CINEMATICA

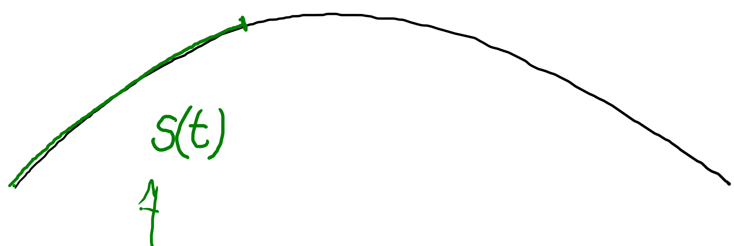
$$\left.\begin{array}{l}\text{cinematica}\\ \text{statica}\\ \text{dinamica}\end{array}\right\}\ \text{meccanica}$$

PUNTO MATERIALE : oggetto puntiforme di massa m

MOTO DEL PUNTO MATERIALE : posizione in funzione di t

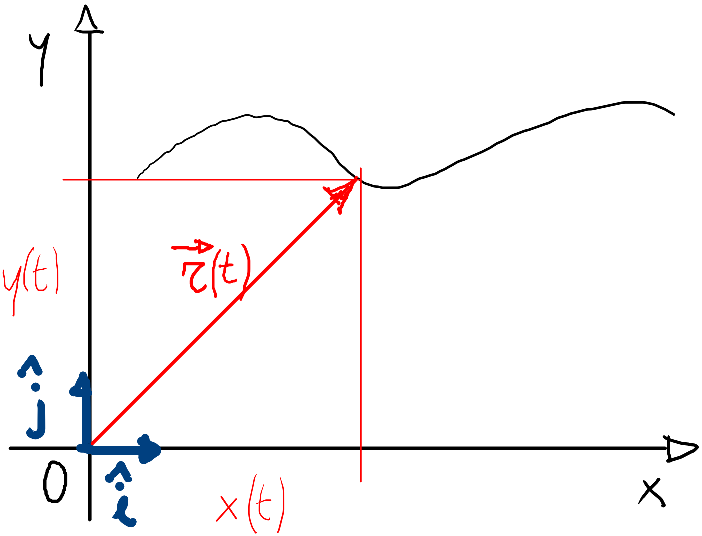
TRAIETTORIA : luogo geometrico delle posizioni

se la traiettoria è nota,



lunghezza del tratto di traiettoria percorso al tempo t

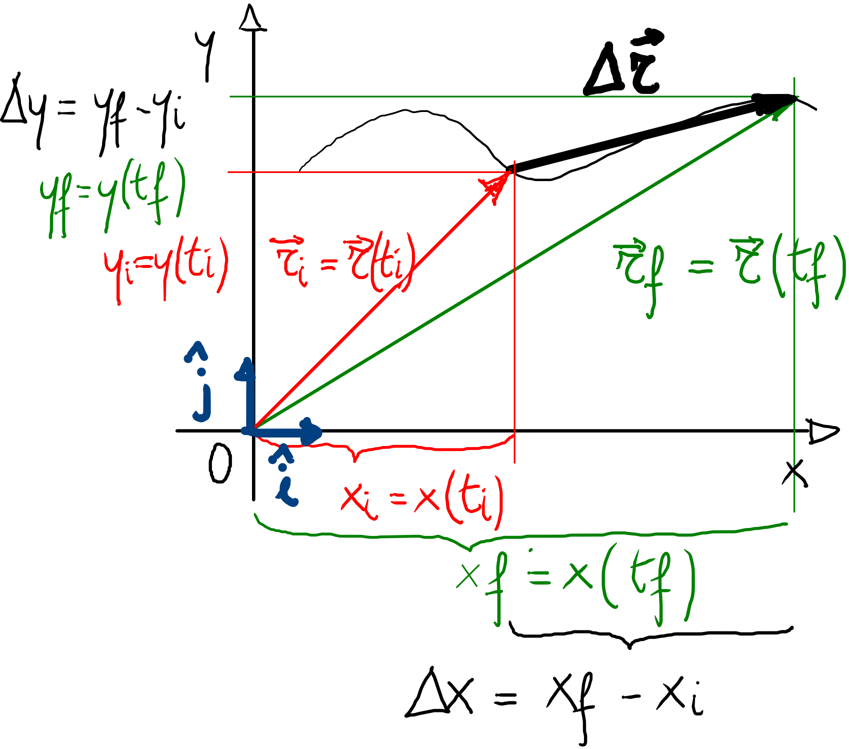
POSIZIONE



$\vec{r}(t)$ vettore posizione
(raggio vettore)

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

SPOSTAMENTO



$t_i \Rightarrow \vec{r}_i = \vec{r}(t_i)$

$t_f \Rightarrow \vec{r}_f = \vec{r}(t_f)$

spostamento:

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i \stackrel{!}{=} \vec{r}(t_f) - \vec{r}(t_i)$$

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$\Delta t = t_f - t_i$$

VELOCITÀ MEDIA

1D

$$\vec{v}_m = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad\qquad v_m = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$\vec{v}_m$ è vettore : $|\vec{v}_m| = \dfrac{|\Delta \vec{r}|}{\Delta t}$

$$\left.\begin{array}{l}\text{direzione}\\ \text{verso}\end{array}\right\} \text{vedi } \Delta\vec{r}$$

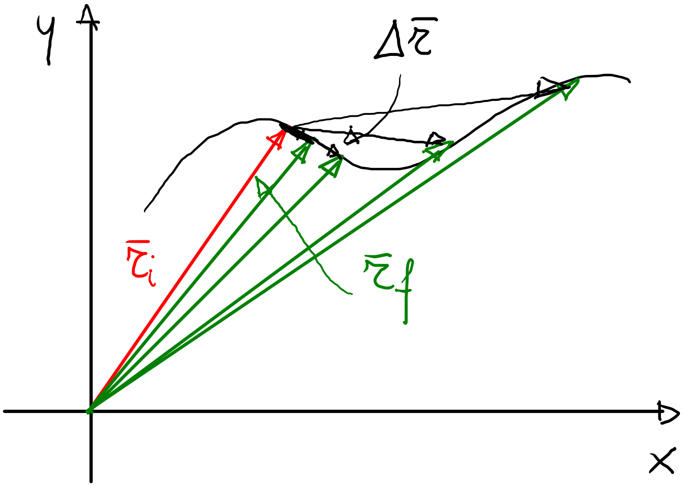
$$[v] = \frac{[L]}{[T]} \qquad \frac{m}{s} \text{ in SI} \qquad \frac{cm}{s} \text{ in cgs}$$

$$1\ \frac{Km}{h} = \frac{1000\ m}{3600\ s} = \frac{1}{3{,}6}\ \frac{m}{s}$$

$$1\ \frac{m}{s} = 3{,}6\ \frac{Km}{h}$$

$$c = 3 \cdot 10^8\ \frac{m}{s} = 30\ cm/ns$$

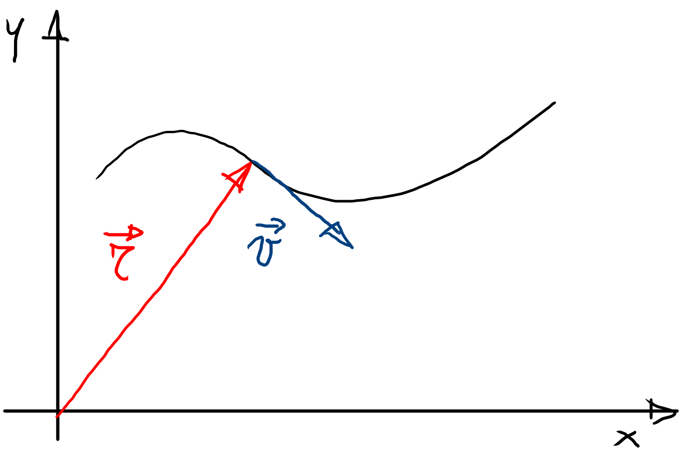
VELOCITÀ ISTANTANEA



Δt piccolo

$\Delta\vec{r}$ "si appoggia" alla traiettoria

$|\Delta\vec{r}|$ piccolo



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$|\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \left| \frac{d\vec{r}}{dt} \right|$$

direzione è tangente alla traiettoria

verso del moto

In pratica si può fare così:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x(t)\,\hat{i} + y(t)\,\hat{j} \right) = \frac{dx(t)}{dt}\,\hat{i} + \frac{dy(t)}{dt}\,\hat{j}$$

in 1D

$$v = \frac{dx(t)}{dt}$$

dove $x(t)$, $y(t)$ rappresentano la legge oraria del moto

$(\longrightarrow)$

ad esempio $x(t) = A \sin(\omega t + \varphi)$

$$v(t) = \frac{d}{dt} x(t) = A\omega \cos(\omega t + \varphi)$$

ACCELERAZIONE

$$\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

$$[a] = \frac{[L]}{[T^2]}$$

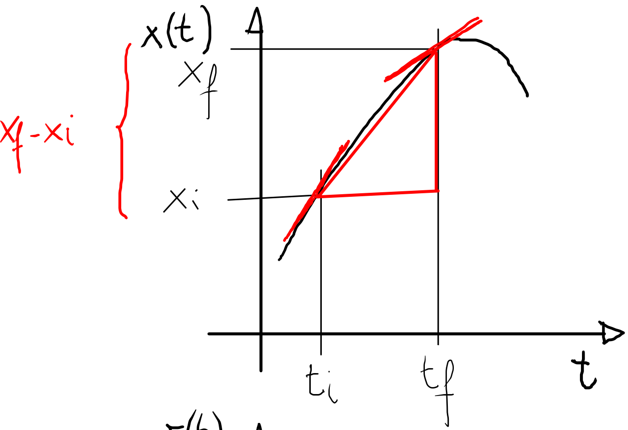
$\frac{m}{s^2}$ in SI $\qquad$ $\frac{cm}{s^2}$ in c.g.s.

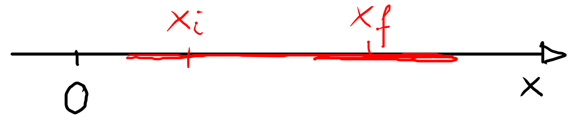
$$|\vec{g}| = 9{,}8 \ \frac{m}{s^2}$$

$$|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right| = \left|\frac{d^2\vec{r}}{dt^2}\right|$$

LEGGE ORARIA DEL MOTO (1D)

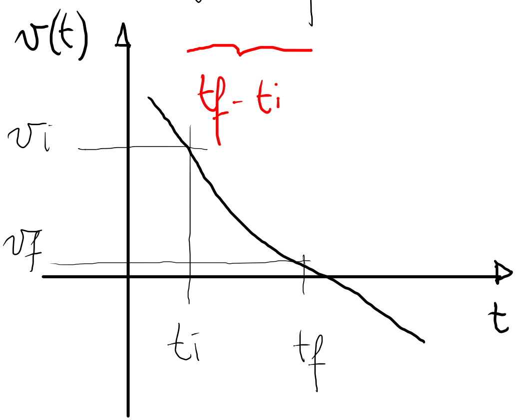
in 1D : $x(t)$





$$v_m = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

v_m è la pendenza media della curva



v è la pendenza della curva nel punto (inclinazione della tangente alla curva nel punto)

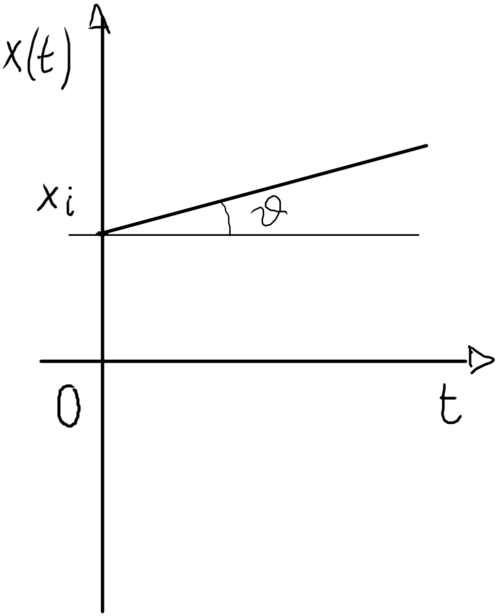
$$a_m = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

MOTO RETTILINEO UNIFORME

RETTILINEO $\downarrow$ 1D

UNIFORME $\downarrow$ v costante

$$v = v_i = v_m = v_f$$



$$v = \frac{\Delta x}{\Delta t} = \operatorname{tg} \vartheta$$

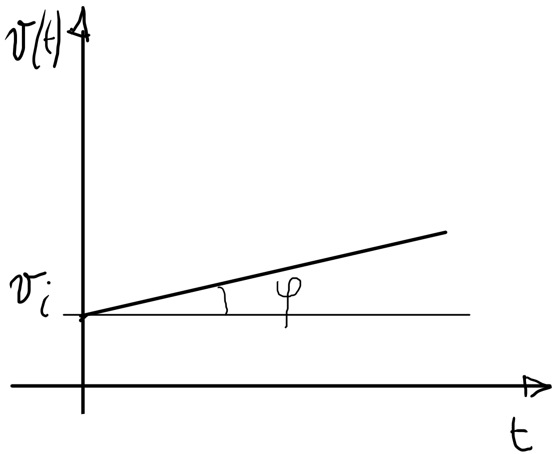
$$x(t) = x_i + v t$$

$$\vec{r}(t) = \vec{r}_i + \vec{v}\, t \quad , \quad \vec{v} \text{ costante}$$

MOTO RETTILINEO UNIFORMEMENTE ACCELERATO

RETTILINEO $\downarrow$ 1D

UNIFORMEMENTE $\downarrow$ a è costante



I) $\vec{v}(t) = \vec{v}_i + \vec{a}\,t$

II) $\vec{x}(t) = \vec{x}_i + \vec{v}_i\,t + \frac{1}{2}\vec{a}\,t^2$

III) TO BE CONTINUED

$$a = \operatorname{tg}\varphi$$

I $\quad \underline{v(t) = v_i + at} \qquad (*)$

II $\quad \underline{x(t) = x_i + v_i t + \frac{1}{2} a t^2}$

$$x(t=0) = x_i$$

$$\frac{dx(t)}{dt} = v_i + \frac{1}{\cancel{2}} a \cancel{2} t \overset{!}{=} v_i + at \quad (*)$$

III $\quad \underline{v^2 = v_i^2 + 2a(x - x_i)}$

$$\begin{cases} \mathrm{I} & v(t) = v_i + at \\ \mathrm{II} & x(t) = x_i + v_i t + \frac{1}{2}at^2 \end{cases}$$

$$\begin{cases} t = \dfrac{v - v_i}{a} \\ - \end{cases}$$

$$x = x_i + v_i\left(\frac{v - v_i}{a}\right) + \frac{1}{2}a\left(\frac{v - v_i}{a}\right)^2$$

$$x - x_i = \frac{vv_i - v_i^2}{a} + \frac{1}{2}\cancel{a}\,\frac{(v^2 - 2vv_i + v_i^2)}{a^{\cancel{2}}}$$

$$x - x_i = \frac{\cancel{2vv_i} - 2v_i^2 + v^2 - \cancel{2vv_i} + v_i^2}{2a}$$

$$x - x_i = \frac{v^2 - v_i^2}{2a}$$

$$2a(x - x_i) = v^2 - v_i^2$$

$$\mathrm{III} \quad \underline{v^2 = v_i^2 + 2a(x - x_i)}$$

MOTO CIRCOLARE UNIFORME

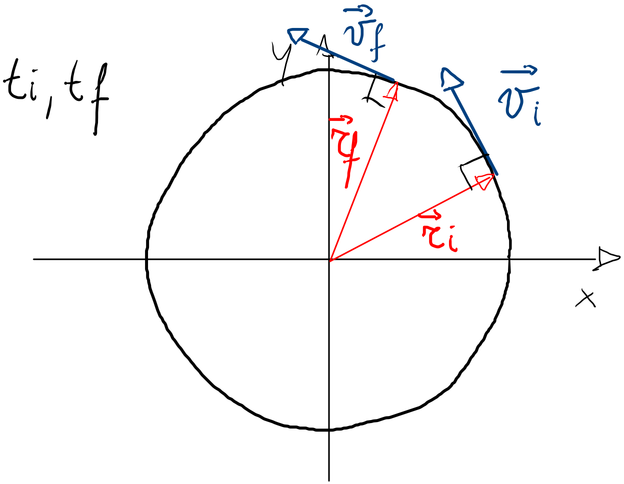
$|\vec{r}_i| = |\vec{r}_f| = R$ $\quad$ $|\vec{v}_i| = |\vec{v}_f| = v$

R è costante

v è costante

$$v = \frac{\Delta s}{\Delta t}$$

t_i, t_f



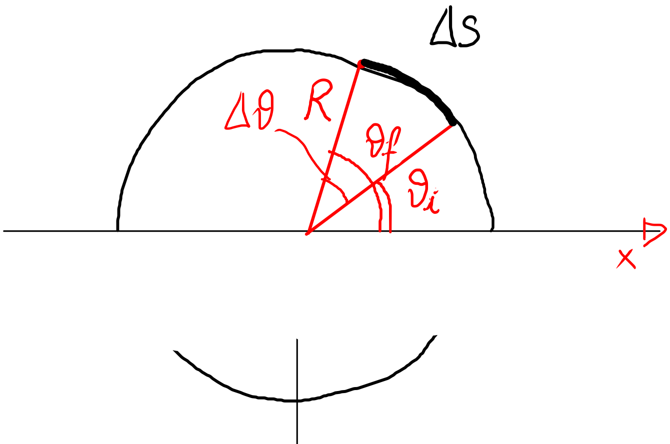
Altra costante: la velocità angolare ω

VEL. ANGOLARE

$$\Delta\vartheta = \vartheta_f - \vartheta_i$$

$$\omega_m = \frac{\Delta\vartheta}{\Delta t} \qquad \frac{\text{rad}}{s}$$ MEDIA

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} \qquad \frac{\text{rad}}{s}$$ ISTANT.



$$\Delta\vartheta = \frac{\Delta s}{R} \qquad \frac{\Delta\vartheta}{\Delta t} = \frac{\Delta s}{\Delta t} \cdot \frac{1}{R}$$

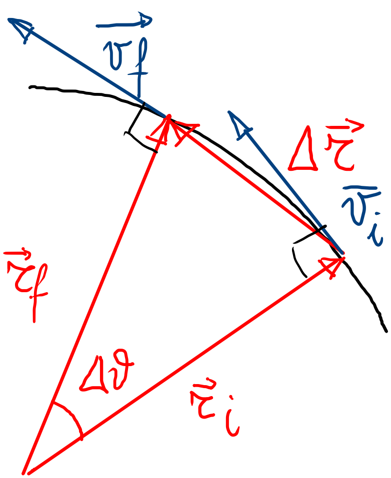
$$\boxed{\omega = \frac{v}{R} \qquad v = \omega R}$$

$$\alpha_m = \frac{\Delta\omega}{\Delta t}$$ ACCELERAZIONE ANGOLARE MEDIA

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$ ISTANTANEA

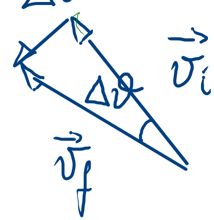
Nel moto circ. uniforme ω è costante $\Rightarrow$ α è nulla

CONSIDERAZIONI GEOMETRICHE SU $\Delta\vec{r}$ e $\Delta\vec{v}$



$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$



$$|\vec{r}_i| = |\vec{r}_f| = R$$
$$|\vec{v}_i| = |\vec{v}_f| = v$$

$$\frac{|\Delta\vec{r}|}{R} = \frac{|\Delta\vec{v}|}{v}$$

ACCELERAZIONE NEL MOTO CIRCOLARE UNIFORME

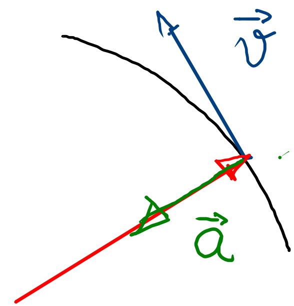
$$\vec{a}_m = \frac{\Delta \vec{v}}{\Delta t} \, .$$

$$|\vec{a}_m| = \frac{|\Delta \vec{v}|}{\Delta t} = v \, \frac{|\Delta \vec{r}|}{R} \cdot \frac{1}{\Delta t} = \frac{v}{R} \cdot \frac{|\Delta \vec{r}|}{\Delta t}$$

$$|\vec{a}| = \lim_{\Delta t \to 0} |\bar{a}_m| = \lim_{\Delta t \to 0} \frac{v}{R} \cdot \frac{|\Delta \vec{r}|}{\Delta t} = \frac{v}{R} \cdot \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t}$$

$$= \frac{v}{R} \underbrace{\left(\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \right)}_{v} = \frac{v}{R} \cdot v = \frac{v^2}{R}$$

$$\boxed{|\vec{a}| = \frac{v^2}{R} = \omega^2 \cdot R}$$



$\vec{a}$ é centripeta

MOTI PERIODICI, PERIODO, PULSAZIONE, FREQUENZA

$$v = \frac{2\pi R}{T}$$ T periodo del moto

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$$

moto circ. uniforme

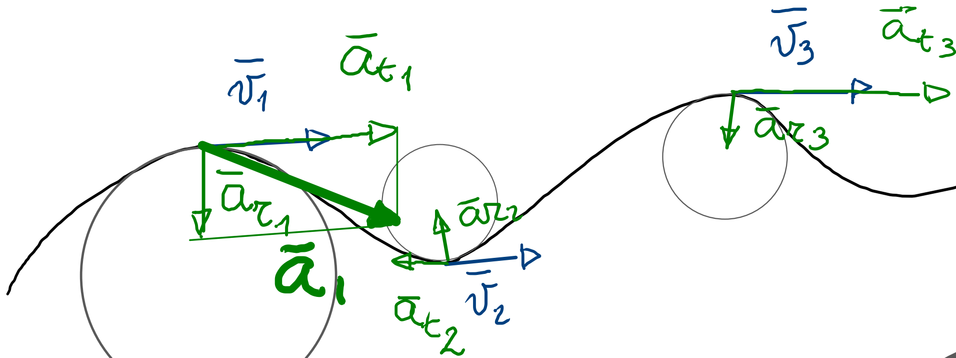
moti periodici (in generale)

$$T = \frac{2\pi}{\omega}$$ pulsazione del moto

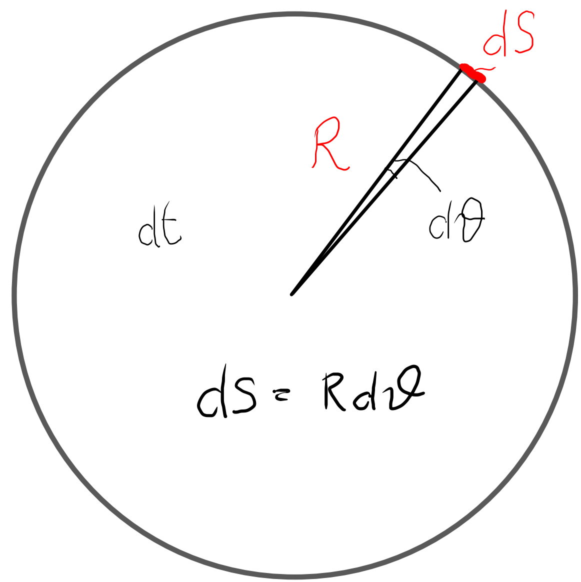
$$\nu = \frac{1}{T}$$ frequenza $(1 \text{ Hertz} = 1\, s^{-1})$

$$T = \frac{1}{\nu} \qquad \nu = \frac{\omega}{2\pi} \qquad \omega = 2\pi\nu$$

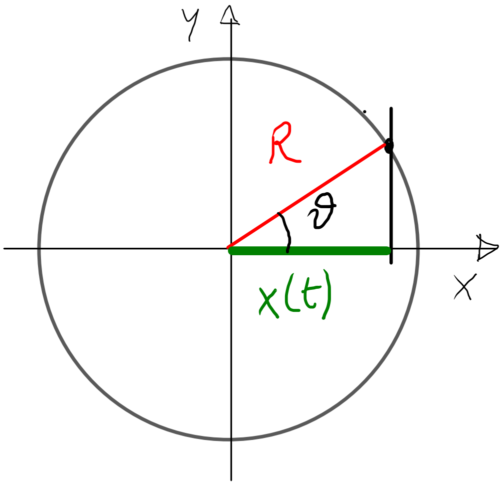
GENERICO MOTO CURVILINEO



$$|\vec{a}_t| = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}\left(\frac{R\,d\vartheta}{dt}\right) = R\,\frac{d}{dt}\underbrace{\left(\frac{d\vartheta}{dt}\right)}_{\omega} = R\,\frac{d\omega}{dt} = R\alpha$$



MOTO ARMONICO



$$\vartheta(t) = \omega t$$

$$x(t) = R\cos(\vartheta) = R\cos(\omega t)$$

$$x(t) = R\cos(\omega t)$$ moto armonico

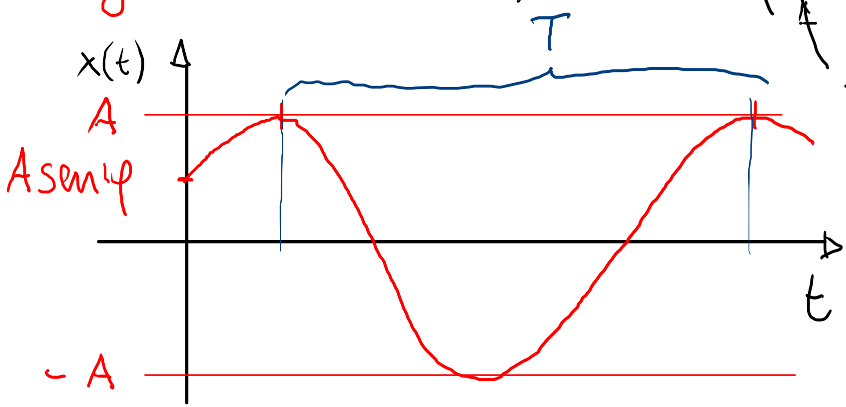
In generale :

$$x(t) = A\sin(\omega t + \varphi)$$

ampiezza

fase (legata alle condizioni iniziali)

pulsazione



$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$

$$\operatorname{sen}\left(\alpha + \frac{\pi}{2}\right) = \cancel{\operatorname{sen}\alpha \cos\frac{\pi}{2}} + \cos\alpha \operatorname{sen}\frac{\pi}{2} = \cos\alpha$$

$$\operatorname{sen}(\omega t + \underbrace{\varphi}_{=\frac{\pi}{2}}) = \cos(\omega t)$$

$$x(t) = A \operatorname{sen}(\omega t + \varphi)$$

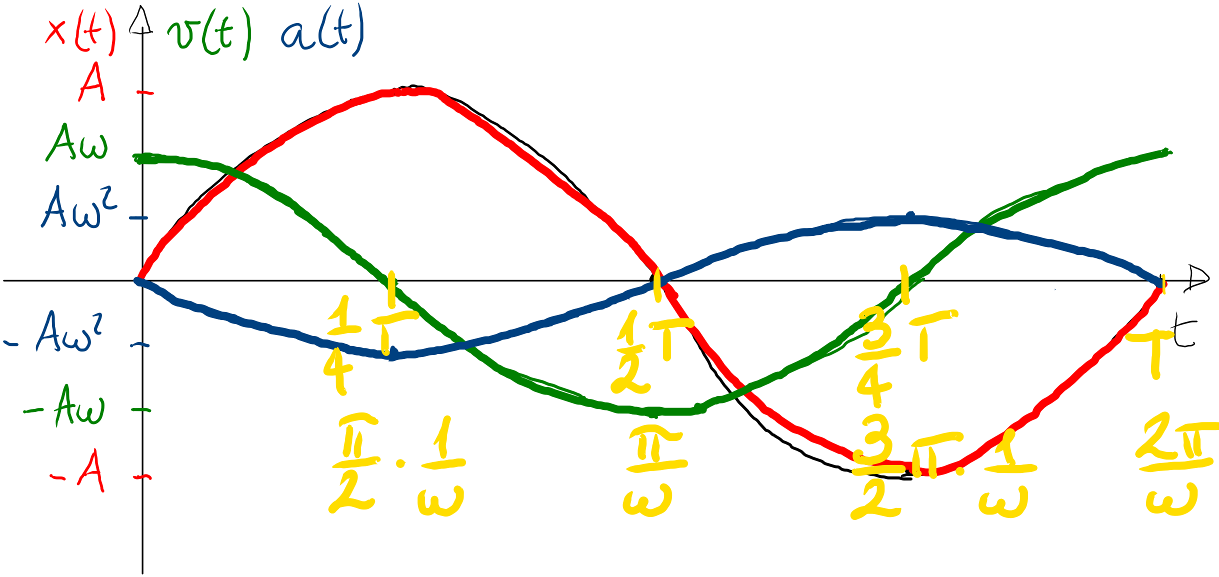
$$v(t) = \frac{dx(t)}{dt} = A\cos(\omega t + \varphi) \cdot \overbrace{\frac{d}{dt}(\omega t + \varphi)}^{\omega} = A\omega\cos(\omega t + \varphi)$$

$$a(t) = \frac{dv(t)}{dt} = A\omega\left(-\sin(\omega t + \varphi)\,\underbrace{\frac{d}{dt}(\omega t + \varphi)}_{\omega}\right) = -A\omega^2 \operatorname{sen}(\omega t + \varphi)$$

$$x(t) = A \operatorname{sen}(\omega t + \varphi)$$
$$v(t) = A\omega \cos(\omega t + \varphi)$$
$$a(t) = -A\omega^2 \operatorname{sen}(\omega t + \varphi)$$

$\varphi = 0$

$$x(t) = A \operatorname{sen}(\omega t)$$
$$v(t) = A\omega \cos(\omega t)$$
$$a(t) = -A\omega^2 \operatorname{sen}(\omega t)$$



$T = \frac{2\pi}{\omega}$

$$\boxed{a(t) = -\omega^2 x(t)}$$