MECCANICA RAZIONALE

S sistema meccanico in equilibrio $\Longrightarrow$ $\begin{cases} \underline{R}^{(e)} = \underline{0} \\ \underline{M}^{(e)}_{(O)} = \underline{0} \end{cases}$

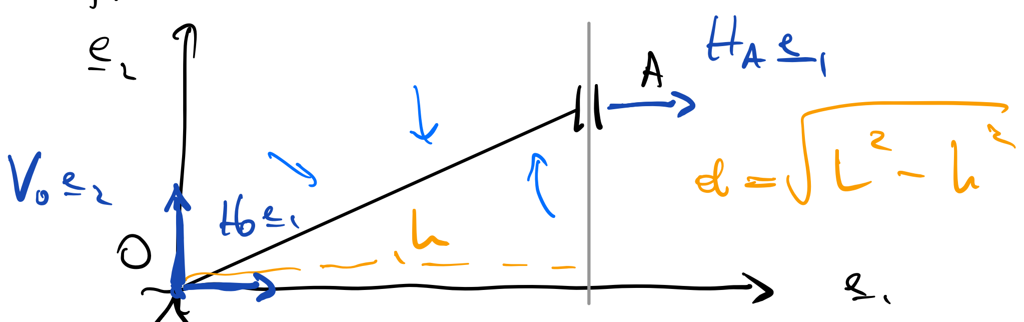
$$\sum_{P \in I} (\underline{x}_P - \underline{x}_O) \wedge \underline{F}_P$$

condizione necessaria per l'equilibrio

Per un corpo rigido

$$LV_{rigido} = \underline{R} \cdot \delta \underline{x}_O + \underline{M}(O) \cdot \underline{\chi}$$

Applicazioni alla statica



$\overline{OA} = L$

$\underline{R}^e = \underline{0}$ $\qquad \underline{e}_1 : \quad H_0 + H_A + \underline{R}^{e,\text{attive}} \cdot \underline{e}_1 = 0$

$\qquad\qquad\qquad \underline{e}_2 : \quad V_0 + \underline{R}^{e,\text{attive}} \cdot \underline{e}_2 = 0$

$\underline{M}^e(O) = \underline{0}$ $\qquad \underline{e}_3 : \quad -H_A\, d + \underline{M}^{e,\text{attive}}_{(O)} \cdot \underline{e}_3 = 0$

se $d \neq 0$ (vincoli indipendenti)

$\Rightarrow H_0, V_0, H_A$ sono determinate dalle ECS per ogni sistema di forze attive.

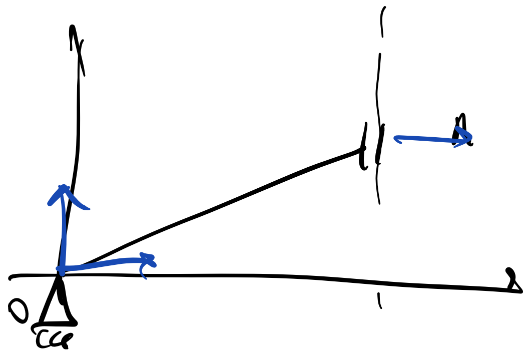
se $d = 0$

$-\cancel{H_A\, d} + \underline{M}^{e,\text{attive}}_{(O)} \cdot \underline{e}_3 = 0$ $\qquad \left\{ \begin{array}{l} \underline{M}^{e,\text{att}} \cdot \underline{e}_3 = 0 \\ \rightarrow H_A \text{ indeterminata} \end{array} \right.$

oppure

$\left\{ \underline{M}^{e,\text{att}}_{(O)} \cdot \underline{e}_3 \neq 0 \right.$

Commento



$$H_O + H_A + \underline{R}^{e,a}\cdot \underline{e}_1 = 0$$

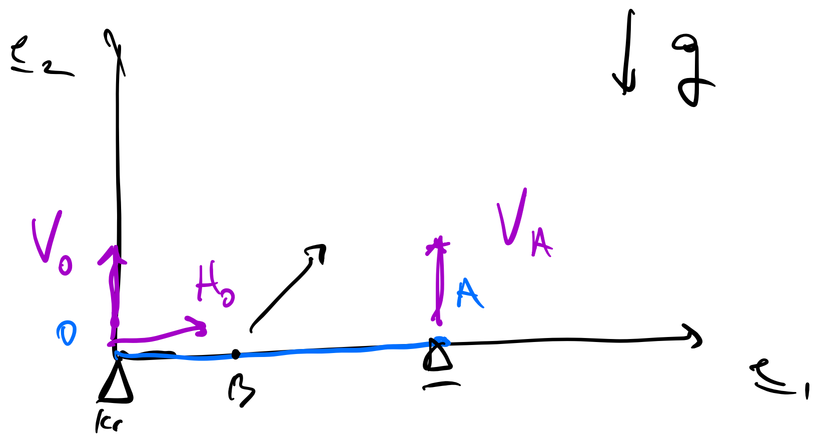
$$V_O + \underline{R}^{e,e}\cdot \underline{e}_2 = 0$$

$$-H_A d + \nu^2 + \underline{M}^{e,0}\cdot \underline{e}_3 = 0$$

$\nu^2 \underline{e}_3$ momento di attrito

→ 3 eq e 4 incognite

Esercizio



$$\underline{F}_B = f\,\underline{e}_1 + 2f\,\underline{e}_2$$

$$\overline{OA} = \ell$$

$$\overline{OB} = \frac{\ell}{4}$$

$$\underline{R}^e = \underline{0} \Rightarrow$$

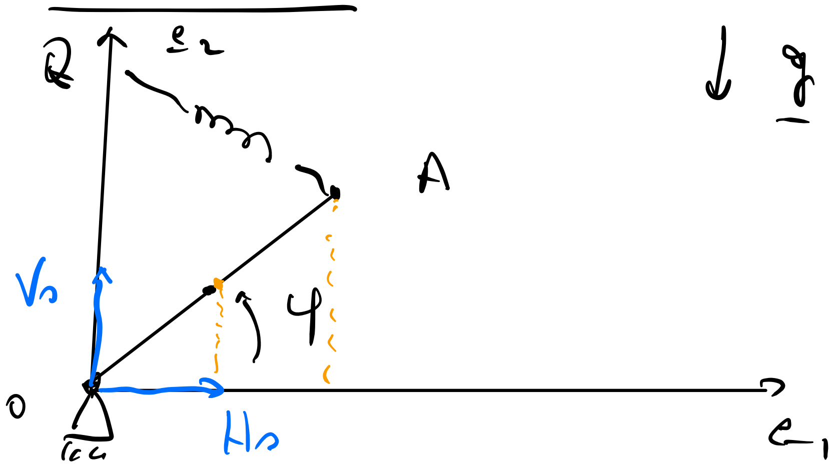
lungo $\underline{e}_1$: $H_O + f = 0$

lungo $\underline{e}_2$: $V_O + V_A - Mg + 2f = 0$

$$\underline{M}'(O) = \underline{0} \quad : \quad \ell V_A - Mg\frac{\ell}{2} + 2f\frac{\ell}{4} = 0$$
$$\uparrow$$
$$t$$

Quindi :
$$\begin{cases} V_A = \dfrac{Mg}{2} - \dfrac{f}{2} \\[2ex] V_o = Mg - 2f - \left(\dfrac{Mg}{2} - \dfrac{f}{2}\right) \\[1ex] \quad\; = \dfrac{Mg}{2} - \dfrac{3}{2}f \\[3ex] H_o = -f \end{cases}$$

Esercizio



$OA = L$, m

$OQ = D$

1. determinare conf eq.

2. reazioni in O all' eq.

1. PLV per forze conservative

$$V = mg\, y_G + \frac{c}{2}\|\underline{x}_A - \underline{x}_Q\|^2$$

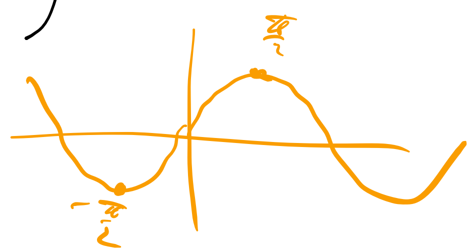
$$= mg\frac{L}{2}\sin\varphi + \frac{c}{2}\left\| L\cos\varphi\, \underline{e}_1 + L\sin\varphi\, \underline{e}_2 - D\,\underline{e}_2\right\|^2$$

$$= mg\frac{L}{2}\sin\varphi + \frac{c}{2}\left\| L\cos\varphi\, \underline{e}_1 + (L\sin\varphi - D)\,\underline{e}_2\right\|^2$$

$$= mg\frac{L}{2}\sin\varphi + \frac{c}{2}\left(L^2\cos^2\varphi + L^2\sin^2\varphi + D^2 - 2LD\sin\varphi\right)$$

$$= mg\frac{L}{2}\sin\varphi + \frac{c}{2}\left(L^2 + D^2 - 2LD\sin\varphi\right)$$

$$= L\sin\varphi\left(\frac{mg}{2} - cD\right) + \text{costante}$$



Conf eq.

- se $\frac{mg}{2} - cD = 0$ $\quad \forall \varphi$

- se $\frac{mg}{2} \neq cD$ $\quad V \sim \underset{L\left(\frac{mg}{2} - cD\right)}{\underline{\neq}} \sin\varphi$

$\frac{mg}{2} > cD$ $\quad -\frac{\pi}{2}$ stabile

$\frac{mg}{2} < cD \quad \frac{\pi}{2}$ stabile

Eq cardinali della statica: $\underline{M}^e(O) = \underline{0}$

$$-mg\frac{L}{2}\cos\varphi + (\underline{x}_A - \underline{x}_O) \wedge \left[-c\,(\underline{x}_A - \underline{x}_Q)\right] \cdot \underline{e}_3$$

$$M = \sum_p (\underline{x}_p - \underline{x}_O) \wedge \underline{F}_p$$

$$\underline{x}_A = L\cos\varphi\,\underline{e}_1 + L\sin\varphi\,\underline{e}_2$$

$$\underline{x}_O = \underline{0}$$

$$\underline{x}_Q = D\,\underline{e}_2$$

$$(L\cos\varphi\,\underline{e}_1 + L\sin\varphi\,\underline{e}_2) \wedge$$

$$\wedge \left(-c\left[L\cos\varphi\,\underline{e}_1 + (L\sin\varphi - D)\,\underline{e}_2\right]\right)$$

$$\underline{e}_1 \wedge \underline{e}_2 = \underline{e}_3 \qquad \underline{e}_2 \wedge \underline{e}_1 = -\underline{e}_3$$

$$\underline{e}_1 \wedge \underline{e}_1 = \underline{e}_2 \wedge \underline{e}_2 = 0$$

$$= (L\cos\varphi)\left[-c\,(L\sin\varphi - D)\right]\underline{e}_1 \wedge \underline{e}_2 +$$

$$+ (L\sin\varphi)\left[-c\,L\cos\varphi\right]\underline{e}_2 \wedge \underline{e}_1$$

$$= \left[- c L^2 \cos\varphi \sin\varphi + c L D \cos\varphi \right] \underbrace{\underline{e}_1 \wedge \underline{e}_2}_{\underline{e}_3}$$

$$- c L^2 \cos\varphi \sin\varphi \quad \underbrace{\underline{e}_2 \wedge \underline{e}_1}_{-\underline{e}_3}$$

$$= \left[- c L^2 \cancel{\cos\varphi \sin\varphi} + c L D \cos\varphi + c L^2 \cancel{\cos\varphi \sin\varphi} \right] \underline{e}_3$$

$$= c L D \cos\varphi \; \underline{e}_3$$

$$- m g \frac{L}{2} \cos\varphi + c L D \cos\varphi = 0$$

questo eq è $-V'(\varphi)$

Non è una sorpresa! $Q_\varphi = - \dfrac{\partial V}{\partial \varphi} =$

$$= \underline{M}(O) \cdot \underline{e}_3$$

2. Equazioni cardinali $\rightarrow$ $\underline{R}^e = \underline{0}$

lungo $\underline{e}_1$: $\quad H_0 + \underline{e}_1 \cdot \left[-c\,(\underline{x}_A - \underline{x}_Q) \right] = 0$

$$H_0 + \underline{e}_1 \left[-c \left(L\cos\varphi\, \underline{e}_1 + \left(L\sin\varphi - D\right) \underline{e}_2 \right) \right] = 0$$

$$\rightarrow \quad H_0 = cL\cos\varphi$$

lungo $\underline{e}_2$:

$$V_0 - mg + \underline{e}_2 \cdot \left[-c\,(\underline{x}_A - \underline{x}_Q) \right] = 0$$

$$V_0 = mg + c\left(L\sin\varphi - D\right)$$

Abbiamo trovato

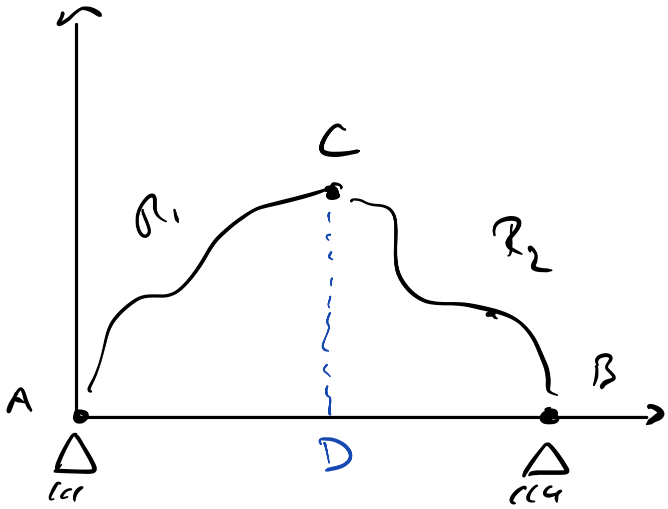
$$\begin{cases} H_0 = cL\cos\varphi \\ V_0 = mg + c\left(L\sin\varphi - D\right) \end{cases} \quad \Bigg\} \text{ se } \frac{mg}{2} = cD$$

$\frac{mg}{2} \neq cD$

$$\varphi = \frac{\pi}{2} \qquad \begin{cases} H_0 = 0 \\ V_0 = \underline{mg + c\,(L - D)} \end{cases}$$

$$\varphi = -\frac{\pi}{2} \qquad \begin{cases} H_0 = 0 \\ V_0 = \underline{mg - c\,(L + D)} \end{cases}$$

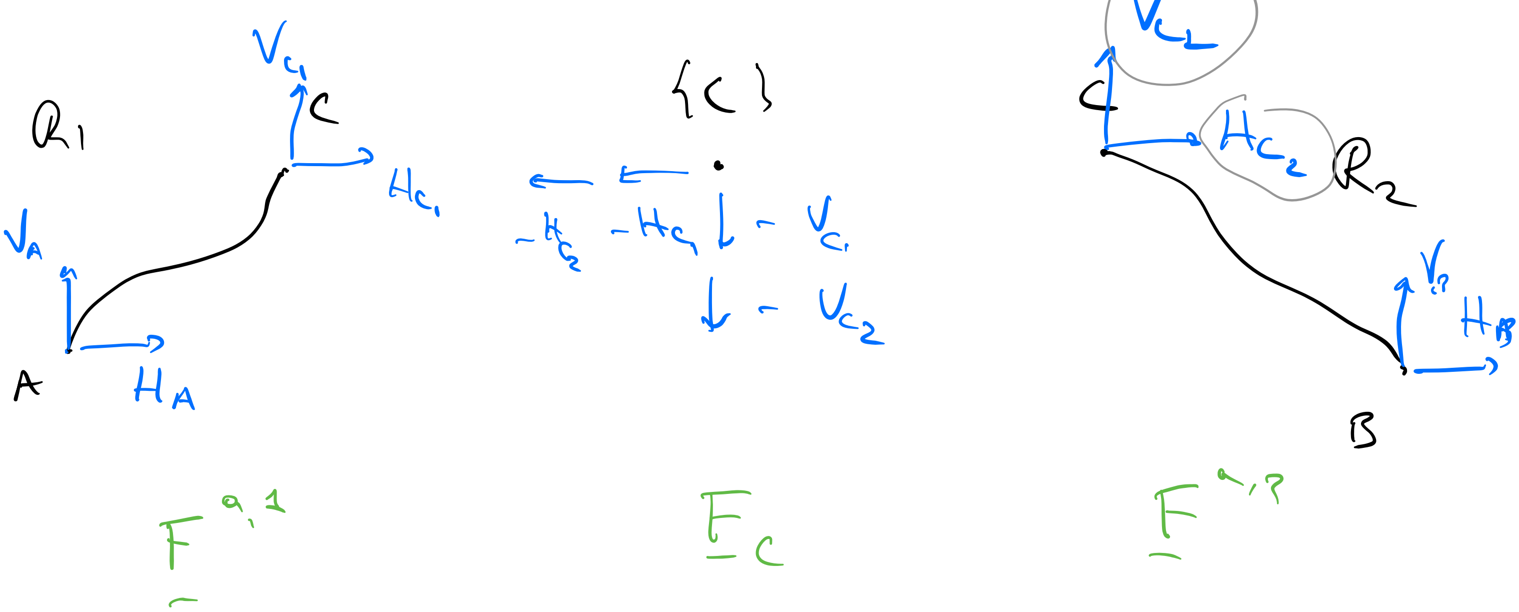
Applicazione alla statica dei sistemi articolati



ARCO A TRE CERNIERE

$R_1 \cup R_2 \cup \{C\}$

Supponiamo forze attive note



Forze attive $\underline{F}^{a,1}$, $\underline{F}_C$, $\underline{F}^{a,2}$

Incognite $(H_A, V_A, H_B, V_B, H_{C_1}, V_{C_1}, H_{C_2}, V_{C_2})$

1) Eq. Cardinali della statica

per il sistema $R_1 \cup R_2 \cup \{C\}$

$$\underline{R}^{(e)} = \underline{0} \Rightarrow \quad \underline{e}_1 \quad H_A + H_B + \underline{R}^{e,a} \cdot \underline{e}_1 = 0$$

$$\underline{e}_2 \quad V_A + V_B + \underline{R}^{e,a} \cdot \underline{e}_2 = 0$$

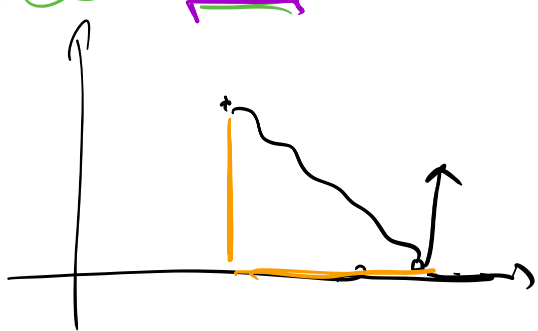
$$\underline{M}^e(A) = \underline{0} \rightarrow \overline{AB}\, V_B + \underline{M}^{e,a}(A) \cdot \underline{e}_3 = 0$$

$\Rightarrow$ V_A, V_B sono noti, rimane un'incognita fra H_A, H_B

2) ECS per R_2

$$\underline{M}^{e,2}(C) = \underline{0} \Rightarrow$$

$$\overline{CD}\, H_B + \overline{DB}\, V_B + \underline{M}^{a,2}(C) \cdot \underline{e}_3 = 0$$



$$\underline{R}^{a,2} = \underline{0} \Rightarrow \quad \underline{e}_1 \quad H_B + H_{C_2} + \underline{F}^{a,2}\cdot\underline{e}_1 = 0$$

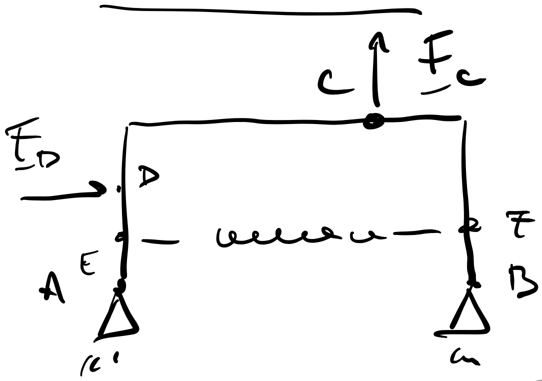
$$\underline{e}_2 \quad V_B + V_{C_2} + \underline{F}^{a,2}\cdot\underline{e}_2 = 0$$

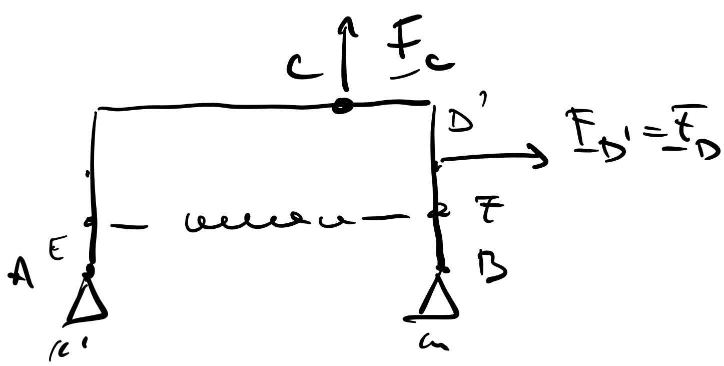
3. $\{C\}$ ECS :

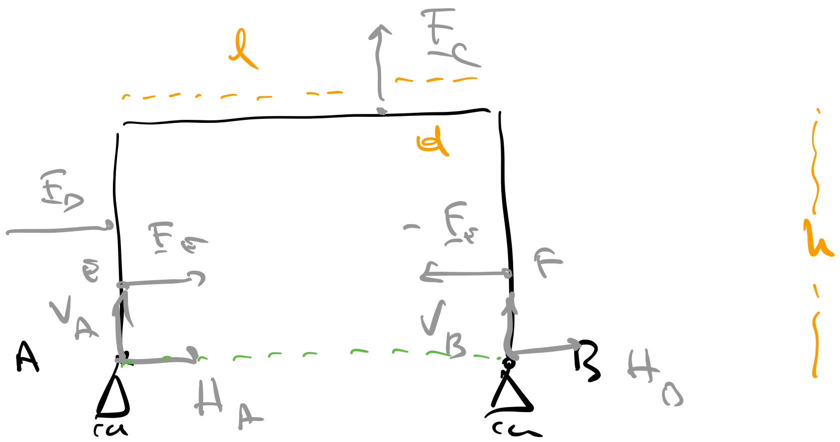
$$\underline{e}_1 \quad -H_{C_1} - H_{C_2} + \underline{F}_C\cdot\underline{e}_1 = 0$$

$$\underline{e}_2 \quad -V_{C_1} - V_{C_2} + \underline{F}_C\cdot\underline{e}_2 = 0$$

Esercizio







$$\underline{F}_D = F_D\,\underline{e}_1$$

$$\underline{F}_C = F_C\,\underline{e}_2$$

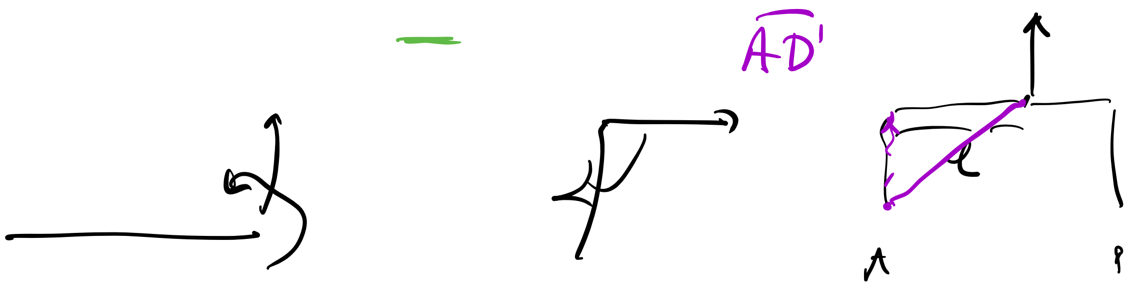
$$\underline{F}_e = -c(\underline{x}_e - \underline{x}_f)$$

— ECS per tutto il sistema

$$\underline{R}^e = \underline{0} \qquad \begin{aligned} \underline{e}_1 &\quad H_A + H_B + F_D = 0 \\ \underline{e}_2 &\quad V_A + V_B + F_C = 0 \end{aligned}$$

$$\underline{M}^e_{(A)} = \underline{0} \; : \quad \overline{AB}\, V_B - F_D\, \overline{AD} + F_C\, l = 0$$

$\overline{AD'}$



$$V_B = \frac{1}{\overline{AB}} \left(F_D\, \overline{AD} - F_C\, l \right)$$

$$V_A = -V_B - F_C = -F_D \frac{\overline{AD}}{\overline{AB}} + F_C \left(\frac{l}{\overline{AB}} - 1 \right)$$

— se vogliamo ad esempio H_B

→ consideriamo

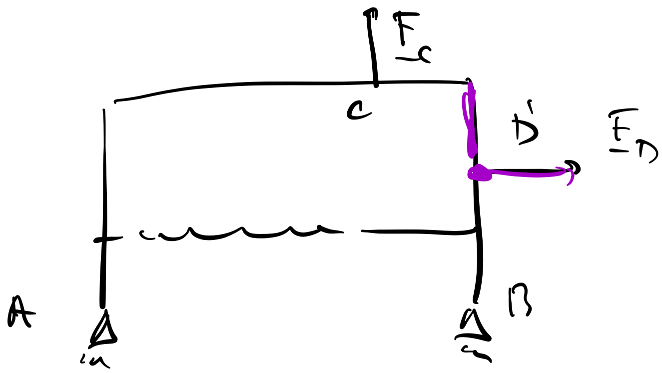
$$\underline{M}_{CB}(C) = \underline{0}$$

$$h H_B + d V_B + (-c\,\overline{AB})(h - \overline{BF}) = 0$$

$$H_B = -\frac{d}{h} V_B + c\,\overline{AB}\left(1 - \frac{\overline{BF}}{h}\right)$$

$$H_A = -H_B - F_D$$

Andiamo a vedere



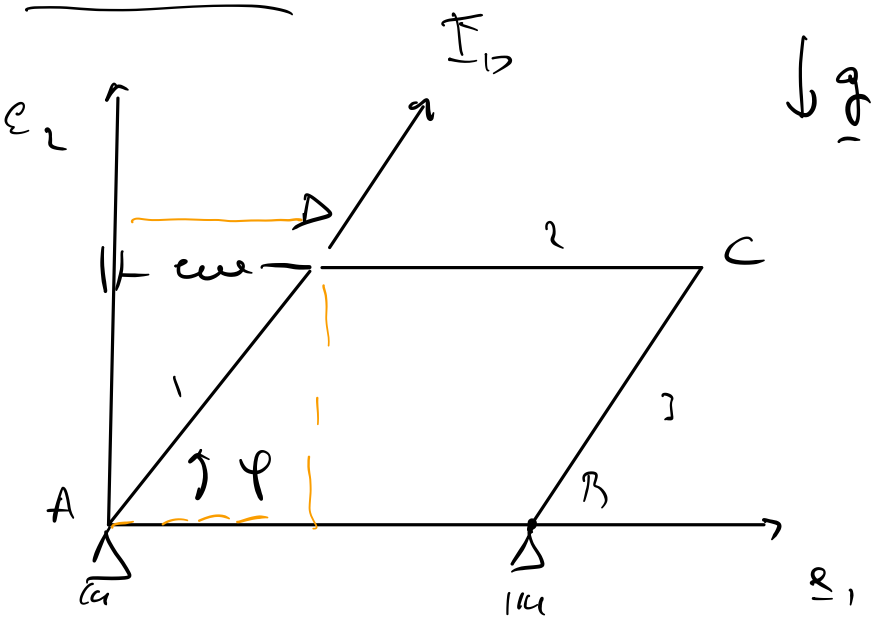
$$\underline{M}_{CB}(C) = \underline{0}$$

$$h H_B + d V_B - c\,\overline{AB}(h - \overline{BF}) + F_D (h - \overline{D'B}) = 0$$

$$H_B = -\frac{d}{h} V_B + c\,\overline{AB}\left(1 - \frac{\overline{BF}}{h}\right) - F_D\left(1 - \frac{\overline{D'B}}{h}\right)$$

$$H_A = -H_B - F_D$$

Esercizio



$\overline{AD} = \overline{DC} = \overline{CB} = \ell$

$m_1 \quad m_2 \quad m_3$

1. conf. eq. e stabilità
2. reazioni in A e B

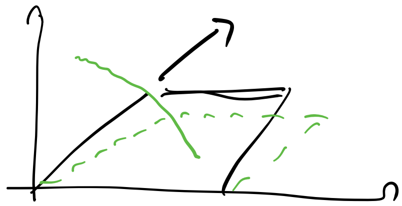
Forze attive :

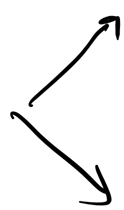
1. Peso $\quad V = m_1 g\, y_{G_1} + m_2 g\, y_{G_2} + m_3 g\, y_{G_3}$

$$= m_1 g \frac{\ell}{2} \sin\varphi + m_2 g \ell \sin\varphi + m_3 g \frac{\ell}{2} \sin\varphi$$

$$= g \ell \sin\varphi \left(\frac{m_1 + m_3}{2} + m_2 \right)$$

2. molla $\quad V = \frac{c}{2} x_D^2 = \frac{c}{2} \ell^2 \cos^2\varphi$

3.  $\quad F_D \cdot \delta x_D = 0$



$$V_{TOT} = g\ell \sin\varphi \left(\frac{m_1 + m_3}{2} + m_2 \right) + \frac{c}{2} \ell^2 \cos^2\varphi$$

$$V'_{(\varphi)} = g\ell \left(\frac{m_1 + m_3}{2} + m_2 \right) \cos\varphi - c\ell^2 \sin\varphi \cos\varphi$$

$$= c\ell^2 \left(\mu - \sin\varphi \right) \cos\varphi$$

$$\mu = g \left(\frac{m_1 + m_3}{2} + m_2 \right) \frac{1}{c\ell}$$

Si $\varphi \in [-\pi, \pi)$

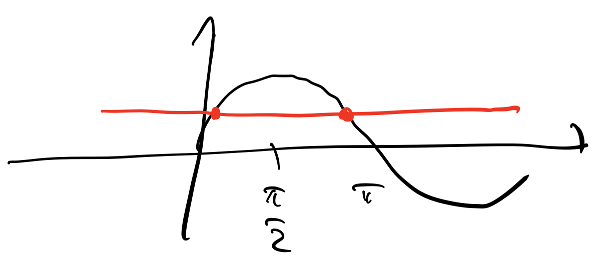
- quando $\mu > 1$ $\qquad \varphi = \pm \frac{\pi}{2}$

  $(\Leftrightarrow \sin\varphi \neq \mu)$

- quando $\mu < 1$ abbiamo $\varphi = \pm \frac{\pi}{2}$

φ_1, φ_2 tali che $\sin\varphi_1 = \mu \quad 0 < \varphi_1 < \frac{\pi}{2}$

$\sin\varphi_2 = \mu \quad \frac{\pi}{2} < \varphi_2 < \pi$



Se $V(\varphi) = c\ell^2 (\mu - \sin\varphi)\cos\varphi$

$$V'(\varphi) = c\ell^2 \left(-\mu\sin\varphi + \sin^2\varphi - \cos^2\varphi\right)$$
$$= c\ell^2 \left(-\mu\sin\varphi + 2\sin^2\varphi - 1\right)$$

- $\mu > 1$ $\quad \varphi = -\frac{\pi}{2}$ stable

  $\varphi = \frac{\pi}{2}$ instable

  $\underbrace{(-\mu + 2 - 1)}_{-\mu + 1}$

- $\mu < 1$ $\quad \varphi = \mp\frac{\pi}{2}$ stabili

  $$V(\varphi_{1,2}) = c\ell^2 \Big(-\mu \underbrace{\sin\varphi_1}_{\mu} + 2 \underbrace{\sin^2\varphi_1}_{\mu^2} - 1\Big) = c\ell^2 \left(-\mu^2 + 2\mu^2 - 1\right)$$
  $$= c\ell^2 (\mu^2 - 1)$$

  instable