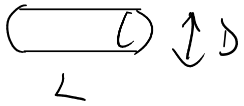
PROPAGAZIONE DELLE INCERTEZZE



$D = (1.35 \pm 0.05)\ \text{cm}$

$L = (5.15 \pm 0.05)\ \text{cm}$

$V = \frac{\pi}{4} D^2 L = A\,L$

$A = \frac{\pi}{4} D^2 = 1.4314\ \text{cm}^2$

$$A_{max} = \frac{\pi}{4}(D + \Delta D)^2 = \frac{\pi}{4}(D^2 + 2D\Delta D + \Delta D^2)$$

$$A_{min} = \frac{\pi}{4}(D - \Delta D)^2 = \frac{\pi}{4}(D^2 - 2D\Delta D + \Delta D^2)$$

$$\Delta A \equiv \frac{A_{max} - A_{min}}{2} = \frac{\pi}{4}\, 2 D \Delta D = A\, 2 \frac{\Delta D}{D} \Rightarrow \frac{\Delta A}{A} = 2 \frac{\Delta D}{D}$$

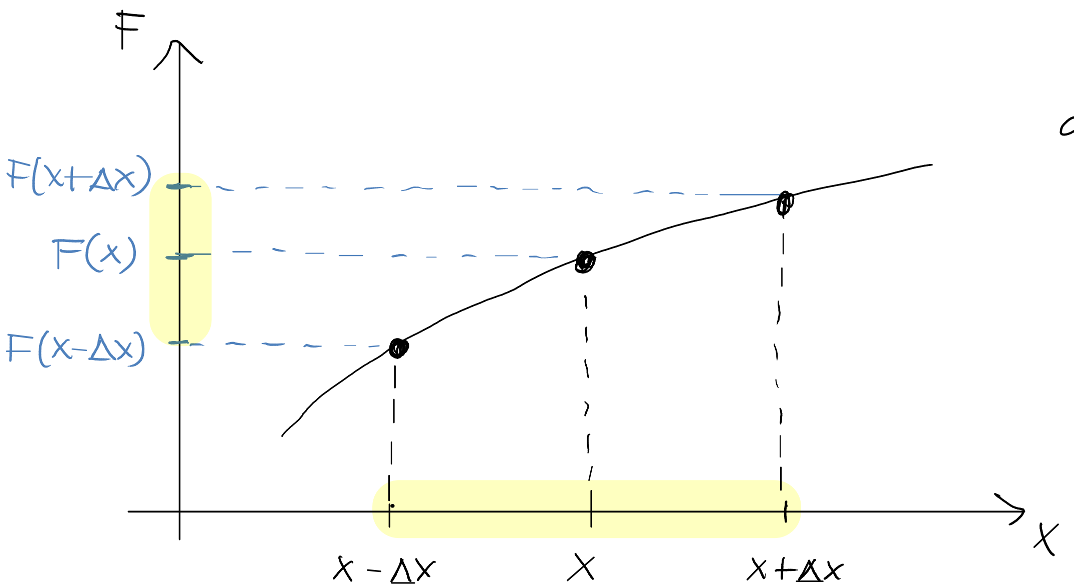
$$\frac{\Delta D}{D} = 0.037 \qquad \frac{\Delta A}{A} = 0.074 \qquad \Delta A = 0.074 \times 1.4314\ \text{cm}^2 = 0.106\ \text{cm}^2 = 0.11\ \text{cm}^2$$

$$A = (1.43 \pm 0.11)\ \text{cm}^2$$

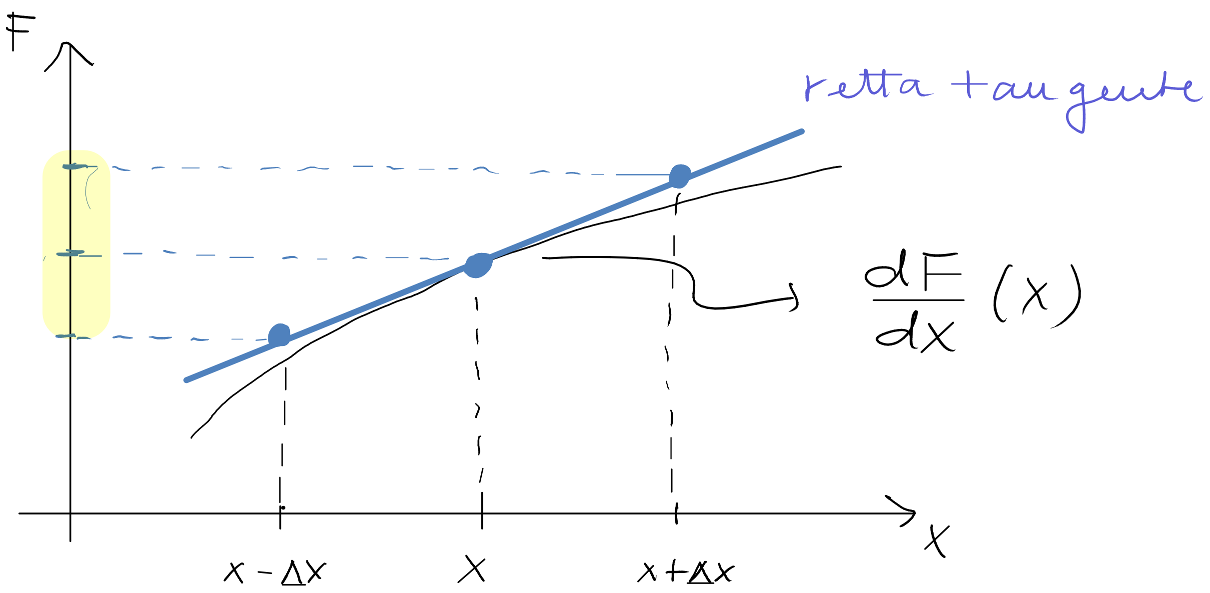
Regola pratica: nel caso di funzioni che coinvolgono coefficienti esatti e/o leggi di potenza, conservo il numero di cifre significative

Propagazione dell'incertezza : funzioni di 1 variabile

$$X \pm \Delta X \qquad F(X) \rightarrow \Delta F = ?$$



approssimazione LINEARE



Sviluppo di Taylor di F nell' intorno di X

$$F(X+\Delta X) = F(X) + \frac{dF}{dX}\Delta X + \frac{1}{2}\frac{d^2F}{dX^2}\Delta X^2 + O(\Delta X^3)$$

$$F(X-\Delta X) = F(X) - \frac{dF}{dX}\Delta X + \frac{1}{2}\frac{d^2F}{dX^2}\Delta X^2 + O(\Delta X^3)$$

regola di propagazione

$$\Delta F \equiv \frac{|F(X+\Delta X) - F(X-\Delta X)|}{2} = \left|\frac{dF}{dX}\right|\Delta X + O(\Delta X^3) \Rightarrow \boxed{\Delta F = \left|\frac{dF}{dX}\right|\Delta X}$$

Es: $F(x) = cx^2$ $\quad x \pm \Delta x \rightarrow \Delta F = ?$

$$\frac{dF}{dx} = 2cx \qquad \Delta F = 2|c||x|\Delta x \rightarrow \frac{\Delta F}{|F|} = 2\frac{\Delta x}{|x|}$$

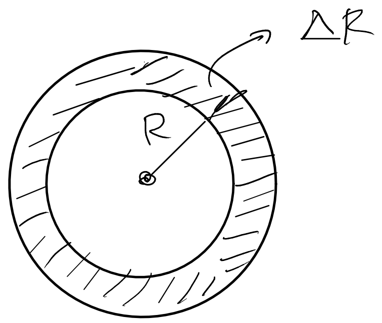
Approssimazioni lineari utili $\quad |x| \ll 1$

1) $(1+x)^\alpha \approx 1 + \alpha x \rightarrow \dfrac{1}{1+x} \approx 1 - x$

2) $\exp(x) \approx 1 + x$

3) $\log(1+x) \approx x$

Es: $\Delta R \ll R$ $\quad V = \frac{4}{3}\pi(R+\Delta R)^3 - \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3\left[\left(1+\frac{\Delta R}{R}\right)^3 - 1\right]$



$$\approx \frac{4}{3}\pi R^3\left(\cancel{1} + 3\frac{\Delta R}{R} - \cancel{1}\right) = 4\pi R^2 \, \Delta R$$

superficie spessore

$$V = \int_0^R dV = \int_0^R 4\pi r^2 dr$$

Propagazione delle incertezze: funzione di 2 variabili

$$X \pm \Delta X \,, \quad Y \pm \Delta Y \rightarrow F(X,Y) \rightarrow \Delta F = ?$$

$$\Delta F \equiv \frac{F_{max} - F_{min}}{2}$$

$$\downarrow$$

1) **Somma**: $F = X + Y$

$$\left.\begin{aligned} F_{max} &= (X+\Delta X) + (Y+\Delta Y) = X+Y+(\Delta X+\Delta Y) \\ F_{min} &= (X-\Delta X) + (Y-\Delta Y) = X+Y-(\Delta X+\Delta Y) \end{aligned}\right\} \Rightarrow \Delta F = \Delta X + \Delta Y$$

2) **Differenza**: $F = X - Y$

$$\left.\begin{aligned} F_{max} &= (X+\Delta X) - (Y-\Delta Y) = X-Y+(\Delta X+\Delta Y) \\ F_{min} &= (X-\Delta X) - (Y+\Delta Y) = X-Y-(\Delta X+\Delta Y) \end{aligned}\right\} \Rightarrow \Delta F = \Delta X + \Delta Y$$

(+) e (−): Sommo le incertezze assolute

3) **Prodotto**: $F = X \cdot Y \qquad X, Y > 0$

$$\left.\begin{aligned} F_{max} &= (X+\Delta X)(Y+\Delta Y) = XY + X\Delta Y + Y\Delta X + \Delta X \Delta Y \\ F_{min} &= (X-\Delta X)(Y-\Delta Y) = XY - X\Delta Y - Y\Delta X + \Delta X \Delta Y \end{aligned}\right\} \Rightarrow \frac{\Delta F}{F} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$$

$$\frac{\Delta F}{F} = \frac{X\Delta Y + Y\Delta X}{XY}$$

4) **Divisione**: $F = X/Y$

$$\begin{aligned} F_{max} &= \ldots \\ F_{min} &= \end{aligned}$$

(×) e (:): Sommo le incertezze relative

5) **Legge di potenza**: $F = X^a Y^b \Rightarrow \dfrac{\Delta F}{F} = |a| \dfrac{\Delta X}{X} + |b| \dfrac{\Delta Y}{Y}$

Applicazione : $V = \frac{\pi}{4} D^2 L = A L$

$A = \frac{\pi}{4} D^2 \qquad \frac{\Delta A}{A} = 2 \frac{\Delta D}{D} = 0.074$

$$\frac{\Delta V}{V} = 2 \frac{\Delta D}{D} + \frac{\Delta L}{L} = \frac{\Delta A}{A} + \frac{\Delta L}{L} = 0.074 + 0.0097 = 0.0837 = 8.37\%$$

$$\Delta V = 0.0837 \times \underbrace{1.4314 \times 5.15}_{V} = 0.617\ cm^3 \qquad V = 7.3\overset{5}{\cancel{717}}\ cm^3$$

$$V \pm \Delta V = (7.4 \pm 0.6)\ cm^3$$

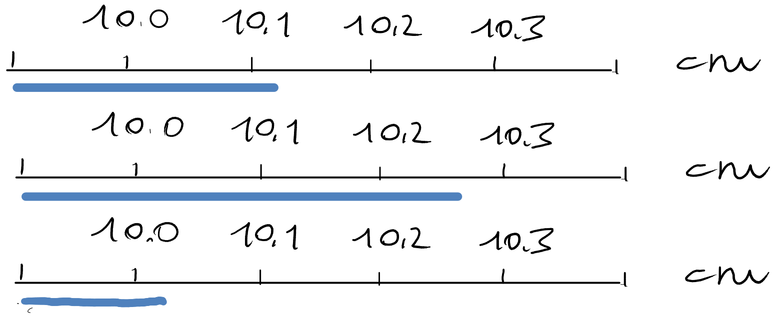
<u>ES.</u>! incertezza su $F = X^2 - Y^2$ con $X \pm \Delta X$, $Y \pm \Delta Y$ $\rightarrow$ $\Delta F = ?$ $\Delta(X^2 - Y^2) = ?$

Applicazione numerica: $X = (1.01 \pm 0.01)$ $\quad Y = (1.00 \pm 0.01)$

$F = 0.02 \pm 0.04$

Incertezze statistiche e loro propagazione

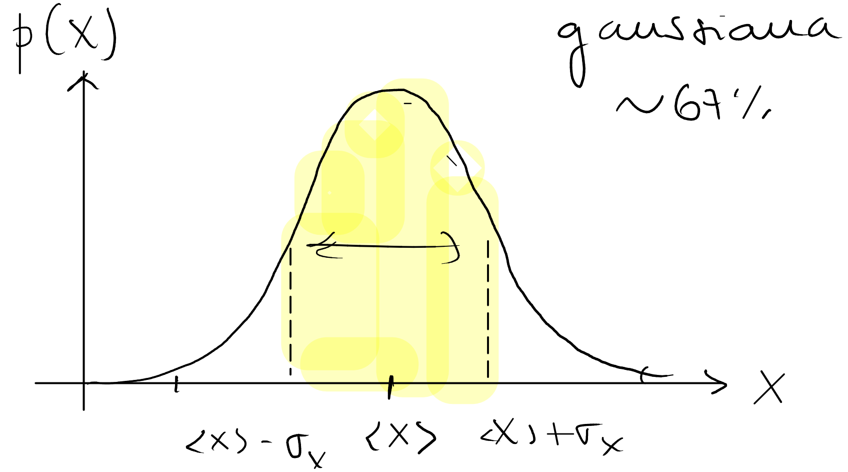
$X_1, X_2, \dots, X_N$



$x = (10{,}15 \pm 0.05)$ cm

$x = (10{,}25 \pm 0.05)$ cm

$x = (10{,}05 \pm 0.05)$ cm



Valore medio : $\langle X \rangle \equiv \frac{1}{N} \sum_{i=1}^{N} X_i$

Deviazione standard: $\sigma_X \equiv \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \langle X_i \rangle)^2}$

1) Somma e differenza : $\sigma_F^2 = \sigma_X^2 + \sigma_Y^2$

2) Prodotto e divisione : $\left(\frac{\sigma_F}{\langle F \rangle}\right)^2 = \left(\frac{\sigma_X}{\langle X \rangle}\right)^2 + \left(\frac{\sigma_Y}{\langle Y \rangle}\right)^2$