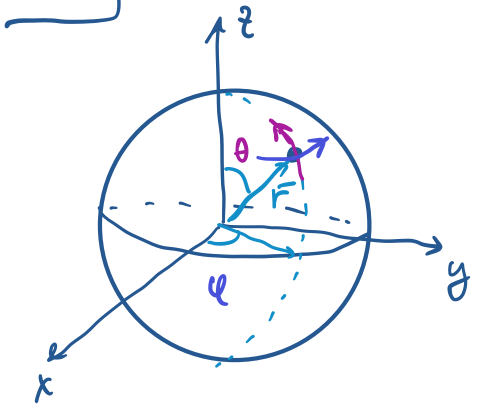
ES. PENDOLO SFERICO (pto materiale vincolato a una sfera (R) soggetto alla forza di gravità)



$$x = R \operatorname{sen}\theta \cos\varphi$$
$$y = R \operatorname{sen}\theta \operatorname{sen}\varphi \quad \leftarrow \quad \bar r(\varphi, \theta)$$
$$z = R\cos\theta$$

$$\frac{\partial \bar r}{\partial \theta}, \quad \frac{\partial \bar r}{\partial \varphi}$$

En. cinetica

$$\dot x = -R \operatorname{sen}\varphi \operatorname{sen}\theta\, \dot\varphi + R\cos\varphi\cos\theta\,\dot\theta$$
$$\dot y = R\cos\varphi \operatorname{sen}\theta\,\dot\varphi + R\operatorname{sen}\varphi\cos\theta\,\dot\theta \quad \leftarrow \quad \bar v(\bar q, \dot{\bar q})$$
$$\dot z = -R\operatorname{sen}\theta\,\dot\theta$$

$$T = \frac{1}{2} m (\dot x^2 + \dot y^2 + \dot z^2) =$$

$$= \frac{1}{2} m \Big[\big(R^2 \operatorname{sen}^2\varphi \operatorname{sen}^2\theta\, \dot\varphi^2 + R^2\cos^2\varphi\cos^2\theta\,\dot\theta^2 - 2R^2 \cancel{\cos\varphi \operatorname{sen}\varphi} \overset{\cos\theta}{} \operatorname{sen}\theta\,\dot\varphi\dot\theta \big)$$
$$+ \big(R^2\cos^2\varphi \operatorname{sen}^2\theta\,\dot\varphi^2 + R^2\operatorname{sen}^2\varphi\cos^2\theta\,\dot\theta^2 + 2R^2 \cancel{\cos\varphi \operatorname{sen}\varphi} \cos\theta\operatorname{sen}\theta\,\dot\varphi\dot\theta \big)$$
$$+ R^2\operatorname{sen}^2\theta\,\dot\theta^2 \Big] =$$

$$= \frac{1}{2} m \left(R^2 \operatorname{sen}^2\theta\,\dot\varphi^2 + R^2\cos^2\theta\,\dot\theta^2 + R^2\operatorname{sen}^2\theta\,\dot\theta^2 \right)$$

$$= \frac{1}{2} m \left(R^2 \operatorname{sen}^2\theta\,\dot\varphi^2 + R^2\dot\theta^2 \right) \leftarrow a = \begin{pmatrix} mR^2\operatorname{sen}^2\theta & 0 \\ 0 & mR^2 \end{pmatrix}$$

dip. da q

$$= \frac{1}{2}\sum_{h,k} a_{hk}\,\dot q_h \dot q_k$$

$$V = mgz = mgR\cos\theta$$

$$L = \frac{mR^2}{2}\left(\operatorname{sen}^2\theta\,\dot\varphi^2 + \dot\theta^2\right) - mgR\cos\theta$$

n=2 gradi di lib. (ci aspettiamo 2 eq. di Lag.)

$$L(\varphi, \theta, \dot\varphi, \dot\theta)$$

Eq. di Lagrange

$$\frac{\partial L}{\partial \dot\varphi} = mR^2 \sin^2\theta\, \dot\varphi \quad \rightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = 2mR^2 \sin\overset{\theta(t)}{\theta} \cos\theta\, \dot\theta\, \overset{\dot\varphi(t)}{\dot\varphi} + mR^2 \sin^2\theta\, \ddot\varphi$$

$$\frac{\partial L}{\partial \dot\theta} = mR^2\dot\theta \quad \rightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = mR^2\ddot\theta$$

$$\frac{\partial L}{\partial \varphi} = 0$$

$$\frac{\partial L}{\partial \theta} = mR^2 \sin\theta\cos\theta\, \dot\varphi^2 + mgR\sin\theta$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} - \frac{\partial L}{\partial \varphi} = 2mR^2\sin\theta\cos\theta\,\dot\theta\dot\varphi + mR^2\sin^2\theta\,\ddot\varphi$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot\theta} - \frac{\partial L}{\partial \theta} = mR^2\ddot\theta - mR^2\sin\theta\cos\theta\,\dot\varphi^2 - mgR\sin\theta$$

$$\hookrightarrow \quad \ddot\theta = \sin\theta\cos\theta\,\dot\varphi^2 + \frac{g}{R}\sin\theta$$

INVARIANZA PER CAMBIAMENTO DI COORDINATE

Sistema vincolato $\bar{r}_i(\bar{q}, t) \rightsquigarrow \bar{v}_i(\bar{q}, \dot{\bar{q}}, t)$

abbiamo fatto una SCELTA di COORDINATE in Q

"Mettiamo qte funzioni" dentro $T(\bar{v}_1, \bar{v}_2, \dots, \bar{v}_N)$ e $V(\bar{r}_1, \dots, \bar{r}_N)$

$\rightsquigarrow L(\bar{q}, \dot{\bar{q}}, t) \rightsquigarrow$ eq. del moto (eq. di Lagrange)

- La scelta di coordinate per Q è ARBITRARIA

posso scegliere $\{q_h\}$ o $\{\tilde{q}_h\}$ (diversi sistemi di coord)

$$
\begin{tikzcd}
\{q_h\} \arrow[d, "\bar{r}_i = \bar{r}_i(q_h, t)"] & & \{\tilde{q}_h\} \arrow[ll, leftrightarrow, "\text{TASFORMAZ. di COORD.}" above, "q_h = q_h(\tilde{\bar{q}}, t)" below] \arrow[d, "\bar{r}_i = \bar{r}_i(\tilde{q}_h, t)"] \\
L(\bar{q}, \dot{\bar{q}}, t) \arrow[d, squiggly] & & \tilde{L}(\tilde{\bar{q}}, \dot{\tilde{\bar{q}}}, t) \arrow[d, squiggly] \\
\text{ep. del moto (eq. diff. nelle } q_h(t)) \arrow[d, squiggly] & & \text{eq. del moto (eq. diff. nelle } \tilde{q}_h(t)) \arrow[d, squiggly] \\
\text{Solut. } q_h(t) \arrow[d] & & \text{Solut. } \tilde{q}_h(t) \arrow[d] \\
\bar{r}_i(t) \equiv \bar{r}_i(\bar{q}(t), t) & = & \bar{r}_i(t) \equiv \bar{r}_i(\tilde{\bar{q}}(t), t)
\end{tikzcd}
$$

$\leftarrow$ Espressione analitica della Lagrangiana è diversa nei due sist. di coord

$$\Rightarrow \quad \bar{q}(t) = \bar{q}(\tilde{\bar{q}}(t), t)$$

Es) Pto materiale vincolato su un piano e legato all'origine con una molla di lungh. a riposo nulla e $k = m\omega^2$.

$$\bar{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Coord. cartesiane

$$x = q_1 \qquad y = q_2 \qquad z = 0 \qquad \bar{r}_i(\bar{q})$$

Coord. polari

$$x = \tilde{q}_1 \cos \tilde{q}_2 \qquad y = \tilde{q}_1 \,\mathrm{sen}\, \tilde{q}_2 \qquad z = 0 \qquad \bar{r}_i(\bar{\tilde{q}})$$

Trasf. di coord:

$$q_1 = \tilde{q}_1 \cos \tilde{q}_2 \qquad q_2 = \tilde{q}_1 \,\mathrm{sen}\, \tilde{q}_2 \qquad \leftarrow \quad \bar{q} = \bar{q}(\bar{\tilde{q}})$$

$$T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \qquad V = \frac{1}{2} m\omega^2 (x^2 + y^2) \qquad \leftarrow T(\bar{v}), V(\bar{r})$$

$$L(\bar{q}, \dot{\bar{q}}, t) = \frac{m}{2}(\dot{q}_1^2 + \dot{q}_2^2) - \frac{m\omega^2}{2}(q_1^2 + q_2^2) \qquad \tilde{L}(\bar{\tilde{q}}, \dot{\bar{\tilde{q}}}, t) = \frac{m}{2}(\dot{\tilde{q}}_1^2 + \tilde{q}_1^2 \dot{\tilde{q}}_2^2) - \frac{m\omega^2}{2} \tilde{q}_1^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} \qquad \frac{\partial L}{\partial q_1}$$

$$\begin{cases} m \ddot{q}_1 = -m\omega^2 q_1 \\ m \ddot{q}_2 = -m\omega^2 q_2 \end{cases}$$

Soluz.

$q_1(t) \quad q_2(t)$

$$\frac{d}{dt}\frac{\partial \tilde{L}}{\partial \dot{\tilde{q}}_1} = m \ddot{\tilde{q}}_1 \qquad \frac{\partial L}{\partial \tilde{q}_1} = m \tilde{q}_1 \dot{\tilde{q}}_2^2 - m\omega^2 \tilde{q}_1$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\tilde{q}}_2} = \frac{d}{dt}\left(m \tilde{q}_1^2 \dot{\tilde{q}}_2\right) = 2m \tilde{q}_1 \dot{\tilde{q}}_1 \dot{\tilde{q}}_2 + m \tilde{q}_1^2 \ddot{\tilde{q}}_2$$

$$\frac{\partial L}{\partial \tilde{q}_2} = 0$$

$$\begin{cases} m \ddot{\tilde{q}}_1 = m \tilde{q}_1 \dot{\tilde{q}}_2^2 - m\omega^2 \tilde{q}_1 \\ m \tilde{q}_1^2 \ddot{\tilde{q}}_2 + 2m \tilde{q}_1 \dot{\tilde{q}}_1 \dot{\tilde{q}}_2 = 0 \end{cases}$$

Soluz.

$\tilde{q}_1(t) \quad \tilde{q}_2(t)$

$$q_1(t) = \tilde{q}_1(t) \cos \tilde{q}_2(t)$$
$$q_2(t) = \tilde{q}_1(t) \,\mathrm{sen}\, \tilde{q}_2(t)$$

Trasf. di coord.

$$q_h = q_h(\tilde q_1, \dots, \tilde q_m, t) \qquad \text{t.c.} \qquad \det\left(\frac{\partial q_h}{\partial \tilde q_k}\right) \neq 0 \qquad (\star)$$

$$\dot q_h = \dot q_h(\bar{\tilde q}, \dot{\bar{\tilde q}}, t) = \sum_{k=1}^{m} \frac{\partial q_h}{\partial \tilde q_k}\dot{\tilde q}_k + \frac{\partial q_h}{\partial t}$$

<u>Prop.</u> Dato un sist. Lagrangiano con Lagrangiana $L(\bar q, \dot{\bar q}, t)$, si consideri il camb. di coord. (regolare e invertibile) $(\star)$ e sia $\tilde L(\bar{\tilde q}, \dot{\bar{\tilde q}}, t)$ la Lagrangiana ottenuta da L per SOSTITUZIONE di $(\star)$, cioè

$$\tilde L(\bar{\tilde q}, \dot{\bar{\tilde q}}, t) \equiv L\left(\bar q(\bar{\tilde q}, t), \dot{\bar q}(\bar{\tilde q}, \dot{\bar{\tilde q}}, t), t\right) \qquad (\bullet)$$

Allora $\bar{\tilde q}(t)$ è soluzione delle eq. di Lagrange con Lagr. $\tilde L$ SE E SOLO SE $\bar q(t)$ è solut. delle eq. di Lagr con Lag L.

<u>Dim.</u>

$$\frac{\partial \tilde L}{\partial \dot{\tilde q}_h} \overset{(\bullet)}{=} \sum_{l=1}^{m} \frac{\partial L}{\partial \dot q_l}\frac{\partial \dot q_l}{\partial \dot{\tilde q}_h} \overset{(\star)}{=} \sum_{l=1}^{m} \frac{\partial L}{\partial \dot q_l} \cancel{\sum_{k=1}^{m}} \frac{\partial q_l}{\partial \tilde q_k} \underbrace{\frac{\partial \dot{\tilde q}_k}{\partial \dot{\tilde q}_h}}_{\delta_{kh}}$$

$$= \sum_{l=1}^{m} \frac{\partial L}{\partial \dot q_l}\frac{\partial q_l}{\partial \tilde q_h}$$

$$\frac{d}{dt}\frac{\partial \tilde L}{\partial \dot{\tilde q}_h} = \sum_{l=1}^{m}\left(\frac{d}{dt}\frac{\partial L}{\partial \dot q_l}\right)\frac{\partial q_l}{\partial \tilde q_h} + \sum_{l=1}^{m}\frac{\partial L}{\partial \dot q_l}\frac{d}{dt}\frac{\partial q_l}{\partial \tilde q_h}(\bar{\tilde q}(t), t)$$

$$= \sum_{m=1}^{m}\frac{\partial^2 q_l}{\partial \tilde q_h \partial \tilde q_m}\dot{\tilde q}_m + \frac{\partial^2 q_l}{\partial \tilde q_h \partial t} = \quad f(\bar{\tilde q}, \dot{\bar{\tilde q}}, t) \text{ valutata in } \bar{\tilde q}(t)$$

$$= \frac{\partial}{\partial \tilde q_h}\left(\sum_{m=1}^{m}\frac{\partial q_l}{\partial \tilde q_m}\dot{\tilde q}_m + \frac{\partial q_l}{\partial t}\right) = \frac{\partial}{\partial \tilde q_h}\dot q_l$$

$$= \sum_{l=1}^{m}\left[\left(\frac{d}{dt}\frac{\partial L}{\partial \dot q_l}\right)\frac{\partial q_l}{\partial \tilde q_h} + \frac{\partial L}{\partial \dot q_l}\frac{\partial \dot q_l}{\partial \tilde q_h}\right]$$

$$\frac{\partial \tilde{L}}{\partial \tilde{q}_h} = \sum_{l=1}^{m} \left[\frac{\partial L}{\partial q_l} \frac{\partial q_l}{\partial \tilde{q}_h} + \frac{\partial L}{\partial \dot{q}_l} \frac{\partial \dot{q}_l}{\partial \tilde{q}_h} \right]$$

$$\Rightarrow \underbrace{\frac{d}{dt}\frac{\partial \tilde{L}}{\partial \dot{\tilde{q}}_h} - \frac{\partial \tilde{L}}{\partial \tilde{q}_h}}_{\text{eq. di Lagr. di } \tilde{L} \ (\tilde{*})} = \overbrace{\sum_{l=1}^{m} \underbrace{\left(\frac{\partial q_l}{\partial \tilde{q}_h}\right)}_{\equiv J_{hl} \text{ invertibile}} \Bigg[\underbrace{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_l} - \frac{\partial L}{\partial q_l}}_{\text{eq. di Lagr. di } L \ (*)} \Bigg]}^{\sum_l J_{hl} V_l} \qquad h=1,\dots,m$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_l} - \frac{\partial L}{\partial q_l} \equiv V_l$$

Se $(*)$ sono sodd. $\forall l \Longrightarrow (\tilde{*})$ sono soddisf. $\forall h$ (manifesto)

Se $(\tilde{*})$ sono sodd. $\forall h \Rightarrow \sum_{l=1}^{m} J_{hl} V_l = 0 \iff J \cdot \bar{V} = 0$

$\underset{J \text{ invertibile}}{\Longrightarrow}$ $(*)$ sono sodd. $\forall l$

(moltiplichiamo a destra e sinistra per J^{-1})

//

Diverse Lagrangiane possono portare alle stesse eq. del moto.

<u>Prop.</u> Per ogni scelta della funzione $F(\bar{q},t)$ e della cost. $c \neq 0$, la Lagrangiana $L(\bar{q},\dot{\bar{q}},t)$ e la Lagrangiana

$$L'(\bar{q},\dot{\bar{q}},t) = c\,L(\bar{q},\dot{\bar{q}},t) + \varphi(\bar{q},\dot{\bar{q}},t),$$

dove $\varphi(q,\dot{\bar{q}},t) = \sum_{k=1}^{m} \frac{\partial F}{\partial q_k}\dot{q}_k + \frac{\partial F}{\partial t}$,

"derivata totale" $\bar{\varphi}(\bar{q}(t),\dot{\bar{q}}(t),t) = \frac{d}{dt}F(\bar{q}(t),t)$

conducono alle stesse eq. di Lagrange.

"Lagrangiane che differiscono per una DERIVATA TOTALE (in t) sono (classicam.) EQUIVALENTI"

Dim. $\displaystyle \frac{d}{dt}\frac{\partial L'}{\partial \dot q_h} = c\,\frac{d}{dt}\frac{\partial L}{\partial \dot q_h} + \underbrace{\frac{d}{dt}\frac{\partial \varphi}{\partial \dot q_h}}$

$$= \frac{d}{dt}\frac{\partial F}{\partial q_h} = \sum_{\ell=1}^{n} \frac{\partial^2 F}{\partial q_h \partial q_\ell}\dot q_\ell + \cancel{\frac{\partial^2 F}{\partial q_h \partial t}}$$

$$\frac{\partial L'}{\partial q_h} = c\,\frac{\partial L}{\partial q_h} + \underbrace{\frac{\partial \varphi}{\partial q_h}}$$

$$= \sum_{k=1}^{n} \frac{\partial^2 F}{\partial q_h \partial q_k}\dot q_k + \cancel{\frac{\partial^2 F}{\partial q_h \partial t}}$$

$$\Rightarrow \frac{d}{dt}\frac{\partial L'}{\partial \dot q_h} - \frac{\partial L'}{\partial q_h} = c\left(\frac{d}{dt}\frac{\partial L}{\partial \dot q_h} - \frac{\partial L}{\partial q_h}\right) \qquad //$$

$c \neq 0$

Potenziali dipendenti da velocità

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q_h} - \frac{\partial T}{\partial q_h} = Q_h \longrightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot q_h} - \frac{\partial L}{\partial q_h} = 0 \qquad (\neq)$$

se $Q_h = -\frac{\partial V}{\partial q_h}$ $\qquad L = T - V$

Ci si può ridurre alla forma $(\neq)$ anche nel caso più generale in cui (*) vedi alla fine per definiz.

$$Q_h = \frac{d}{dt}\frac{\partial V}{\partial \dot q_h} - \frac{\partial V}{\partial q_h}$$

con $V = V(\bar q, \dot{\bar q}, t)$

Assumiamo che V dip. LINEARM. da $\dot{\bar q}$

ES | FORZA DI CORIOLIS

$$\bar F = 2m\, \dot{\bar q} \times \bar\omega$$

(Forza apparente in un sist. di rif. rotante con $\bar\omega$ che poi prendiamo cost.)

(Forza centrifuga è puram. posizionale da

$$V_c(\bar q) = -\frac{1}{2} m \omega^2 d^2(\bar q)$$

distanza da asse di rotaz.)

Prodotto vett. in $\mathbb{R}^3$

$$\bar v \times \bar u = \bar e_1 (v_2 u_3 - v_3 u_2) + \bar e_2 (v_3 u_1 - v_1 u_3) + \bar e_3 (v_1 u_2 - v_2 u_1) =$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \rightsquigarrow \det \begin{pmatrix} \bar e_1 & v_1 & u_1 \\ \bar e_2 & v_2 & u_2 \\ \bar e_3 & v_3 & u_3 \end{pmatrix}$$

$$= \sum_{ijk} \epsilon_{ijk}\, \bar e_i\, v_j\, u_k$$

tensore totalm. antisimm.

$(\Rightarrow \epsilon_{iij} = 0) \qquad \epsilon_{123} = 1$

$$\left(\epsilon_{123} = 1 \rightarrow \epsilon_{213} = -1 \rightarrow \epsilon_{231} = 1 \rightarrow \rightarrow \epsilon_{321} = -1 \right)$$

Prop. La forza di Coriolis con $\bar\omega$ cost. si ricava attraverso

$$F_h = \frac{d}{dt}\frac{\partial V_1}{\partial \dot q_h} - \frac{\partial V_1}{\partial q_h} \quad \text{con} \quad V_1(\bar q, \dot{\bar q}) = -m(\dot{\bar q}\times\bar\omega)\cdot\bar q$$

Dim.

$$(\bar a\times\bar b)\cdot\bar c = \Big(\sum_{ijk}\overset{\epsilon_{ijk}}{\bar e_i}\, a_j b_k\Big)\cdot\Big(\sum_l c_l \bar e_l\Big) =$$

$$= \sum_i\Big(\sum_{jk} a_j \overset{\epsilon_{ijk}}{b_k}\Big) c_i =$$

$$= \sum_{ijk}\epsilon_{ijk}\, a_j b_k c_i = \sum_{ijk}\epsilon_{kij}\, b_k c_i a_j =$$

$$= \sum_j\Big(\sum_{ki}\epsilon_{jki}\, b_k c_i\Big) a_j = (\bar b\times\bar c)\cdot\bar a =$$

$$= \bar a\cdot(\bar b\times\bar c)$$

$$V_1 = -m(\dot{\bar q}\times\bar\omega)\cdot\bar q = -m\,\dot{\bar q}\cdot(\bar\omega\times\bar q)$$

$$\frac{\partial V_1}{\partial q_h} = -m(\dot{\bar q}\times\bar\omega)_h$$

$$\frac{\partial V_1}{\partial \dot q_h} = -m(\bar\omega\times\bar q)_h \qquad \frac{d}{dt}\frac{\partial V_1}{\partial \dot q_h} = -m(\bar\omega\times\dot{\bar q})_h = m(\dot{\bar q}\times\bar\omega)_h$$

$$F_h = \frac{d}{dt}\frac{\partial V_1}{\partial \dot q_h} - \frac{\partial V_1}{\partial q_h} = 2m(\dot{\bar q}\times\bar\omega)_h \quad //$$

$L = T - V_c - V_1$

forza centrifuga

forza Coriolis

en cinetica in coord $\bar{q}$ solidali al sistema di rif. rotante

In un sist. di rif. inerziale con coord x, y, z

$$L = T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Trasf. di coord.:

$$x = q_1 \cos\omega t - q_2 \operatorname{sen}\omega t \qquad \leftarrow x = x(\bar{q}, t)$$
$$y = q_1 \operatorname{sen}\omega t + q_2 \cos\omega t \qquad \vdots$$
$$z = q_3$$

$$\tilde{L}(\bar{q}, \dot{\bar{q}}, t) = L(x(\bar{q},t), y(\bar{q},t), z(\bar{q},t), \dot{x}(\bar{q},\dot{\bar{q}},t), \dot{y}(\bar{q},\dot{\bar{q}},t), \dot{z}(\bar{q},\dot{\bar{q}},t), t)$$

$$\dot{x} = \dot{q}_1 \cos\omega t - \dot{q}_2 \operatorname{sen}\omega t - q_1 \omega \operatorname{sen}\omega t - q_2 \omega \cos\omega t$$
$$\dot{y} = \dot{q}_1 \operatorname{sen}\omega t + \dot{q}_2 \cos\omega t + q_1 \omega \cos\omega t - q_2 \omega \operatorname{sen}\omega t$$
$$\dot{z} = \dot{q}_3$$

$$= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) =$$

$T(\dot{q})$ en. cin. nel sist. rotante

$-V_c(\bar{q})$

$$= \frac{1}{2} m (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) + \frac{m\omega^2}{2} (\underbrace{q_1^2 + q_2^2}) +$$

↳ dist. dall'asse z

$$+ m\omega \underbrace{(q_1 \dot{q}_2 - q_2 \cdot \dot{q}_1)}_{(\bar{q} \times \dot{\bar{q}})_3} \qquad \bar{\omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$$

$$m\bar{\omega} \cdot (\bar{q} \times \dot{\bar{q}}) = -m\bar{\omega} \cdot (\dot{\bar{q}} \times \bar{q}) =$$
$$= -m(\bar{\omega} \times \dot{\bar{q}}) \cdot \bar{q} = -V_1(\bar{q}, \dot{\bar{q}})$$

(*) Per $\frac{d}{dt}\frac{\partial V_1}{\partial \dot{q}_h}$, dato $V_1 = V_1(\bar{q}, \dot{\bar{q}}, t)$, si intende

$$\frac{d}{dt}\frac{\partial V_1}{\partial \dot{q}_h} \equiv \sum_{\ell=1}^{n} \frac{\partial^2 V_1}{\partial q_\ell \partial \dot{q}_h} \dot{q}_\ell + \frac{\partial^2 V_1}{\partial t \partial \dot{q}_h}$$