2}$

$|b| = 2\sqrt{3}$



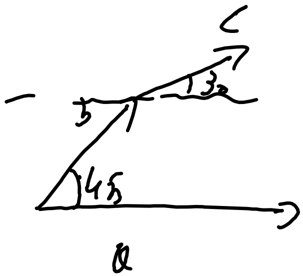
$$\bar{a}\cdot\bar{b} = |a||b|\cos\theta = 2\sqrt{2}\cdot 2\sqrt{3}\cdot\cos(105) = 4\sqrt{6}\cos(105) =$$

$$= -4\sqrt{6}\sin(15) = -\cancel{4}\sqrt{6}\,\frac{\sqrt{6}-\sqrt{2}}{\cancel{4}} = (2\sqrt{3}-6)$$

$$\bar{a}\times\bar{b} = 4\sqrt{6}\cos 15 = \cancel{4}\sqrt{6}\cdot\frac{\sqrt{6}+\sqrt{2}}{\cancel{4}} = (6+2\sqrt{3})\,k$$

entrante verso il foglio.

Ej 5



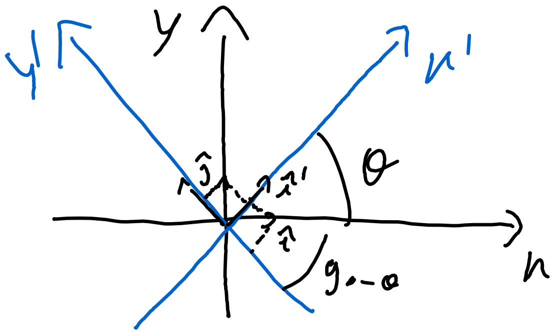
$$\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot c = ab\cos 45 + ac\cos 30 = \frac{a}{2}\left(\sqrt{2}\,b+\sqrt{3}\,c\right)$$

$$\bar{a}\times(\bar{b}+\bar{c}) = \bar{a}\times\bar{b} + \bar{a}\times\bar{c} = (ab\sin 45 + ac\sin 30)\hat{x} = \frac{a}{2}(b\sqrt{2}+c)$$

Es 6

$a = (1,1)$

$b = (2,0)$ in Oxy



$$x' = x \cos\theta + y \sin\theta$$

$$y' = -x \sin\theta + y \cos\theta$$

$$\Rightarrow \quad x' = \frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y$$

$$y' = -\frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y$$

$a' = (\sqrt{2}, 0)$; $b' = (\sqrt{2}, -\sqrt{2})$

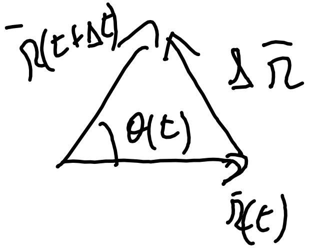
Es 7

$$|v(t)| = \text{cost} \Rightarrow \bar{v}(t) \cdot \bar{v}(t) = \text{cost}$$

$$\frac{d}{dt}\left(\bar{v}(t) \cdot \bar{v}(t)\right) = \frac{d\bar{v}(t)}{dt} \cdot \bar{v}(t) + \bar{v}(t)\,\frac{d\bar{v}(t)}{dt} = \frac{d}{dt}(\text{cost}) = 0$$

$$\Rightarrow \bar{v}(t) \cdot \frac{d\bar{v}(t)}{dt} = 0 \Rightarrow \bar{v}(t) \perp \frac{d\bar{v}(t)}{dt}$$

Es 8



$\Rightarrow$ $$\frac{\Delta \bar R}{\Delta t} = \frac{\Delta R}{\Delta \theta} \cdot \frac{\Delta \theta}{\Delta t}$$

$$|\Delta \bar R| = 2|\bar R(t)| \sin\left(\frac{\Delta\theta}{2}\right)$$

$$\frac{\Delta \bar R(t)}{\Delta t} = \frac{|R(t)| \sin(\Delta\theta/2)}{\frac{\Delta\theta}{2}} \cdot \frac{\Delta\theta}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\Delta R(t)}{\Delta t} = \frac{d\theta}{dt} \hat n$$