MECCANICA RAZIONALE

Problema della statica → equazioni

cardinali della statica,

S all' equilibrio → $\begin{cases} \underline{R}^{(e)} = \underline{0} \\ \underline{M}^{(e)}_{(O)} = \underline{0} \end{cases}$

(equazioni di bilancio)

Singolo corpo rigido → PLV

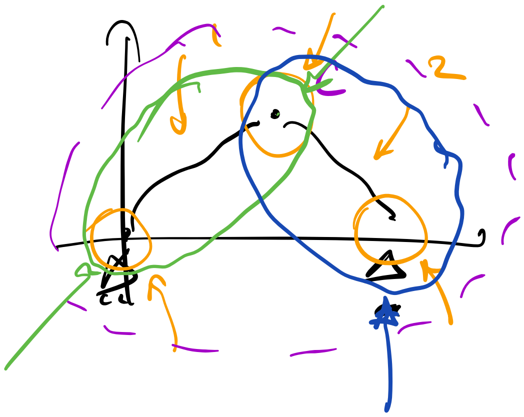
$$LV_{rigido} = \underline{R} \cdot \delta \underline{x}_O + \underline{M}(O) \cdot \underline{\chi}$$

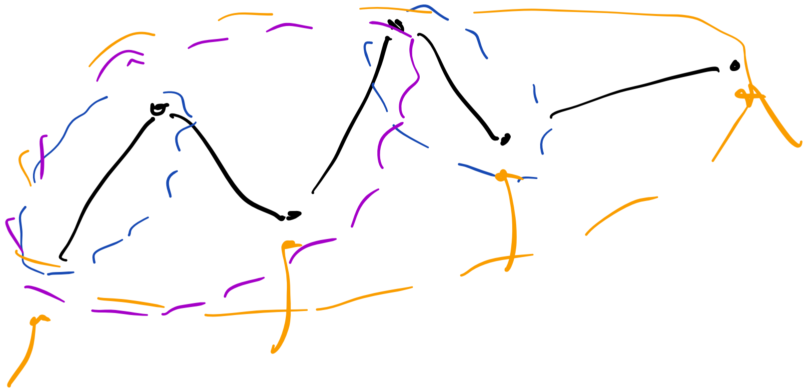
$$\downarrow$$

$$\underline{R}^{(e)} = \underline{0} \qquad \underline{M}^{e}(O) = \underline{0}$$

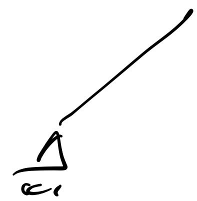
→ calcolare le reazioni vincolari all' equilibrio.

→ Statica dei corpi articolati





↪ sistema olonomo



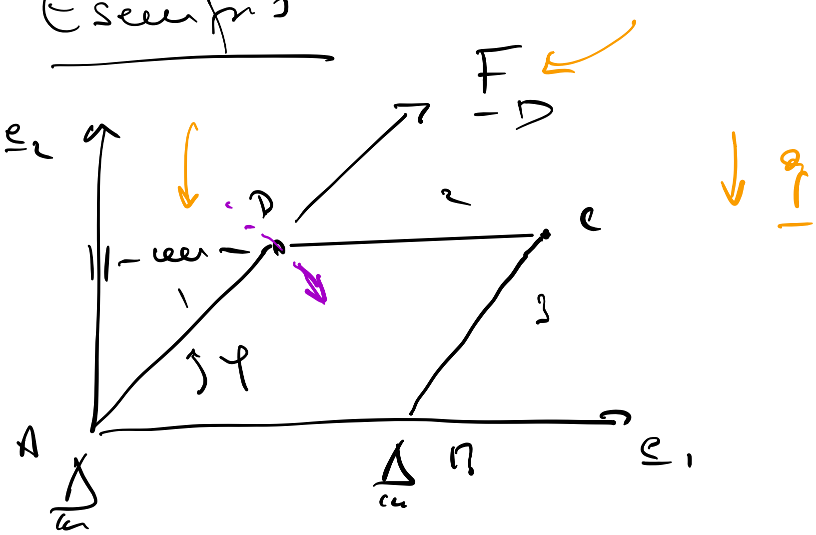
→ PLV ⇝ $\underline{V}$

$dV = 0$ → Hess V

punti di stazionarietà, stabili o instabili

→ ECS per le reazioni vincolari all'equilibrio

Esempio



lunghezza ℓ

omogenee m_1 m_2 m_3

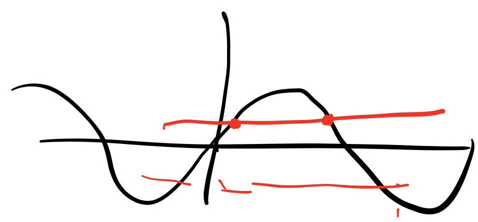
- eq. e stabilità
- reazioni vincolari all'equilibrio in A e in B

$$V_{tot} = g\, l \sin\varphi \left(\frac{m_1 + m_3}{2} + m_2 \right) + \frac{c}{2} l^2 \cos^2\varphi$$

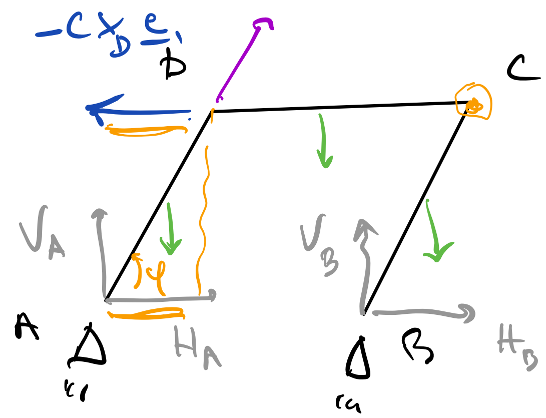
eq:

- $\mu > 1$ — $\varphi = -\frac{\pi}{2}$ stabile; $\varphi = +\frac{\pi}{2}$ instabile

- $\mu < 1$ — $\varphi = -\frac{\pi}{2}, \frac{\pi}{2}$ stabile; φ_1, φ_2 t.c. $\sin\varphi_{1,2} = \mu$ instabili



Reazioni vincolari:



$$\underline{F}_D = F \,\text{vers}(\underline{x}_D - \underline{x}_A)$$

$$= F(\cos\varphi\, e_1 + \sin\varphi\, e_2)$$

$$\underline{F} = -c(\underline{x}_{\text{att. molla}} - \underline{x}_{\text{ancora}})$$

$$\hookrightarrow -c\, x_D\, \underline{e}_1$$

$$V = \frac{c}{\cancel{2}} |\underline{x}_{ap} - \underline{x}_{anc}|^{\cancel{2}}$$

ECS a tutto il sistema

$$\underline{R}^e = \underline{0} \qquad \underline{e}_1 : \quad H_A + H_B - c\,x_D + F\cos\varphi = 0$$

$$H_A + H_B - c\,l\cos\varphi + F\cos\varphi = 0$$

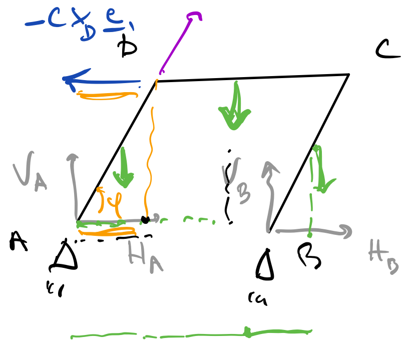
$$\underline{e}_2 : \quad V_A + V_B + F\sin\varphi - (m_1 + m_2 + m_3)\,g = 0$$

$$\underline{M}^e(A) = \underline{0} \qquad l\,V_B - m_1 g \frac{l}{2}\cos\varphi$$

$$- m_2 g\left(l\cos\varphi + \frac{l}{2}\right)$$

$$- m_3 g\left(\frac{l}{2}\cos\varphi + l\right)$$

$$+ c\,(l\cos\varphi)\,l\sin\varphi = 0$$



$$(\underline{x}_{G_3} - \underline{x}_A) \wedge (-m g\, \underline{e}_2)$$

$$\left[\left(\frac{l}{2}\cos\varphi + l\right)\underline{e}_1 + \frac{l}{2}\sin\varphi\, \underline{e}_2\right] \wedge (-m_3 g\, \underline{e}_2)$$

$$= -m_3 g\left(\frac{l}{2}\cos\varphi + l\right)$$

$$V_B = m_1 g \frac{1}{2}\cos\varphi + m_2 g\left(l\cos\varphi + \frac{1}{2}\right) + m_3 g\left(\frac{1}{2}\cos\varphi + 1\right)$$
$$- c\,(\cos\varphi)\, l\sin\varphi$$

$$= g\left(\frac{m_2}{2} + m_3\right) + g\left(\frac{m_1+m_3}{2} + m_2\right)\cos\varphi$$

$$- c\,l\cos\varphi\sin\varphi$$

$$= g\left(\frac{m_2}{2} + m_3\right) + c\,l\cos\varphi\left\{\underbrace{g\left(\frac{m_1+m_3}{2} + m_2\right)\frac{1}{cl}}_{\mu} - \sin\varphi\right\}$$

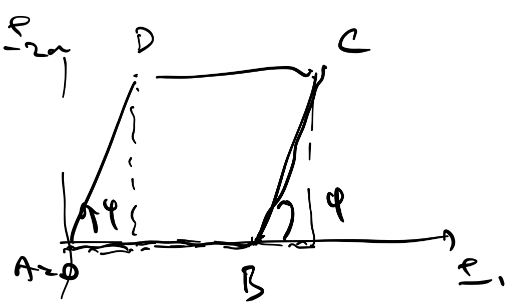
$$\boxed{V_B = g\left(\frac{m_2}{2} + m_3\right) + c\,l\cos\varphi\,(\mu - \sin\varphi)}$$

$$\boxed{V_A = (m_1+m_2+m_3)\,g - F\sin\varphi - V_B}$$

Andiamo a considerare ECS per l'asta BC e imponiamo

$$\underline{M}^e_{BC}(C) = \underline{0} = (\underline{x}_B - \underline{x}_C)\wedge(H_B\,\underline{e}_1 + V_B\,\underline{e}_2)$$

$$+ (\underline{x}_{G_3} - \underline{x}_C)\wedge(-m g\,\underline{e}_2)$$



$$\underline{x}_B = l\,\underline{e}_1$$

$$\underline{x}_C = (l + l\cos\varphi)\,\underline{e}_1 + l\sin\varphi\,\underline{e}_2$$

$$\underline{x}_{G_3} = \left(l + \frac{l}{2}\cos\varphi\right)\underline{e}_1 + \frac{l}{2}\sin\varphi\ \underline{e}_2$$

$$(\underline{x}_B - \underline{x}_C) \wedge (H_B\,\underline{e}_1 + V_B\,\underline{e}_2) =$$

$$= \left[l\,\underline{e}_1 - (l + l\cos\varphi)\underline{e}_1 - l\sin\varphi\ \underline{e}_2\right] \wedge (H_B\,\underline{e}_1 + V_B\,\underline{e}_2)$$

$$= \left[-l\cos\varphi\ \underline{e}_1 - l\sin\varphi\ \underline{e}_2\right] \wedge (H_B\,\underline{e}_1 + V_B\,\underline{e}_2)$$

$$= -l\cos\varphi\ V_B\ \underline{e}_3 + l\sin\varphi\ H_B\ \underline{e}_3$$

$$(\underline{e}_2 \wedge \underline{e}_1 = -\underline{e}_3)$$

$$(\underline{x}_{G_3} - \underline{x}_C) \wedge (-m_3 g\,\underline{e}_2) =$$

$$=\left\{\left[\left(l + \frac{l}{2}\cos\varphi\right)\underline{e}_1 + \frac{l}{2}\sin\varphi\ \underline{e}_2\right] - \left[(l + l\cos\varphi)\underline{e}_1 + l\sin\varphi\ \underline{e}_2\right]\right\} \wedge (-m_3 g\,\underline{e}_2)$$

$$+ (\ldots)\underline{e}_2$$

$$= \left(-\frac{l}{2}\cos\varphi\ \underline{e}_1\right) \wedge (-m_3 g\ \underline{e}_2)$$

$$\underline{e}_2 \wedge \underline{e}_2 = \underline{0}$$

$$= m_3 g\,\frac{l}{2}\cos\varphi\ \underline{e}_3$$

$$\underline{M}^{c}_{BC}(C) = \underline{0} \Rightarrow -\ell\cos\varphi\, V_B + \ell\sin\varphi\, H_B + m_3 g \frac{\ell}{2}\cos\varphi = 0$$

$$\rightarrow \boxed{H_B = \left(V_B - \frac{m_3 g}{2}\right)\cotan\varphi}$$

$$\boxed{H_A = -H_B + (c\ell - F)\cos\varphi}$$

$$\begin{cases} V_B = g\left(\frac{m_2}{2} + m_3\right) + c\ell\cos\varphi\,(\varphi - \sin\varphi) \\ V_A = (m_1 + m_2 + m_3)\,g - F\sin\varphi - V_B \\ H_B = \left(V_B - \frac{m_3 g}{2}\right)\cotan\varphi \\ H_A = -H_B + (c\ell - F)\cos\varphi \end{cases}$$

Quindi per i valori di equilibrio ($\varphi = \pm\frac{\pi}{2}$, $\varphi_{1,2}$ sin $\varphi_{1,2} = \gamma$)

- $\boxed{\varphi = +\frac{\pi}{2}}$ $\rightarrow$ $V_B = g\left(\frac{m_2}{2} + m_3\right)$

$$V_A = (m_1 + m_2 + m_3)\,g - F - g\left(\frac{m_2}{2} + m_3\right)$$

$$= \left(m_1 + \frac{m_2}{2} \right) g - F$$

$H_B = 0$, $H_A = 0$

- $\boxed{\varphi = -\frac{\pi}{2}}$ ----

- $\varphi = \varphi_1$ $V_B = g\left(\frac{m_2}{2} + m_3 \right)$

$$V_A = g\left(m_1 + \frac{m_2}{2} \right) - F\mu$$

$\sin\varphi_1 = \mu$

$$H_B = \frac{\sqrt{1-\mu^2}}{\mu} \left(V_B - m_3 \frac{g}{2} \right)$$

$\cos^2\varphi + \sin^2\varphi = 1 \rightarrow \cos^2\varphi_1 + \mu^2 = 1$

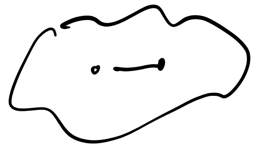
$\cos^2\varphi_1 = 1 - \mu^2 \rightarrow \cos\varphi_1 = \sqrt{1-\mu^2}$

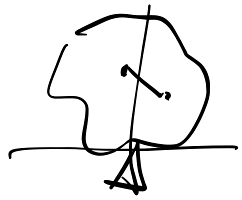
$\left(\cos\varphi_2 = -\sqrt{1-\mu^2} \right)$

$$H_A = -H_B + (cl - F)\sqrt{1-\mu^2}$$

- $\varphi = \varphi_2 \longrightarrow \cos\varphi_2 = -\sqrt{1-\mu^2}$

→ CORPO RIGIDO



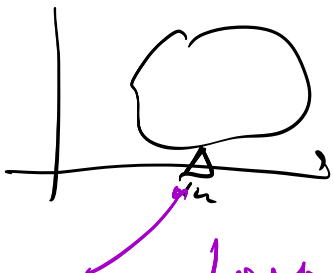


↓

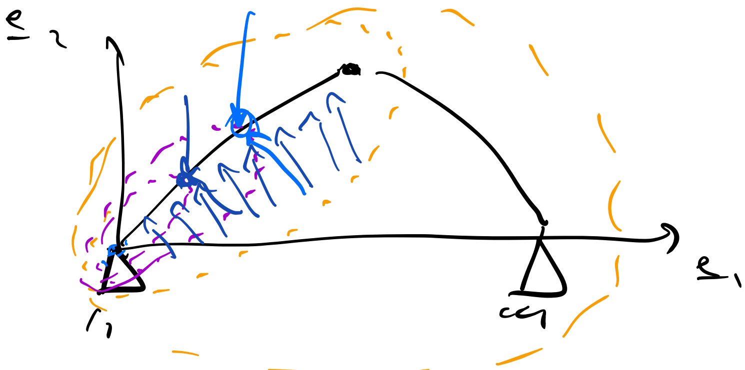
PLV — ECS

Coordinate libere (vincoli + corpi articolati) ; $LV = \sum_{i=1}^{l} Q_i \delta q_i = -dV$

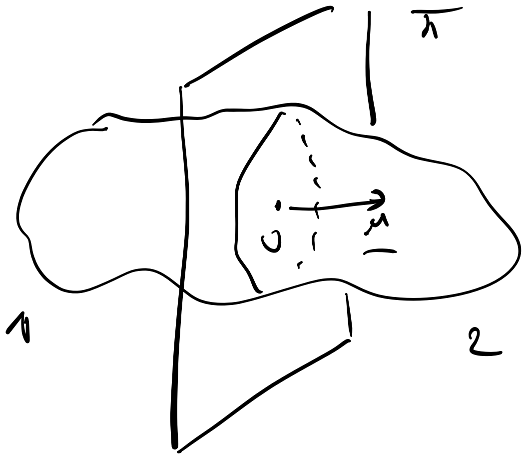
ECS :



forza di reazione all'equilibrio



SFORZI INTERNI AL RIGIDO



3 dim

$\underline{n}$ normale al piano π, direzione $1 \longrightarrow 2$

Parte 1: è in equilibrio sotto l'azione delle forze esterne applicate su 1 e delle forze interne di 2 che agiscono attraverso la sezione

ECS su 1

- $\underline{R}_1^{(e)} + \underline{R}^{(i)} = \underline{0}$

$\underline{R}_1^{(e)}$: risultante delle forze esterne su 1

$\underline{R}^{(i)}$: risultante delle forze interne di 2 su 1

- $\underline{M}_1^{(e)}(O) + \underline{M}^{(i)}(O) = \underline{0}$

$\underline{M}_1^{(e)}(O)$: momenti forze esterne su 1

$\underline{M}^{(i)}(O)$: momenti forze interne di 2 su 1

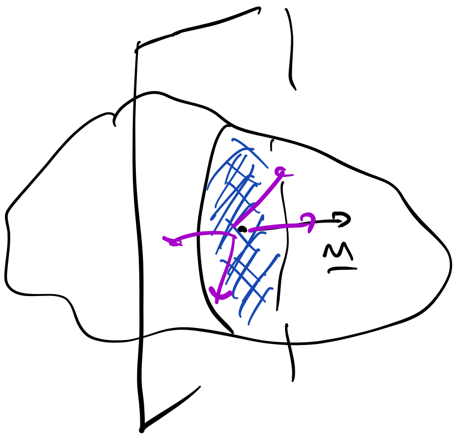
Sforzi interni

$$\underline{R}^{(i)} = N\underline{n} + \underline{T}$$

$N = \underline{R}^{(i)} \cdot \underline{n}$

sforzo normale
scalare

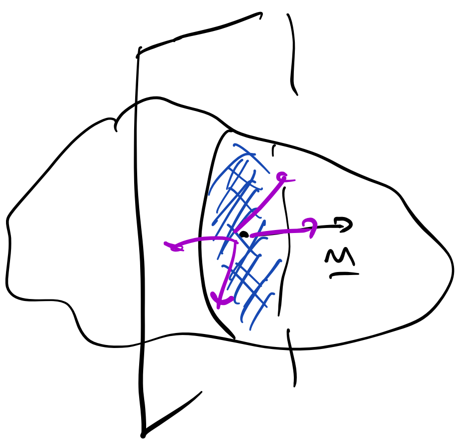
$N > 0$ TRAZIONE
$N < 0$ COMPRESSIONE



$$\underline{T} = \underline{R}^{(i)} - \underline{n} N$$

sforzo di Taglio
(vettore)

$$\underline{M}^{i}(O) = M_{T(O)}\underline{n} + \underline{M}_f(O)$$



$M_T = \underline{M}^{(i)}(O) \cdot \underline{n}$

momento Torcente
(scalare)

$$\underline{M}_f(O) = \underline{M}^{i}(O) - M_{T(O)}\underline{n}$$

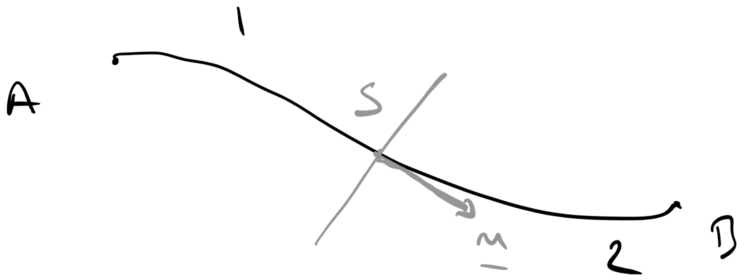
vettore
momento flettente

Archi Rigidi piani

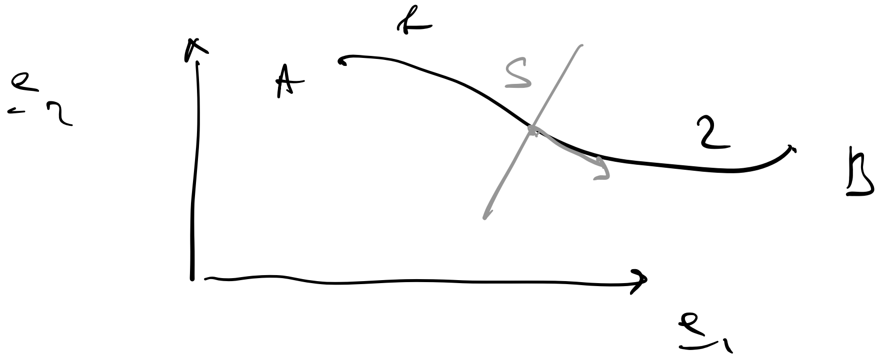
Consideriamo rigidi 1-dim piani

= archi di curve.

Tutte le forze sul piano



la normale $\underline{n}$ al piano di sezione è la tangente geometrica in S. $(1 \to 2)$



$\underline{x} = \underline{x}(S)$

$S \in [0, L]$

$\underline{x}(0) = \underline{x}_A$

$\underline{x}(L) = \underline{x}_B$

Scegliamo di misurare l'arco a partire da A: verso di percorrenza della curva per S crescente

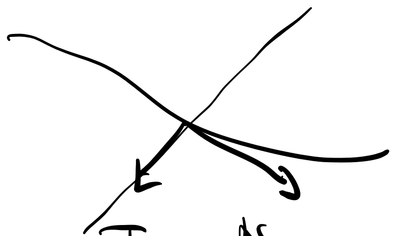
1 prima di S

2 dopo di S

la Tangente è : $\underline{n} = \dfrac{d}{ds}\underline{x}(s)$

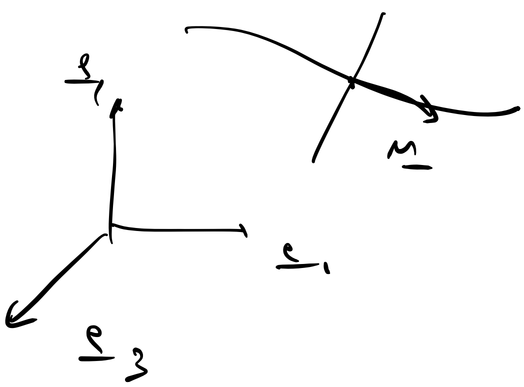
Semplificazione :

$\underline{R}^{(i)}$ appartiene al piano $(\underline{e}_1, \underline{e}_2)$



$\underline{T}$ $N\underline{n}$ diretto lungo n

$\underline{M}^{(i)}$



$\underline{M_T} \overset{=0}{}$ lungo $\underline{n}$

M_f ortogonale a $\underline{n}$

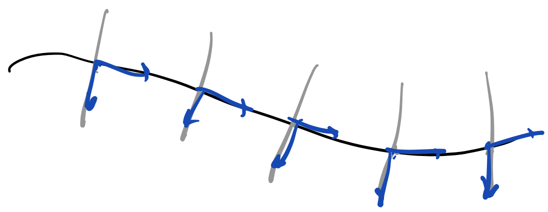
M_f $\underline{e}_3$

le equazioni di bilancio per 1

$$
\begin{cases}
\underline{R}^{(e)}_1 + N_{(s)}\,\underline{n}_{(s)} + \underline{T}_{(s)} = \underline{0} \\
\underline{M}^{(e),1}_{(s)} + \underline{M}_f(s) = \underline{0}
\end{cases}
$$

Le incognite sono 3 funzioni

($N(s)\underline{n}(s)$, $\underline{T}(s)$, $M_f(s)\underline{e}_3$)

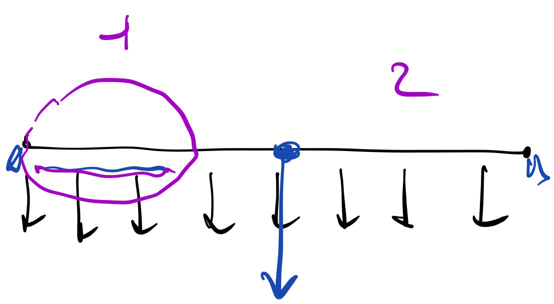


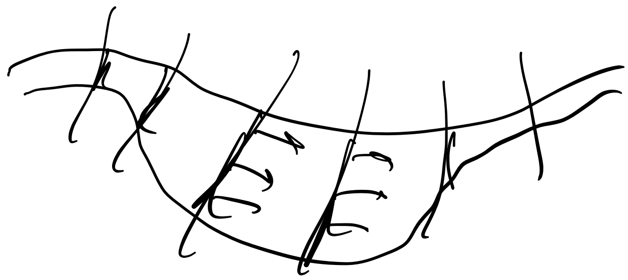
Forze esterne (carichi)

- forze concentrate : applicate in un punto

- forze distribuite : ad esempio la forza peso.

Per gli sforzi interni NON possiamo modellizzare il peso come una forza concentrata nel centro di massa.





Forze distribuite: forza specifica

$\underline{f}(s)$

Ad esempio forza peso: $\underline{f}(s) = \rho(s)\,\underline{g}$