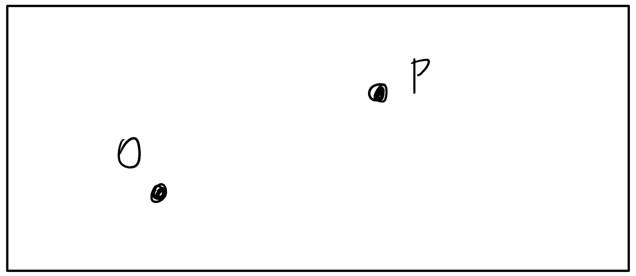
CINEMATICA 2D

Moto di un corpo su un piano $\longrightarrow$ punto $\equiv$ particella

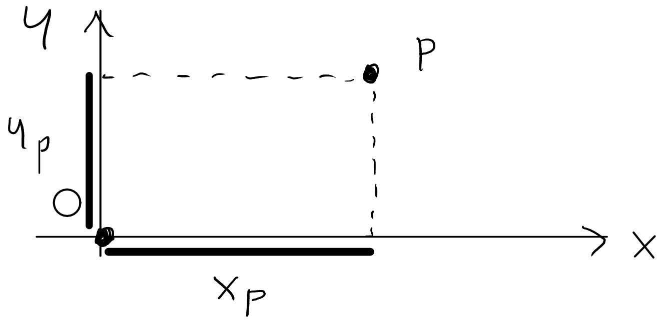
Sistema di coordinate



punto di riferimento $\equiv$ origine

2 assi

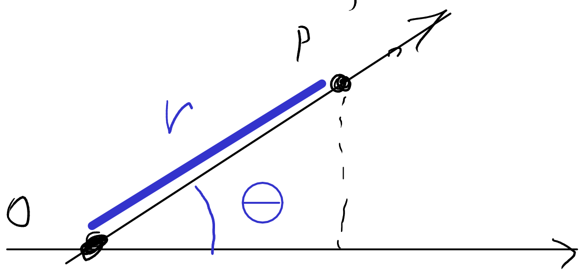
Coordinate cartesiane (x_P, y_P)



$x_P \equiv$ lunghezza della proiezione di P su asse x
segno – se dalla parte opposta della freccia wrt O

$y_P \equiv$ —//— asse y —//—

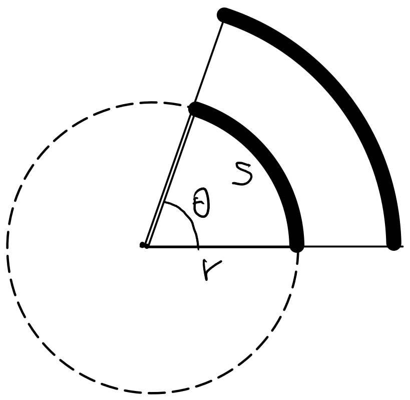
Coordinate polari (r, θ)



$r \equiv$ lunghezza segmento OP

$\theta \equiv$ angolo formato dai 2 assi

Richiami di trigonometria



$\theta \equiv \frac{s}{r}$ angolo $[\theta] = 1$

unità di misura:

1 rad

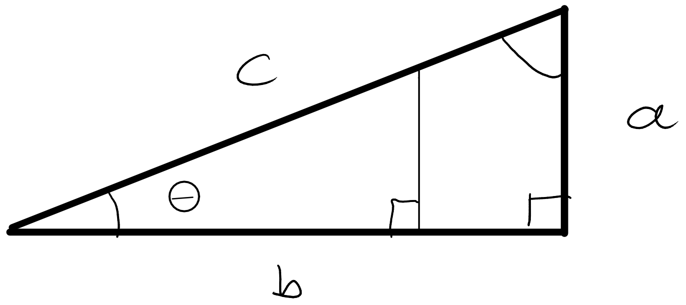
$\theta = 360°$ $\quad s = 2\pi r$ $\quad \theta = 2\pi$

$\theta = 180°$ $\quad s = \pi r$ $\quad \theta = \pi$

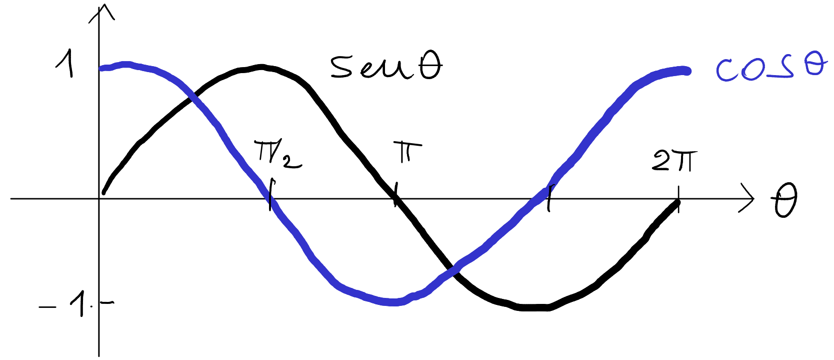
$\theta = 90°$ $\quad s = \frac{\pi}{2} r$ $\quad \theta = \frac{\pi}{2}$

...

$\theta = 1°$ $\quad s = \frac{2\pi r}{360}$ $\quad \theta \approx 0.0174$



$$\begin{cases} \text{sen}\,\theta \equiv \frac{a}{c} \\ \cos\theta \equiv \frac{b}{c} \\ \tan\theta \equiv \frac{a}{b} \end{cases} \Rightarrow \begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \tan\theta = \frac{\sin\theta}{\cos\theta} \end{cases}$$



Angoli notevoli

30°, 45°, 60°, 90°

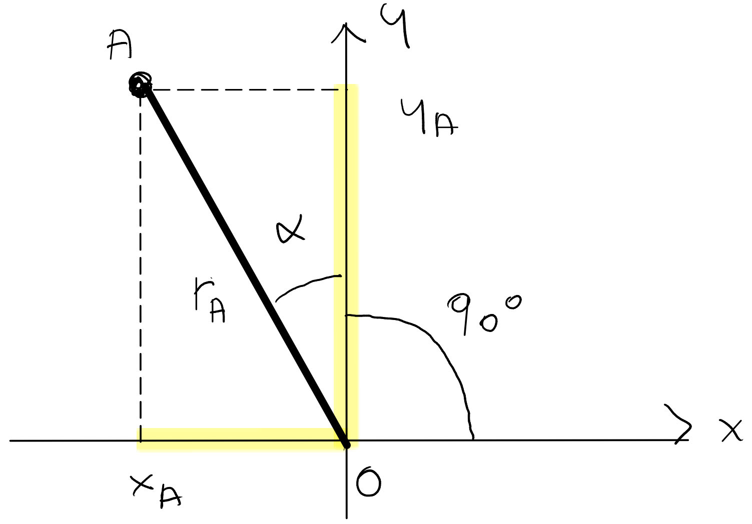
π/6, π/4, π/3, π/2

	0°	30°	60°	90°
sen	0	1/2	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	1/2	$1/\sqrt{2}$
tan	0	$1/\sqrt{3}$	$\sqrt{3}$	∞

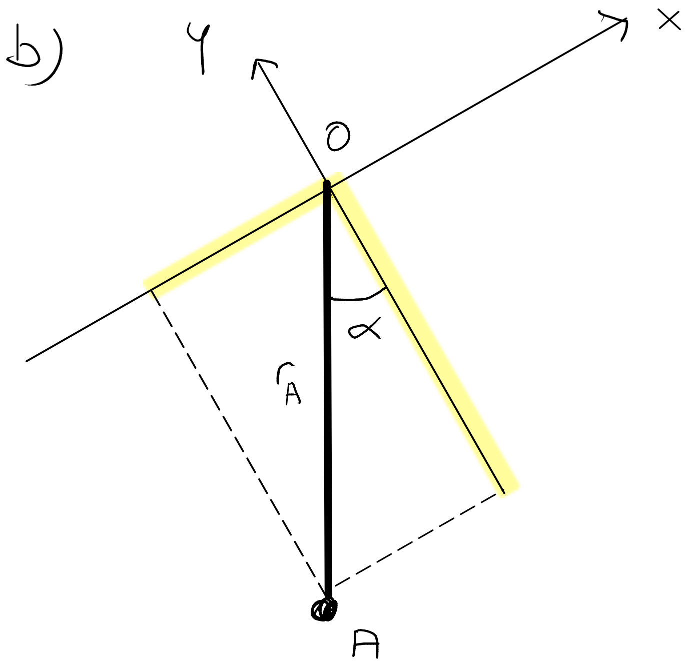
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2} \\ \tan\theta = \dfrac{y}{x} \end{cases} \rightarrow \quad \theta = \arctan\left(\dfrac{y}{x}\right)$$

Esempi

a)



b)



$$\begin{cases} x_A = r_A\cos\left(\frac{\pi}{2}+\alpha\right) \\ y_A = r_A\sin\left(\frac{\pi}{2}+\alpha\right) \end{cases} \qquad \begin{cases} x_A = -r_A\sin\alpha \\ y_A = r_A\cos\alpha \end{cases}$$

(es.)

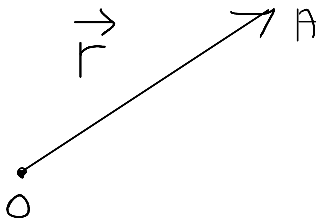
Vettori

Grandezze **scalari** : lunghezze, masse, ... → **1** numero + unità

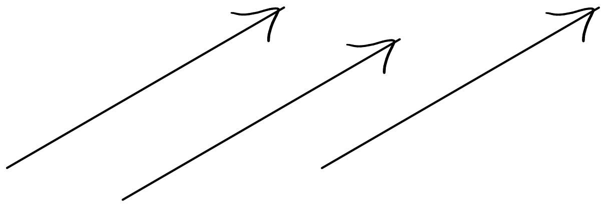
Grandezze **vettoriali** : posizione particella su piano → **2** numeri (o più)

Definizione geometrica

$\vec{r}$, $\bar{r}$, $\underline{r}$, $\mathbf{r}$

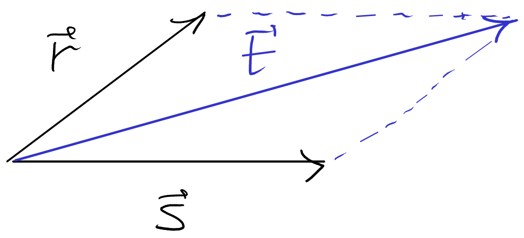


- direzione
- verso
- modulo (norma) $|\vec{r}|$



tutti uguali

* addizione $\vec{t} = \vec{r} + \vec{s}$



$$|\vec{t}| \neq |\vec{r}| + |\vec{s}|$$

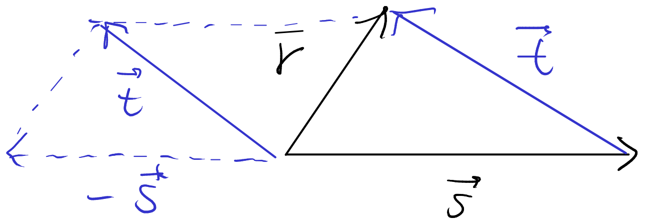
⚠

* opposto $-\vec{r}$

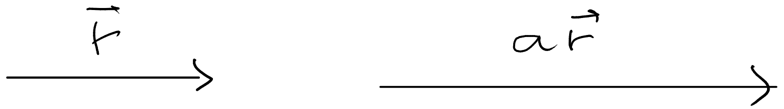
$$\vec{r} + (-\vec{r}) = \vec{0}$$

$$\vec{r} - \vec{r} = \vec{0}$$

* sottrazione $\vec{t} = \vec{r} - \vec{s}$



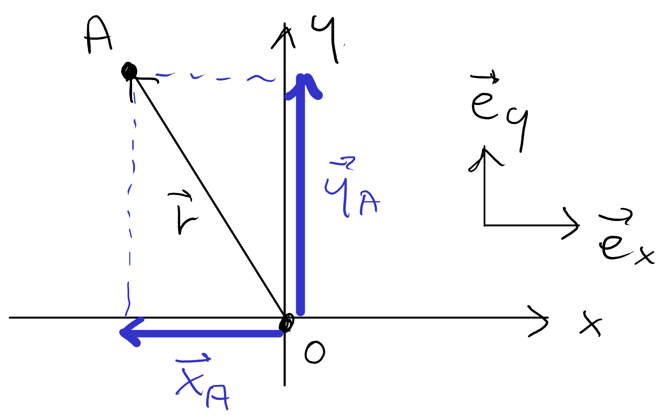
* moltiplicazione per scalare $a \in \mathbb{R}$



$a\vec{r} + a\vec{s}$ ↑

Commutativa: $\vec{r} + \vec{s} = \vec{s} + \vec{r}$ Associativa $(\vec{r}+\vec{s}) + \vec{t} = \vec{r} + (\vec{s}+\vec{t})$ Distributiva: $a(\vec{r}+\vec{s}) =$

Definizione in componenti



$\vec{x}_A$, $\vec{y}_A$ vettori componenti paralleli agli assi

$$\vec{r} = \vec{x}_A + \vec{y}_A = x_A \vec{e}_x + y_A \vec{e}_y$$

$|\vec{e}_x| = 1$ $\qquad \vec{x}_A = x_A \vec{e}_x$ $\qquad \{\vec{e}_x, \vec{e}_y\} \equiv$ base cartesiana (fissi)

$|\vec{e}_y| = 1$ $\qquad \vec{y}_A = y_A \vec{e}_y$

|...| modulo

VERSORI

grandezze scalari : componenti cartesiane del vettore $\vec{r}$

Modulo vettore:

$$|\vec{r}| = \sqrt{x_A^2 + y_A^2} \quad (= r) \qquad |\vec{r}|^2 = |\vec{x}_A|^2 + |\vec{y}_A|^2 = |x_A \vec{e}_x|^2 + |y_A \vec{e}_x|^2 = x_A^2 \overbrace{|\vec{e}_x|^2}^{=1} + y_A^2 \overbrace{|\vec{e}_y|}^{=1}$$

Somma di vettori : $\vec{r}_A = x_A \vec{e}_x + y_A \vec{e}_y$ $\qquad \vec{r}_B = x_B \vec{e}_x + y_B \vec{e}_y \rightarrow \vec{r}_C = \vec{r}_A + \vec{r}_B$

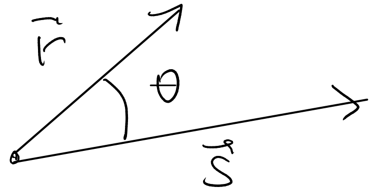
$$\vec{r}_A + \vec{r}_B = x_A \vec{e}_x + y_A \vec{e}_y + x_B \vec{e}_x + y_B \vec{e}_y = (\underbrace{x_A + x_B}_{x_C}) \vec{e}_x + (\underbrace{y_A + y_B}_{y_C}) \vec{e}_y$$

$$\vec{r}_C = x_C \vec{e}_x + y_C \vec{e}_y$$

Moltiplicazione per scalare $a \in \mathbb{R}$

$$a\vec{r} = a(x_A \vec{e}_x + y_A \vec{e}_y) = a x_A \vec{e}_x + a y_A \vec{e}_y$$

Prodotto scalare



$$\vec{r} \cdot \vec{s} = |\vec{r}| \cdot |\vec{s}| \cdot \cos\theta \rightarrow \cos\theta = \frac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|}$$

$$\vec{r} \cdot \vec{s} = x_A x_B + y_A y_B$$

$$\begin{cases} \vec{r} = x_A \vec{e}_x + y_A \vec{e}_y \\ \vec{s} = x_B \vec{e}_x + y_B \vec{e}_y \end{cases}$$

Analisi dimensionale

Componenti cartesiane di una grandezza vettoriale devono essere OMOGENEE fra loro.

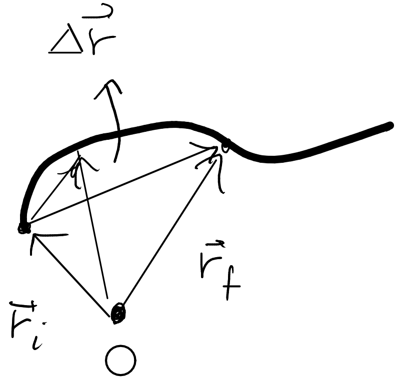
$$\vec{r} = \underset{[x_A] = L}{\underset{\uparrow}{x_A}} \vec{e}_x + \underset{[y_A] = L}{\underset{\uparrow}{y_A}} \vec{e}_y \qquad \cancel{[\vec{r}] = L} \qquad \vec{e}_x = \underset{\uparrow}{1}\, \vec{e}_x + \underset{\uparrow}{0}\, \vec{e}_y$$

adimensionali

Cinematica sul piano

Posizione

$$\vec r = x\,\vec e_y + y\,\vec e_y$$



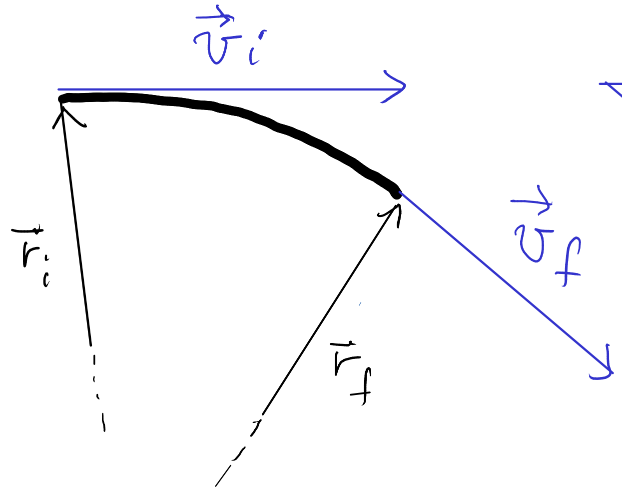
Spostamento

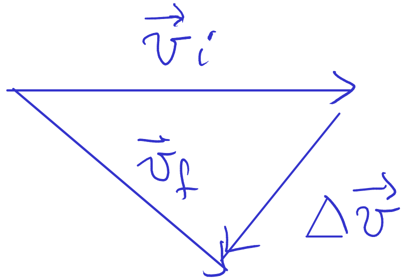
$$\Delta\vec r \equiv \vec r_f - \vec r_i$$

Velocità media

$$\vec v_m \equiv \frac{\Delta\vec r}{\Delta t} = \frac{1}{\Delta t}\,\Delta\vec r = \frac{1}{\Delta t}\left[\Delta x\,\vec e_x + \Delta y\,e_y\right]$$

$$= \frac{\Delta x}{\Delta t}\,\vec e_x + \frac{\Delta y}{\Delta t}\,\vec e_y = v_{mx}\,\vec e_x + v_{my}\,\vec e_y$$





Velocità

$$v_x\,\bar e_x + v_y\,\vec e_y$$

$$\vec v \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec r}{\Delta t} = \frac{d\vec r}{dt} = \frac{d}{dt}\left(x\,\vec e_x + y\,\vec e_y\right) = \frac{d}{dt}(x\,\vec e_x) + \frac{d}{dt}(y\,\vec e_y) = \frac{dx}{dt}\,\vec e_x + \frac{dy}{dt}\,\vec e_y$$

Accelerazione media

$$\vec a_m \equiv \frac{\Delta\vec v}{\Delta t}$$

Accelerazione

$$\vec a \equiv \lim_{\Delta t\to 0}\frac{\Delta\vec v}{\Delta t} = \frac{d^2\vec r}{dt^2} = \frac{d}{dt}\left(\overset{\to\ \vec v}{\frac{d\vec r}{dt}}\right) = \frac{d^2x}{dt^2}\,e_x + \frac{d^2y}{dt^2}\,\vec e_y$$