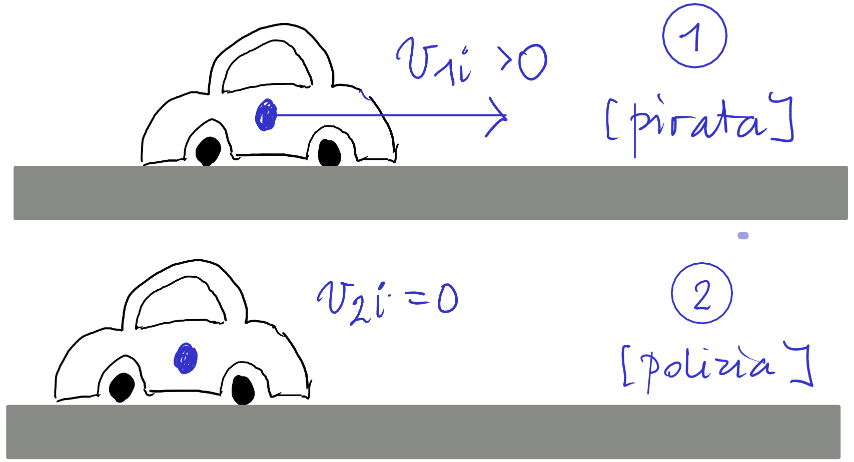
# CINEMATICA

Moto dei corpi, indipendentemente dalle cause del moto → modello: particella

## CINEMATICA 1D



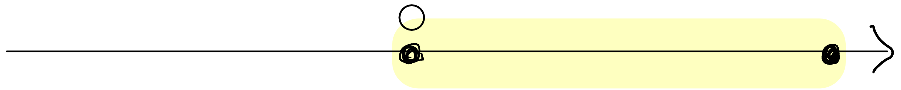

velocità costante
accelerazione costante

→ Dopo quanto tempo 2 raggiunge 1 ?

$(t_i, x_i)$ $(t_A, x_A)$

$(t_f, x_f)$ $(t_B, x_B)$

Metro, cronometro



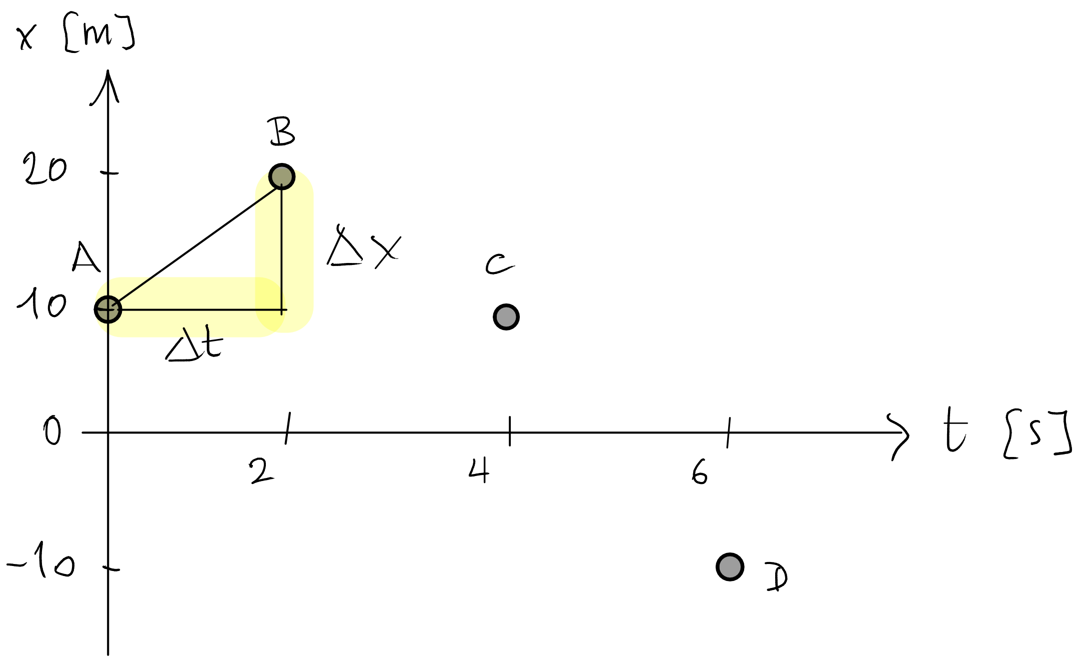

**Posizione**: distanza tra la particella e un punto di riferimento (0) con segno meno se dalla parte opposta della freccia — relative

**Istante di tempo**: lettura del cronometro

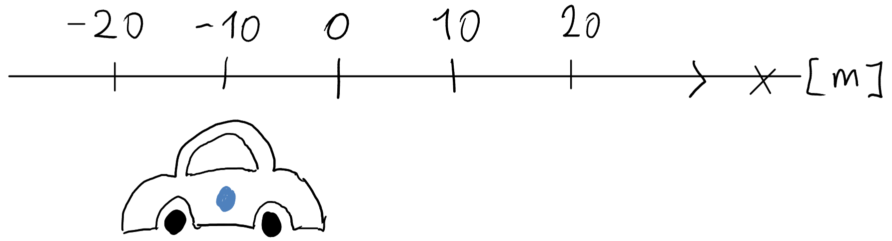
**Intervallo di tempo**: $\Delta t \equiv t_f - t_i$

**Spostamento**: $\Delta x \equiv x_f - x_i$

**Velocità** : quanto rapidamente cambia la posizione?



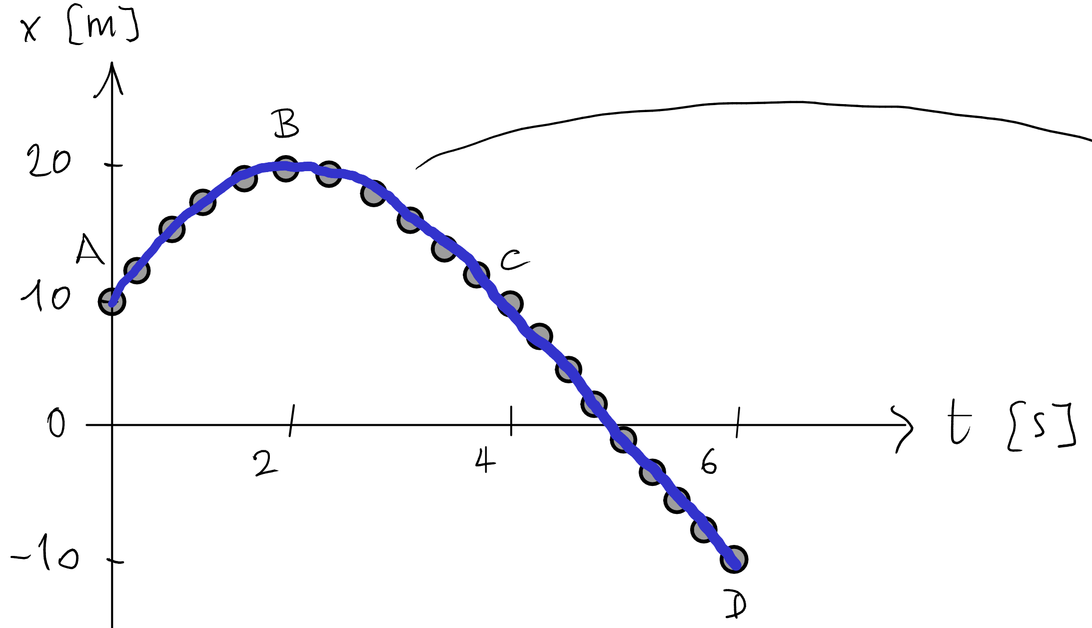

| | t [s] | x [m] |
|---|---|---|
| A | 0.0 | 10 |
| B | 2.0 | 20 |
| C | 4.0 | 10 |
| D | 6.0 | -10 |

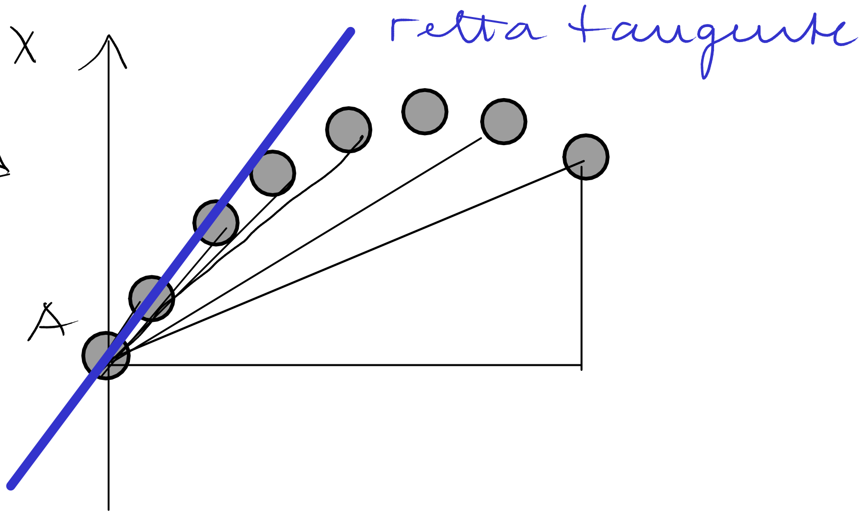



$$AB: \begin{cases} \Delta x = 10\ m \\ \Delta t = 2\ s \end{cases} \qquad CD: \begin{cases} \Delta x = x_D - x_C = -10\,m - 10\,m = -20\ m \\ \Delta t = 2\,s \end{cases}$$

$$\frac{\Delta x}{\Delta t} \equiv v_{mx}$$ **velocità media** $$[v_x] = \frac{L}{T} \qquad SI: \frac{m}{s}$$

$$AB: v_{mx} = \frac{10}{2}\,\frac{m}{s} = 5\,\frac{m}{s} \qquad CD: v_{mx} = -\frac{20}{2}\,\frac{m}{s} = -10\,\frac{m}{s}$$



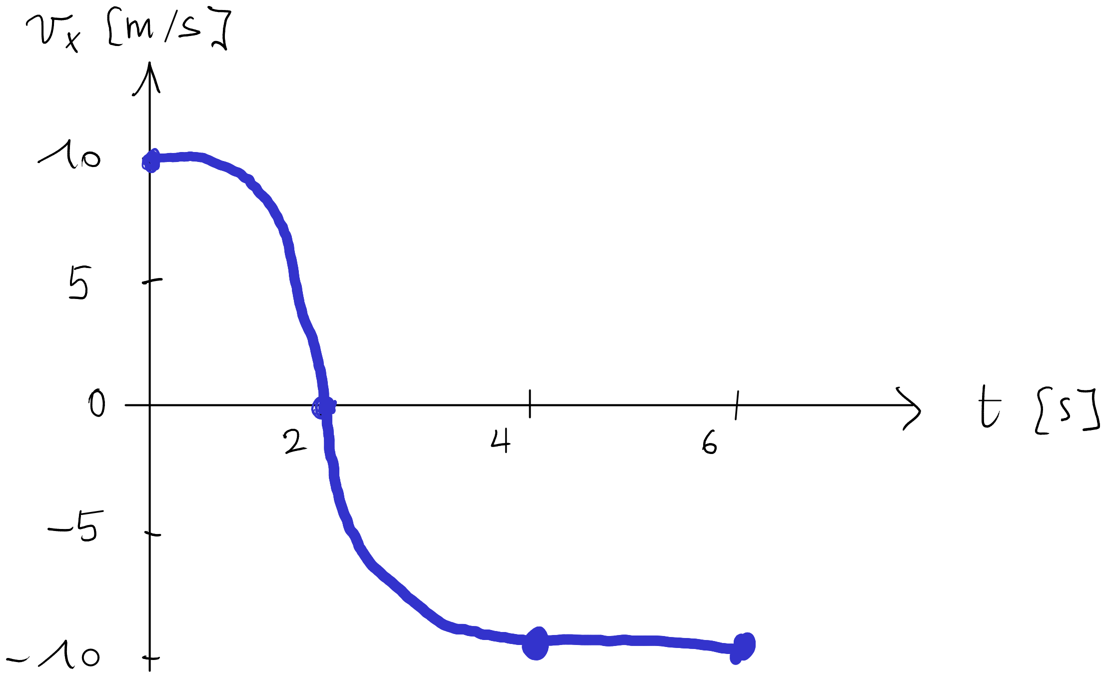




$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv v_x$$ velocità (istantanea)

$$\frac{dx}{dt} = v_x$$ SI: $\frac{m}{s}$

↑

"derivata"




→ $x(t)$ funzione del tempo $t$

→ $v_x(t)$

leggi orarie del moto

# Moto rettilineo uniforme

Particella si muove su una retta (1d)

In ogni istante tra $t_i$ e $t_f$:

$v_x = \text{cost}$

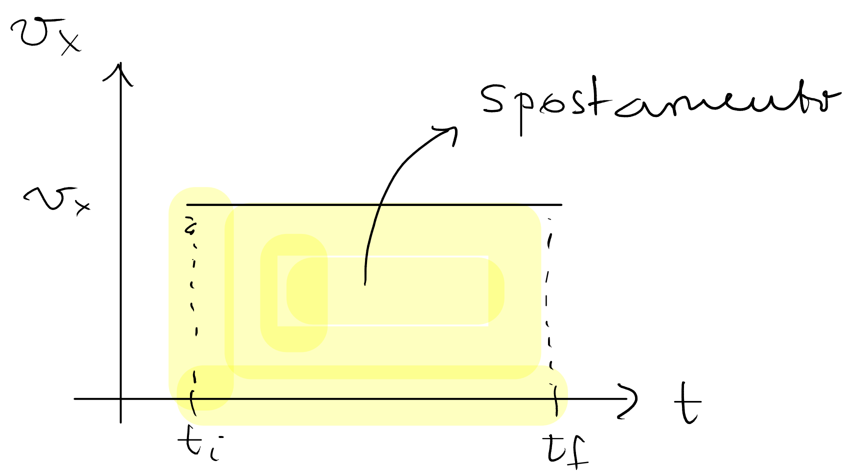
① $v_{mx} = v_x = \dfrac{x_f - x_i}{t_f - t_i}$

$x_f - x_i = v_x (t_f - t_i)$

$x_f = x_i + v_x (t_f - t_i)$

$$\begin{cases} x = x_i + v_x (t - t_i) \\ v_x = \text{cost} \end{cases}$$

leggi orarie del moto




② Calcolo integrale:

$\dfrac{dx}{dt} = v_x$

$v_x = \text{cost}$

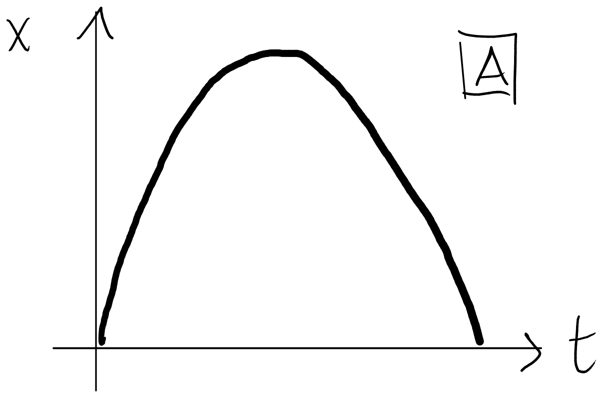
$$\int_{t_i}^{t_f} \frac{dx}{dt}\, dt = \int_{t_i}^{t_f} v_x \, dt \rightarrow x_f - x_i = v_x \int_{t_i}^{t_f} dt = v_x (t_f - t_i) \rightarrow x = x_i + v_x (t - t_i)$$

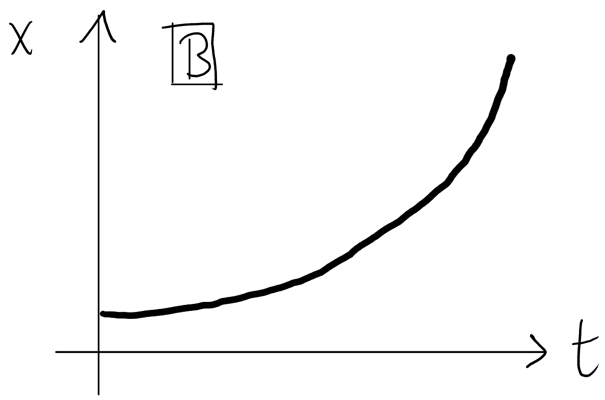
Esercizio : velocità media e istantanea

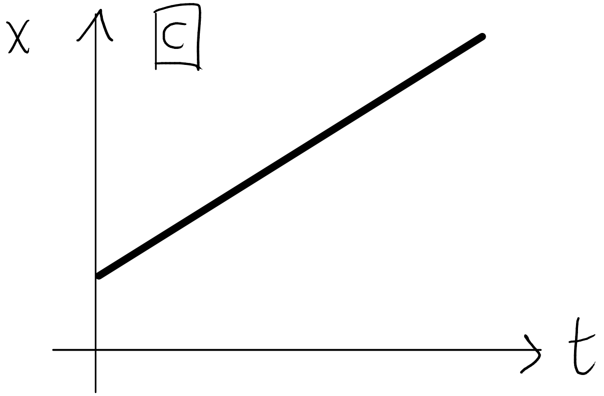
[A] palla lanciata in aria che ricade a terra

[B] auto che aumenta la sua velocità da 0 km/h a 100 km/h

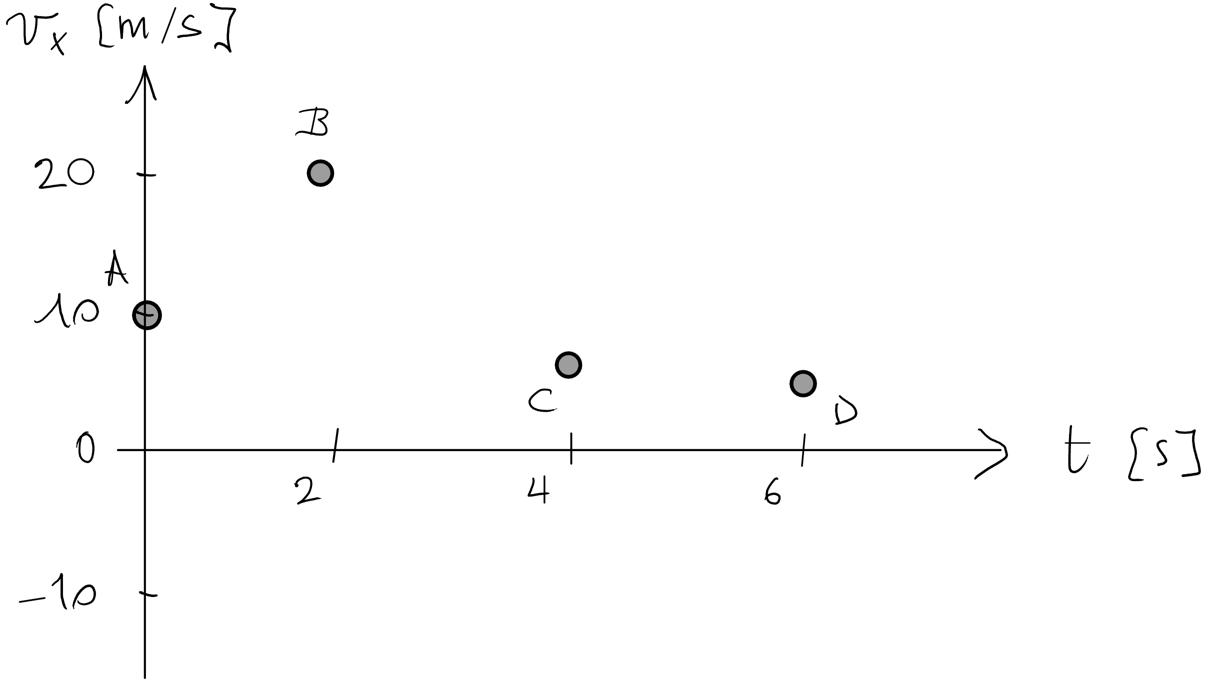
[C] sonda spaziale che si muove nel vuoto a velocità costante



**Accelerazione** : quanto rapidamente varia la velocità?




| | t[s] | $v_x$ [m/s] |
|---|---|---|
| A | 0.0 | 10 |
| B | 2,0 | 20 |
| C | 4.0 | 6 |
| D | 6.0 | 5 |

AB : $\begin{cases} \Delta v_x = v_{xB} - v_{xA} = 10 \text{ m/s} \\ \Delta t = 2 \text{ s} \end{cases}$

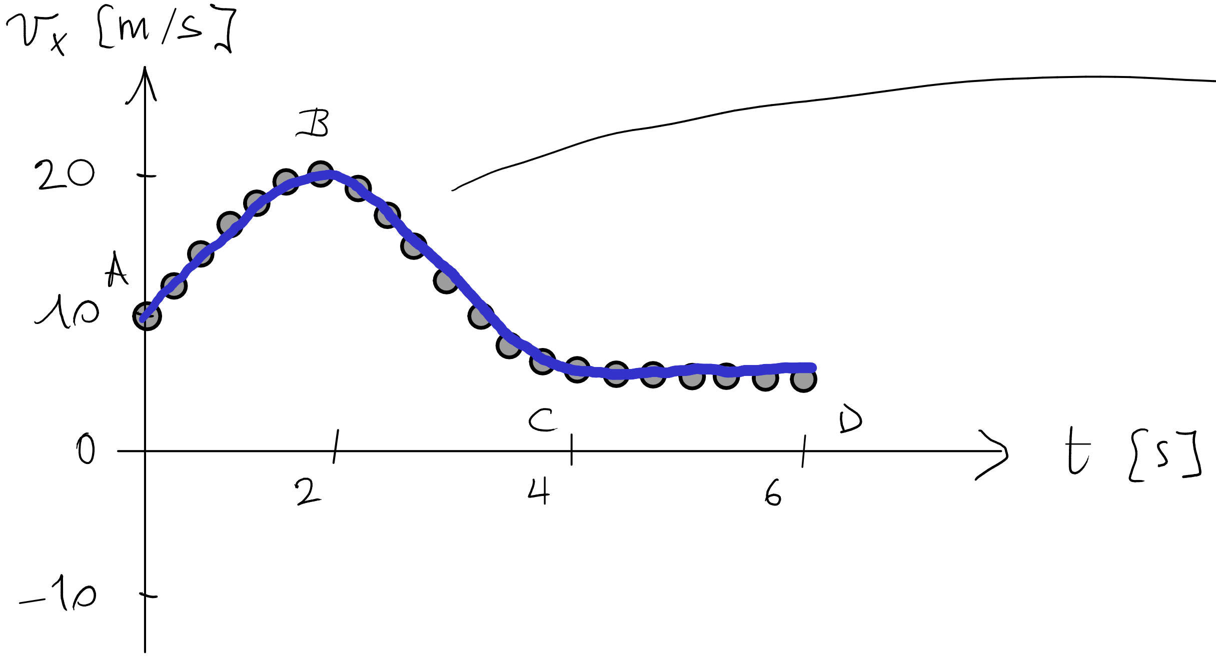
CD : $\begin{cases} \Delta v_x = 5 \text{ m/s} - 6 \text{ m/s} = -1 \text{ m/s} \\ \Delta t = 2 \text{ s} \end{cases}$

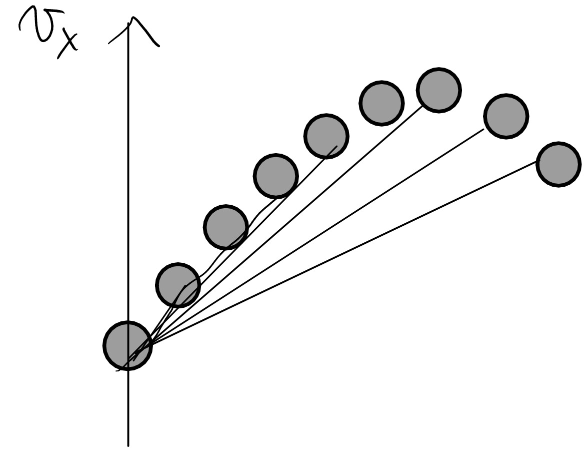
$$\frac{\Delta v_x}{\Delta t} \equiv a_{mx}$$ **accelerazione media**

$[a_{mx}] = \frac{L}{T^2}$ SI : $\frac{m}{s^2}$

AB : $a_{mx} = \dfrac{10 \frac{m}{s}}{2 \ s} = 5 \ \dfrac{m}{s^2}$

CD : $a_{mx} = \dfrac{-1 \frac{m}{s}}{2 \ s} = -0.5 \ \dfrac{m}{s^2}$



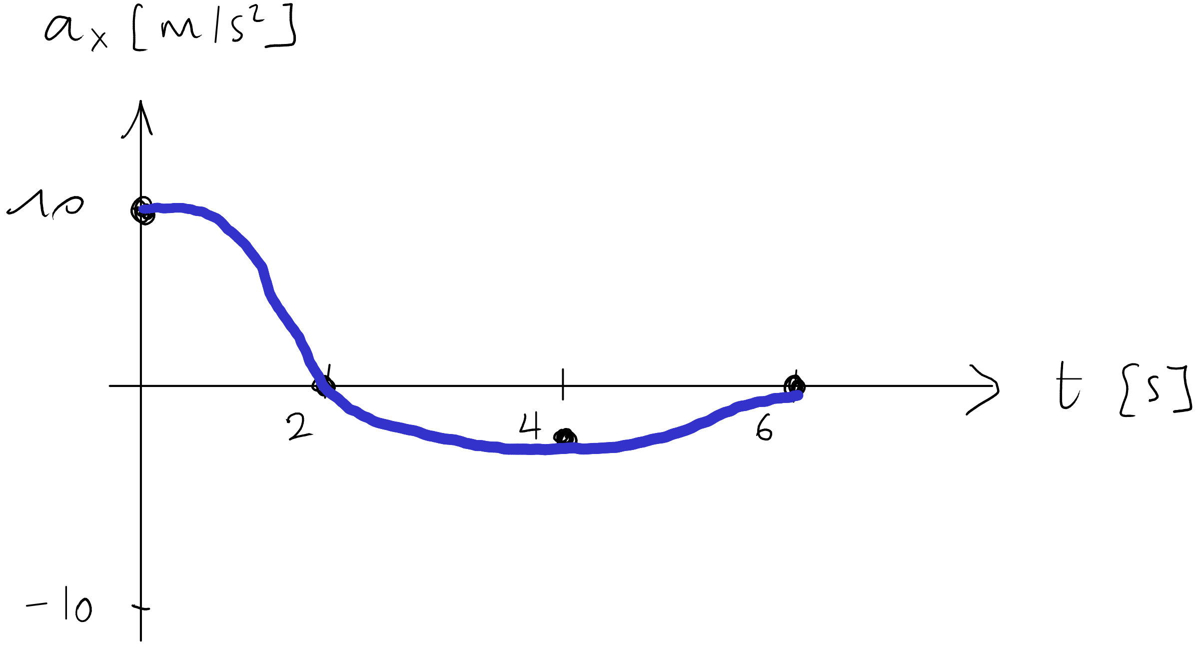




$$\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv a_x$$ accelerazione (istastanea)

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

↑

derivata seconda



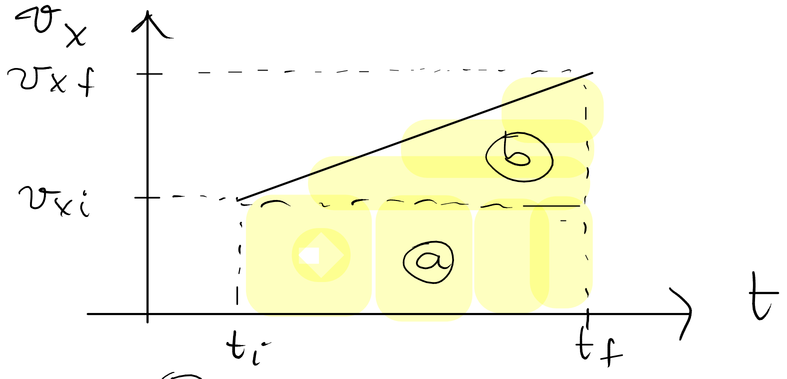

$v_x(t)$ →
$a_x(t)$ →
leggi orarie

## Moto uniformemente accelerato

Particella, 1d

In ogni istante tra $t_i$ e $t_f$

$$a_x = \text{cost}$$




(1) $$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

$$v_{xf} - v_{xi} = a_x (t_f - t_i)$$

$$v_x = v_{xi} + a_x (t - t_i)$$

$$\Delta x = x_f - x_i = \underbrace{v_{xi}(t_f - t_i)}_{a} + \underbrace{\frac{1}{2}(v_{xf} - v_{xi})(t_f - t_i)}_{b}$$

$$= v_{xi}(t_f - t_i) + \frac{1}{2} a_x (t_f - t_i)^2$$

$$x_f = x_i + v_x (t_f - t_i) + \frac{1}{2} a_x (t_f - t_i)^2$$

$$\begin{cases} x = x_i + v_x (t - t_i) + \frac{1}{2} a_x (t - t_i)^2 \\ v_x = v_{xi} + a_x (t - t_i) \\ a_x = \text{cost} \end{cases}$$

$\rightarrow$ leggi orarie del moto

(2) Calcolo integrale

$$\frac{dv_x}{dt} = a_x = cost$$

$$\int_{t_i}^{t_f} \frac{dv_x}{dt}\, dt = \int_{t_i}^{t_f} a_x\, dt$$

$$v_{xf} - v_{xi} \overset{a_x = cost}{=} a_x (t_f - t_i)$$

$$v_x = v_{xi} + a_x (t - t_i)$$

$$\frac{dx}{dt} = v_{xi} + a_x (t - t_i)$$

$$\int_{t_i}^{t_f} \frac{dx}{dt}\, dt = \int_{t_i}^{t_f} v_{xi}\, dt + \int_{t_i}^{t_f} a_x (t - t_i)\, dt$$

$$\int_{t_i}^{t_f} t\, dt - \int_{t_i}^{t_f} t_i\, dt = \ldots \quad (\underline{es}.)$$

$$x_f - x_i = v_{xi} (t_f - t_i) + a_x \int_{t_i}^{t_f} (t - t_i)\, dt$$

$$t' = t - t_i \qquad dt = dt'$$

$$= v_{xi} (t_f - t_i) + a_x \int_0^{t_f - t_i} t'\, dt' = v_{xi} (t - t_i) + a_x \left[ \frac{1}{2} t'^2 \right]_0^{t_f - t_i}$$

$$x_f = x_i + v_{xi} (t_f - t_i) + \frac{1}{2} a_x (t_f - t_i)^2 \rightarrow x = x_i + v_{xi} (t - t_i) + \frac{1}{2} a_x (t - t_i)^2 \quad \square$$