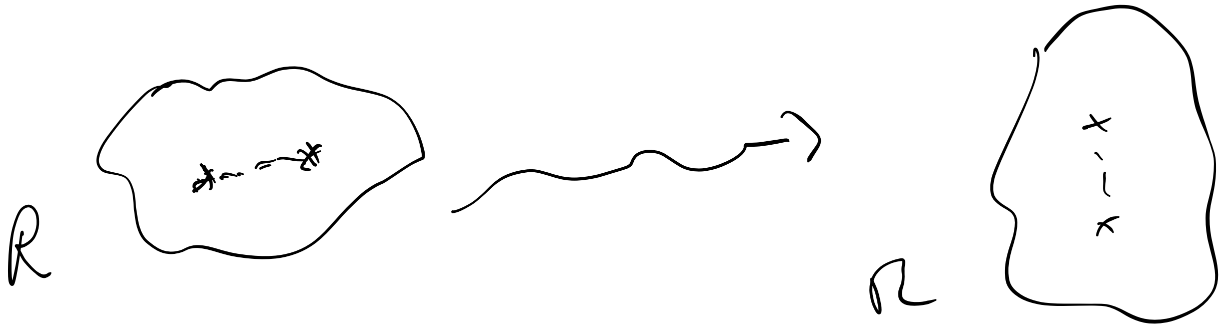
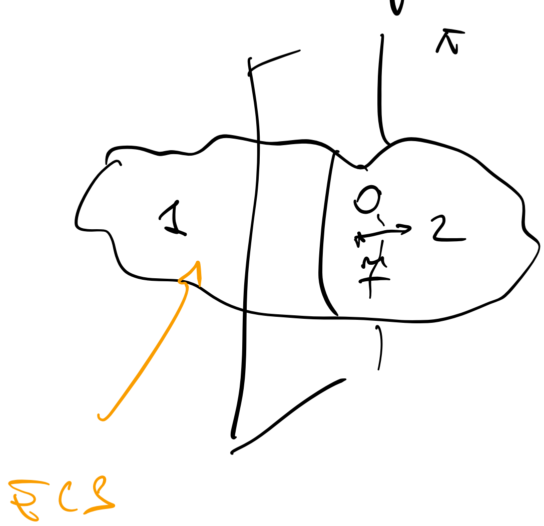
MECCANICA RAZIONALE



→ ∃ sollecitazioni interne che lo mantengono rigido.



Corpo rigido

eq → ECS

$$\begin{cases} \underline{R}^{(e)} = \underline{0} \\ \underline{M}^{(e)}_{(O)} = \underline{0} \end{cases}$$

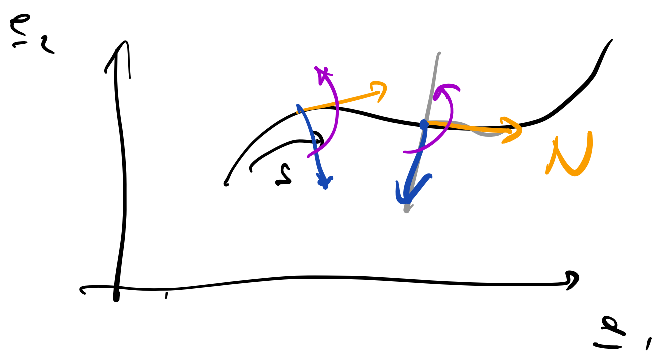
condizioni necessarie.

→ applicate a tutti i sottosistemi.

$$\begin{cases} \underline{R}_1^{(e)} + \underline{R}^{(i)} = \underline{0} \\ \underline{M}_1^{(e)}(O) + \underline{M}^{(i)}(O) = \underline{0} \end{cases}$$

→ N, $\underline{T}$, $M_T(O)$, $\underline{M}_f(O)$

Sistemi piani : archi piani



$\underline{F} \in (\underline{e}_1, \underline{e}_2)$

$\underline{M} \parallel \underline{e}_3$

$\underline{n} = \dfrac{dx(s)}{ds}$

$$\boxed{N(s)\,\underline{n}(s),\ \underline{T}(s),\ M_f(s)\,\underline{e}_3}$$

incognite → 3 funzioni

↳ forze concentrate

↳ forze distribuite
(specifica : $\underline{f}(s) = \rho(s)\,\underline{g}$)

↑

Teorema (Relazioni differenziali per gli sforzi interni)

Supponiamo che

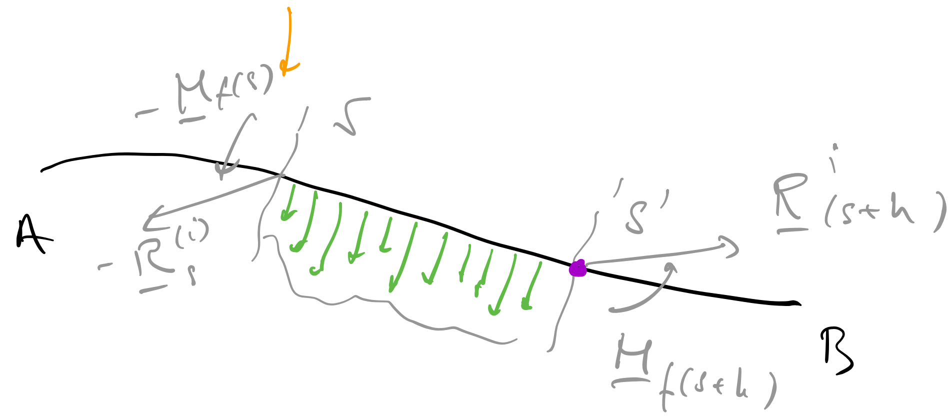
- non ci siano forze concentrate

- forza specifica regolare
- arco regolare

Allora

$$\begin{cases} \dfrac{d}{ds}\left(N(s)\,\underline{m}(s) + \underline{T}(s) \right) = - f(s) \\[2ex] \dfrac{d}{ds}\,\underline{M}_f(s) = \underline{T}(s) \wedge \underline{m}(s) \end{cases}$$

Dimostrazione



Arco tra S e S' è determinato dai valori s e s+h (h>0)

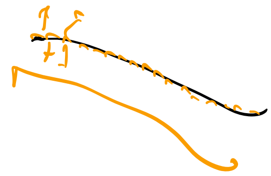
$\underline{R}^{(i)} = N\,\underline{m} + \underline{T}$

$\underline{M}_f$

} sono definiti in ogni sezione come gli sforzi che la parte successiva alla sezione esercita sulla parte precedente.

Forze che agiscono su SS'

- forze esterne (con forza specifica $\underline{f}$)



$$\underline{R}^{SS'} = \int_s^{s+h} \underline{f}(\xi)\, d\xi$$

$$\underline{M}^{SS'}_{(s)} = \int_s^{s+h} \left(\underline{x}(\xi) - \underline{x}(s) \right) \wedge \underline{f}(\xi)\, d\xi$$

$$\left(\sum_P (\underline{x}_P - \underline{x}_O) \wedge \underline{F}_P \right)$$

- sforzi interni di S'B su SS'

  tramite S'

$$\underline{R}^{i}(s+h)$$

$$\underline{M}_f(s+h)$$

- sforzi interni che AS esercita

  su SS' (tramite S)

$$-\underline{R}^{(i)}(s)$$

$$-\underline{M}_f^{(i)}(s)$$

Equazione di bilancio:

$$\Rightarrow \quad \underline{R}^{SS'} + \underline{R}^{(i)}(s+h) - \underline{R}^{(i)}(s) = \underline{0}$$

$$\rightarrow \quad \underline{M}^{(ss')} + \underline{M}_f^{(i)}(s+h) - \underline{M}_f^{(i)}(s) +$$

$$+ \left(\underline{x}(s+h) - \underline{x}(s) \right) \wedge \underline{R}^{(i)}_{(s+h)} = \underline{0}$$

(momenti $\rightarrow$ s è il polo)

Dividiamo per h e poi $h \rightarrow 0$

$$\frac{\underline{R}^{(i)}(s+h) - \underline{R}^{(i)}(s)}{h} = -\frac{1}{h} \underline{R}^{(ss')}$$

$$\frac{\underline{M}_f(s+h) - \underline{M}_f(s)}{h} = -\frac{1}{h} \underline{M}^{ss'}(s)$$

$$+ \underline{R}^{(i)}(s+h) \wedge \frac{\underline{x}(s+h) - \underline{x}(s)}{h}$$

- Tutto è regolare. Possiamo assumere che la forza $\underline{f}$ sia limitata $\| \underline{f} \| < F$: condizione fisica.

$$\bullet \quad \underline{R}^{ss'}(s) = \int_s^{s+h} \underline{f}(\xi)\, d\xi \xrightarrow[h\to 0]{} 0$$

immaginiamo di avere g tale

che $\frac{dg}{ds} = f$ $\qquad \int \frac{dg}{ds} = g|$

$$g(s) - g(s+h) \xrightarrow[h\to 0]{} 0$$

$$\bullet \quad \frac{\underline{R}^{ss'}}{h} = \frac{1}{h}\int_s^{s+h} \underline{f}(\xi)\, d\xi \longrightarrow \underline{f}(s)$$

$$= \frac{1}{h}\left[g(s+h) - g(s)\right] \xrightarrow[h\to 0]{} \frac{dg}{ds}$$

$$= \underline{f}(s)$$

$$\frac{\underline{R}^{(i)}(s+h) - \underline{R}^{(i)}(s)}{h} = -\frac{1}{h}\underline{R}^{(ss')}$$

$$\frac{d}{ds}\left(N\underline{m} + \underline{T}\right) = -\underline{f}(s)$$

$$\bullet \quad \left\| \frac{\underline{M}^{SS}(s)}{h} \right\| = \frac{1}{h} \left\| \int_s^{s+h} (\underline{x}(\xi) - \underline{x}(s)) \wedge \underline{f}(\xi)\, d\xi \right\|$$

$$\leq \frac{1}{h} \int_s^{s+h} \underbrace{\| \underline{x}(\xi) - \underline{x}(s) \|}_{\leq h} \underbrace{\| \underline{f} \|}_{\leq F} \, d\xi$$

$$\leq \frac{1}{h} \, h \, F \cdot \underset{\substack{\uparrow \\ \int_s^{s+h}}}{h} = F h \xrightarrow[h \to 0]{} 0$$

$$\bullet \quad \frac{\underline{x}(s+h) - \underline{x}(s)}{h} \xrightarrow[h \to 0]{} \frac{d\underline{x}}{ds} = \underline{M}(s)$$

$$\frac{\underline{M}_f(s+h) - \underline{M}_f(s)}{h} = -\frac{1}{h} \underline{M}^{SS'}(s)$$

$$+ \underline{R}^{(i)}(s+h) \wedge \frac{\underline{x}(s+h) - \underline{x}(s)}{h}$$

$$\frac{d}{ds} \underline{M}_f(s) = \underline{R}^{(i')} \wedge \underline{M}(s) = \overline{T}_{(s)} \wedge \underline{M}_{(s)}$$

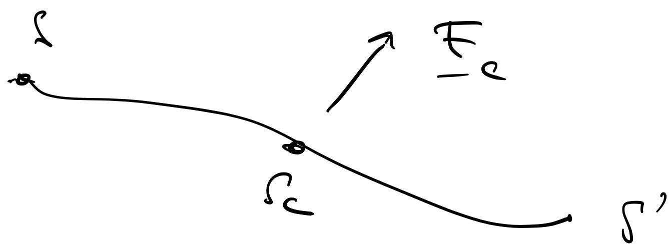
$$L_1 (N \cancel{M} + \overline{T}) \wedge \cancel{M}$$

Abbiamo imposto

$$\begin{cases} \dfrac{d}{ds}\left(N(s)\,\underline{u}(s) + \underline{T}(s) \right) = -\underline{f}(s) \\ \dfrac{d}{ds}\,\underline{M}_f(s) = \underline{T}(s) \wedge \underline{u}(s) \end{cases}$$

Se invece abbiamo $\underline{F}_c$ applicato in

S_c



Eq. di bilancio

- $\underline{R}^{ss'} \longrightarrow$ fuori $\underline{F}_c$
- $\underline{M}^{ss'} \longrightarrow$ $\left(\underline{x}(s_c) - \underline{x}(s)\right) \wedge \underline{F}_c$

$\rightarrow$ $\underline{R}^{(i)}$ ha un salto in S_c

$$\underline{F}_c + \overbrace{\left[\underline{R}^{(i)}\right]_{s=s_c}}^{\lim_{s \to s_c^+} \underline{R}^{(i)}(s) - \lim_{s \to s_c^-} \underline{R}^{(i)}(s)} = \underline{0}$$

$$\rightarrow \quad \underline{M}^{ss'} \xrightarrow[h\to 0]{} \underline{0}$$

• s_c $\quad s \neq s_c \quad \frac{d}{ds}\underline{M}_f(s) = \underline{T}(s) \wedge \underline{u}(s)$

discontinua in C

M_f è continuo

M_f non è derivabile in C

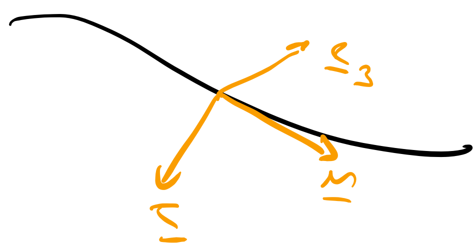
Commento: diagramma degli sforzi interni

prima: $\quad N\underline{n} \longrightarrow \underline{\underline{N(s)}} \leftarrow$

$\underline{M}_f(s) \longrightarrow M_f\,\underline{e}_3 \leftarrow$

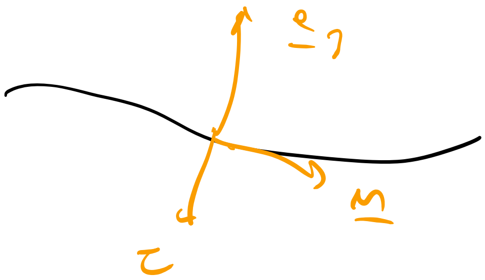
$\underline{T} \longrightarrow T(s)\,\underline{t}$

Convenzione



$\underline{t} \wedge \underline{n} = \underline{e}_3$

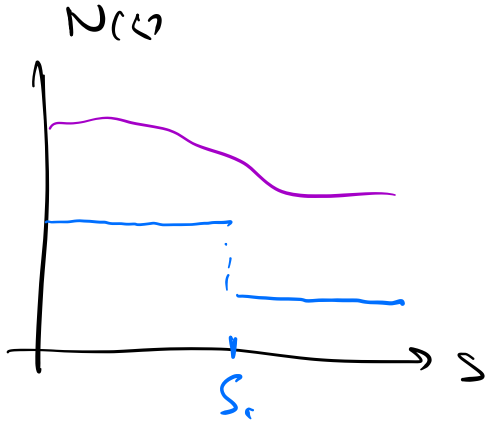
$\longrightarrow \boxed{\underline{t} = \underline{n} \wedge \underline{e}_3}$

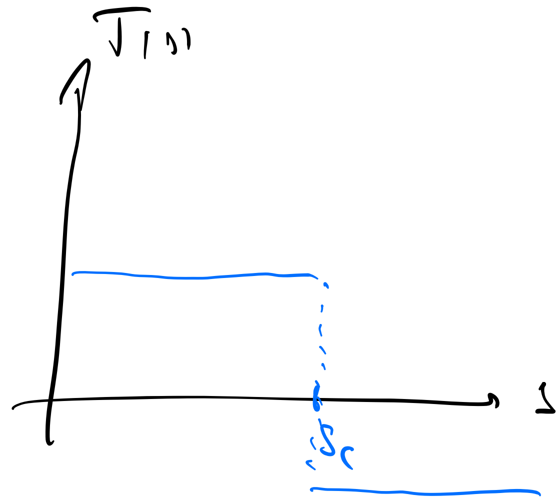


$N(s)$

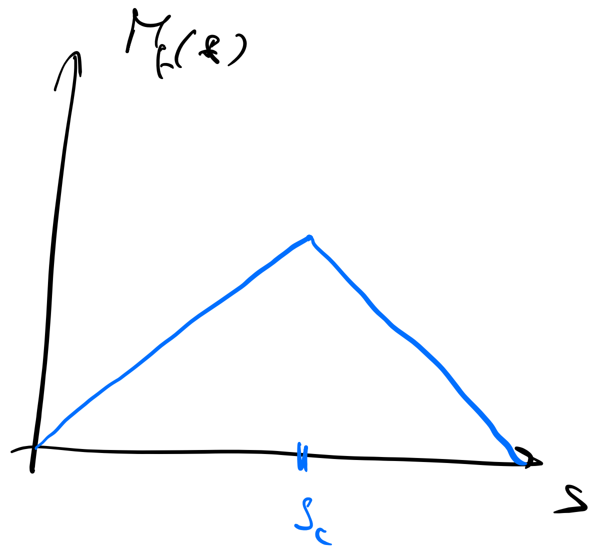
$T(s)$

$M_f(s)$





$f(s)$



carichi concentrati:

$$\boxed{\frac{dM_f}{ds} = \underline{T(s)}}$$

$$\frac{d}{ds} M_f \, \underline{e}_3 = T \, \underline{\tau} \wedge \underline{n}$$

$$= T \, \underline{e}_3$$