Moto di un proiettile

Senza resistenza con l'aria: | Con resistenza con l'aria:

Senza resistenza con l'aria	Con resistenza con l'aria
Traiettoria non dipende da m e/o d	X
Traiettoria parabolica	V ?
Gittata è massima se $\theta = 45°$	V ?
Gittata aumenta come v_i^2 [?]	?
Altezza max se $\theta = 90°$	V
Gittata simmetrica rispetto a $\theta = 45°$	X ?

Modello: moto unif. accelerato in 2d

Corpo ≡ particella

2d

$\vec{a} = \text{cost}$

Condizioni iniziali: $\vec{r}_i$, $\vec{v}_i$ Base $\{\vec{e}_x, \vec{e}_y\}$

$\vec{r}_i = x_i \vec{e}_x + y_i \vec{e}_y$

$\vec{v}_i = v_{xi} \vec{e}_x + v_{yi} \vec{e}_y$

$$\frac{d\vec{v}}{dt} = \vec{a} \qquad \vec{v} = \vec{v}_i + \vec{a}(t - t_i) \quad \ldots$$

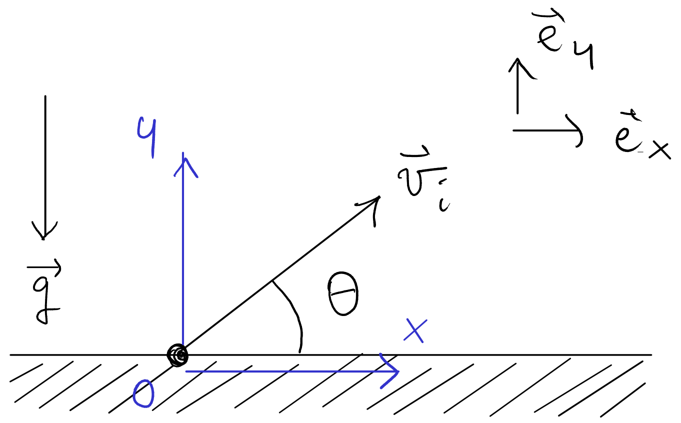
moto unif. acc. 1d

$$\frac{dv_x}{dt}\vec{e}_x + \frac{dv_y}{dt}\vec{e}_y = a_x \vec{e}_x + a_y \vec{e}_y$$

$$\begin{cases} \dfrac{dv_x}{dt} = a_x \\[2mm] \dfrac{dv_y}{dt} = a_y \end{cases} \rightarrow \begin{cases} v_x = v_{xi} + a_x(t - t_i) \\ v_y = v_{yi} + a_y(t - t_i) \end{cases}$$

$$\begin{cases} \vec v = v_x \vec e_x + v_y \vec e_y = (v_{xi}\,\vec e_x + v_{yi}\,\vec e_y) + (a_x \vec e_x + a_y \vec e_y)(t-t_i) = \vec v_i + \vec a\,(t-t_i) \\ \vec r = x\,\vec e_x + y\,\vec e_y = \vec r_i + \vec v_i (t-t_i) + \frac{1}{2}\vec a\,(t-t_i)^2 \end{cases}$$

→ leggi orarie del moto



$$\vec a = \vec g = 0\,\vec e_x - g\,\vec e_y = -g\,\vec e_y \qquad \left(g = 9{,}81\ \frac{m}{s^2}\right)$$

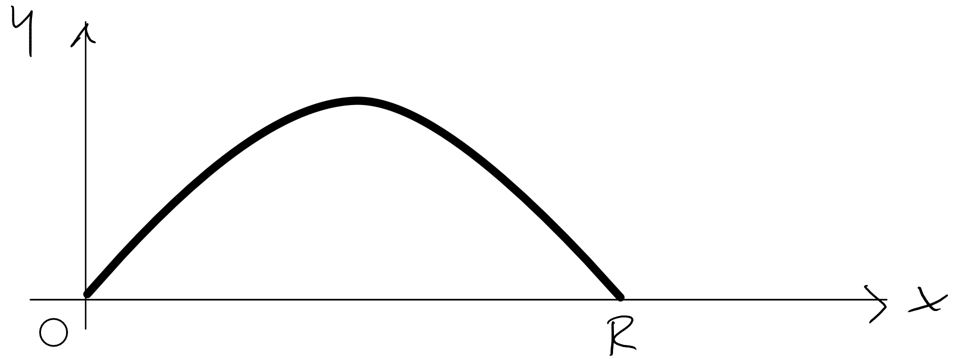
$$\vec r_i = \vec 0 = 0\,\vec e_x + 0\,\vec e_y \qquad t_i = 0$$

$$\vec v_i = |\vec v_i| \cos\theta\,\vec e_x + |\vec v_i| \sin\theta\,\vec e_y$$

$$\vec r = \vec v_i\, t + \frac{1}{2}\vec g\, t^2$$

$$x\,\vec e_x + y\,\vec e_y = |\vec v_i| \cos\theta\, t\,\vec e_x + |\vec v_i| \sin\theta\, t\,\vec e_y - \frac{1}{2} g t^2\,\vec e_y$$

$$\begin{cases} x = |\vec v_i| \cos\theta\, t \\ y = |\vec v_i| \sin\theta\, t - \frac{1}{2} g t^2 \end{cases}$$ leggi orarie del moto



Traiettoria : $y = y(x)$

$$t = \frac{x}{|\vec v_i| \cos\theta}$$

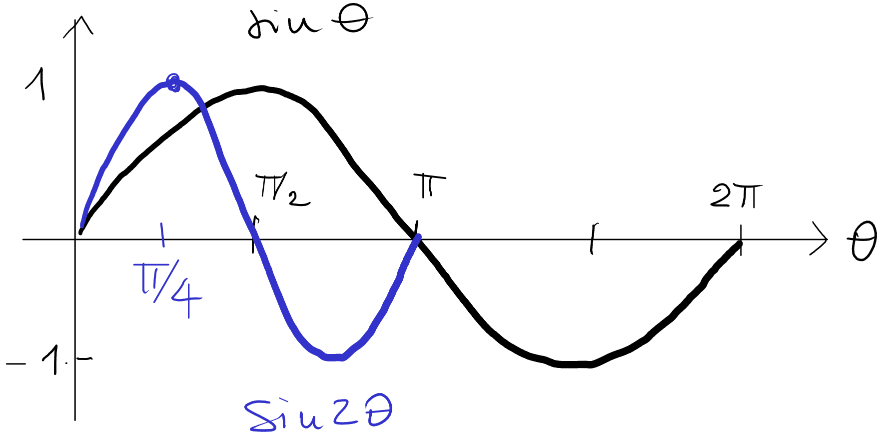
$$y = \frac{\cancel{|\vec v_i|} \sin\theta}{\cancel{|\vec v_i|} \cos\theta}\, x - \frac{1}{2} g \frac{x^2}{|\vec v_i|^2 \cos^2\theta} = \tan\theta\, x - \frac{1}{2} \frac{g}{|\vec v_i|^2 \cos^2\theta}\, x^2$$

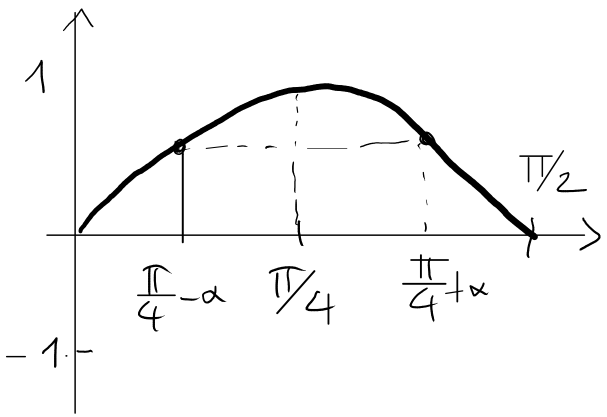
Gittata $\quad 0 = y(R)$

$$0 = \tan\theta \, R - \frac{1}{2}\frac{g}{|\vec{v}_i|^2\cos^2\theta} R^2 = R\overbrace{\left(\tan\theta - \frac{1}{2}\frac{g}{|\vec{v}_i|^2\cos^2\theta}R\right)}^{=0}$$

$$R = \frac{2\tan\theta\,|\vec{v}_i|^2\cos^2\theta}{g} = \frac{2\sin\theta\cos\theta}{g}|\vec{v}_i|^2 = \frac{\sin 2\theta}{g}|\vec{v}_i|^2$$

$R \sim |\vec{v}_i|^2 \quad \boxed{V} \qquad R \sim \frac{1}{g} \qquad R \sim \sin 2\theta \qquad R_{max} \rightarrow \theta = \frac{\pi}{4} = 45^\circ \quad \boxed{V}$



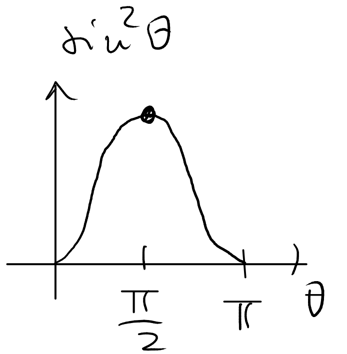


Altezza massima : H

$$\begin{cases} v_x = |\vec{v}_i|\cos\theta \\ v_y = |\vec{v}_i|\sin\theta - g t \end{cases}$$

$$0 = |\vec{v}_i|\sin\theta - g t^*$$

$$t^* = \frac{|\vec{v}_i|\sin\theta}{g}$$



$$H = |\vec{v}_i|\sin\theta\, t^* - \frac{1}{2}g(t^*)^2 = \frac{|\vec{v}_i|^2\sin^2\theta}{g} - \frac{1}{2}\cancel{g}\frac{|\vec{v}_i|^2\sin^2\theta}{g^{\cancel{2}}} = \frac{1}{2}\frac{|\vec{v}_i|^2\sin^2\theta}{g} \sim \sin^2\theta \quad \boxed{V}$$

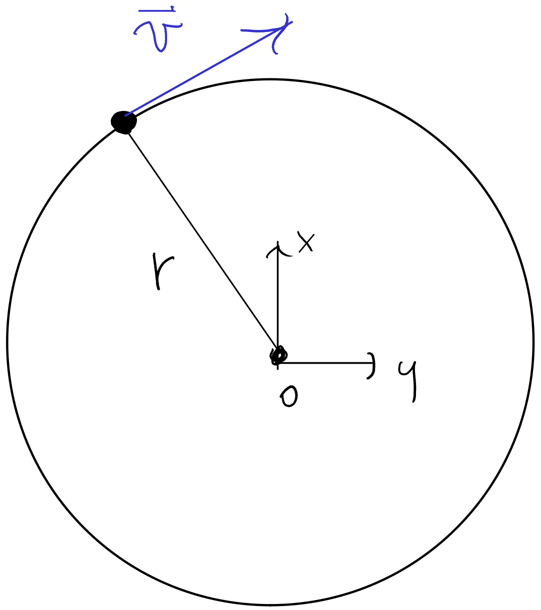
Moto rettilineo unif. nel piano

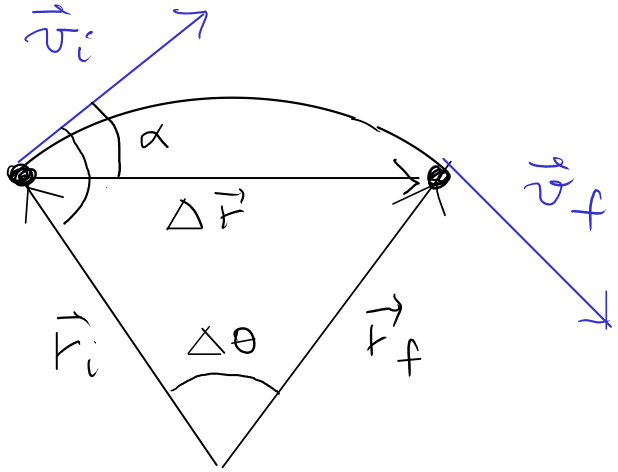
$$\begin{cases} \vec{v} = \vec{v}_i \\ \vec{r} = \vec{r}_i + \vec{v}_i\,(t - t_i) \end{cases}$$

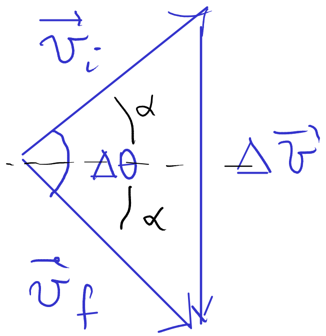
$$\vec{a} = \vec{0}$$

Moto circolare uniforme

$|\vec{r}| = r = \text{cost} \qquad |\vec{v}| = v = \text{cost} \quad \rightarrow \quad$ ⚠ accelerato







$$\Delta\theta + 2\left(\frac{\pi}{2} - \alpha\right) = \pi$$

$$\Delta\theta - 2\alpha = 0$$

$$\Delta\theta = 2\alpha$$

$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \qquad \Delta\vec{v} = \vec{v}_f - \vec{v}_i \qquad \Delta t = t_f - t_i$

$$\frac{|\Delta\vec{r}|}{r} = \frac{|\Delta\vec{v}|}{v} \qquad \frac{|\Delta\vec{r}|}{\Delta t}\frac{1}{r} = \frac{|\Delta\vec{v}|}{\Delta t}\frac{1}{v} \qquad \left|\frac{\Delta\vec{r}}{\Delta t}\right|\frac{1}{r} = \left|\frac{\Delta\vec{v}}{\Delta t}\right|\frac{1}{v}$$

Limite $\Delta t \to 0$: $\quad \frac{\Delta\vec{r}}{\Delta t} \to \vec{v} \qquad \frac{\Delta\vec{v}}{\Delta t} \to \vec{a} \quad \Rightarrow \quad \frac{|\vec{v}|}{r} = \frac{|\vec{a}|}{v} \quad \Rightarrow \quad |\vec{a}| = \frac{v^2}{r}$

$a_c \equiv |\vec{a}| = \frac{v^2}{r}$ **accelerazione centripeta**

Periodo τ : $\quad 2\pi r = v\tau$

$$\tau = \frac{2\pi r}{v}$$