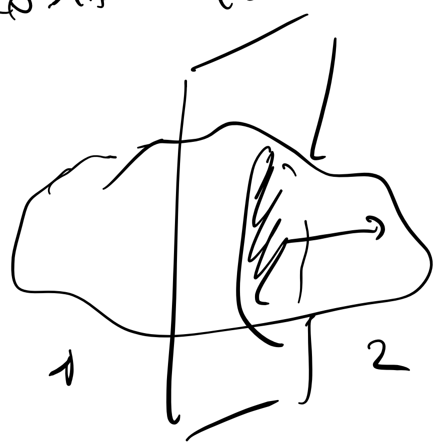
MECCANICA RAZIONALE

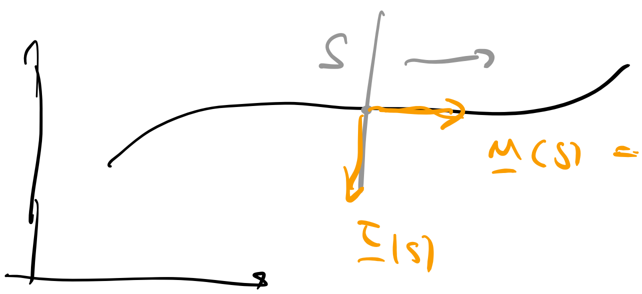
Sforzi interni ad un rigido



Equilibrio → E&S

Caso piano

Archi



$\underline{m}(s) = \frac{d\underline{x}}{ds}(s)$

$\underline{m} \wedge \underline{e}_3 = \underline{\tau}$

$$\rightarrow \left(N(s), \; T(s), \; M_f(s) \right)$$

$$\underline{m} \qquad \underline{\tau} \qquad \underline{e}_3$$

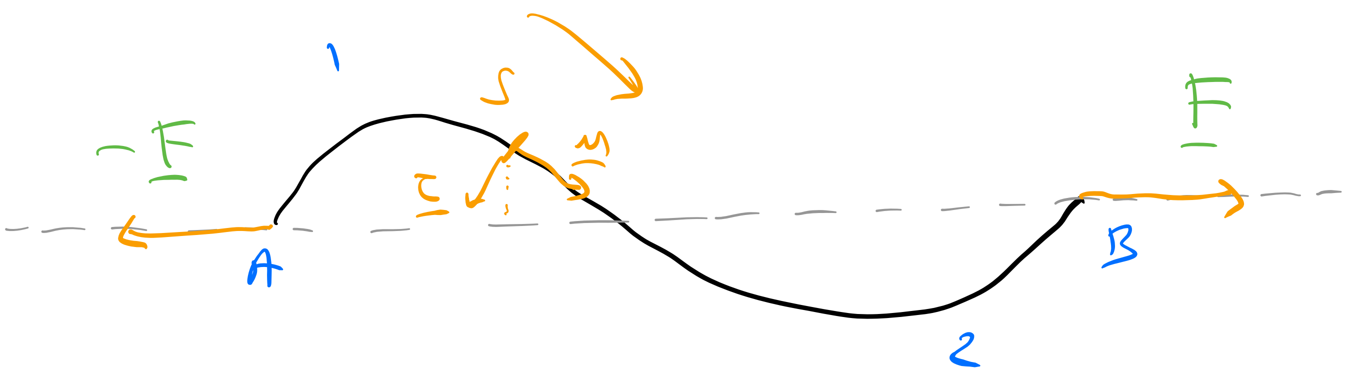
funzioni incognite

$$\begin{cases} \dfrac{d}{ds}\left(N(s)\,\underline{m}(s) + T(s)\,\underline{\tau}(s) \right) = -\underline{f}(s) \\[2ex] \dfrac{d}{ds}\underline{M}_f(s) = \underline{T}(s) \wedge \underline{m}(s) \end{cases}$$

carichi concentrati:

- $N(s)$, $T(s)$ possono essere discontinui
- $\underline{M}_f(s)$ è continuo, può non essere derivabile

Archi scarichi: un arco si dice scarico quando è soggetto solo a due forze applicate alle estremità dell'arco, di braccio nullo.



Equilibrio di 1

$$-\underline{F} + N\underline{n} + T\underline{t} = \underline{0}$$

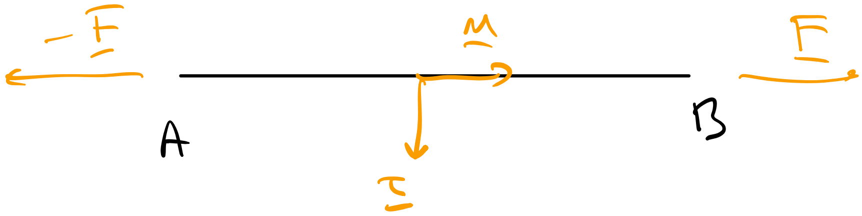
$$(\underline{x}_A - \underline{x}_s) \wedge (-\underline{F}) + \underline{M}_f = \underline{0}$$

$$\Rightarrow \boxed{\underbrace{(\underline{x}_A - \underline{x}_S) \wedge (-\underline{F}) \cdot \underline{e}_3}\ + \underline{M}_f \cdot \underline{e}_3 = 0}$$

N e T funzioni non costanti.

$$|M_f| = \|\underline{F}\| \cdot \left(\begin{array}{c} \text{distanza di } S \\ \text{dalla retta } AB \end{array} \right)$$

Segmenti:

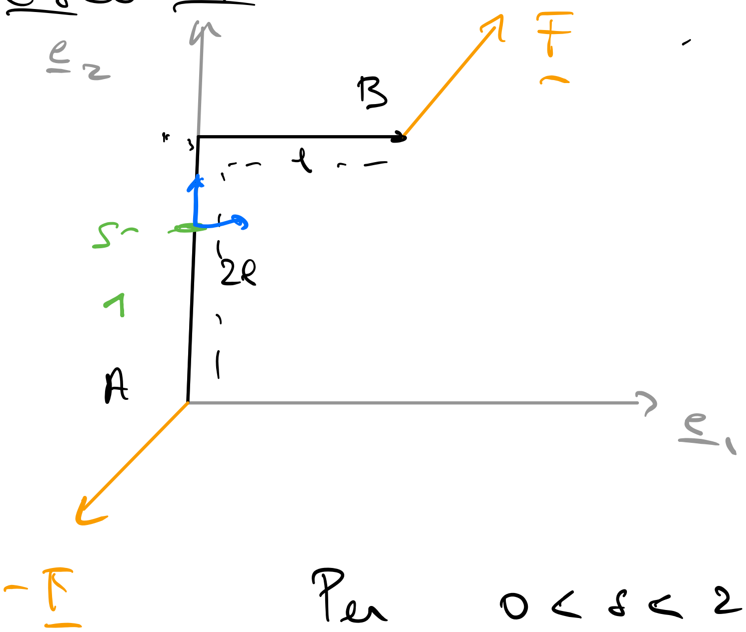


$T = 0$

$M_f = 0$

$$N = \begin{cases} \|F\| > 0 & \text{trazione} \\ \quad \text{oppure} & \\ -\|F\| < 0 & \text{compressione} \end{cases}$$

Esercizio



$$\underline{F} = F \operatorname{vers}(\underline{x}_B - \underline{x}_A)$$

$F > 0$

Calcolare gli sforzi interni misurati a partire da A

Per $0 < s < 2\ell$

$\underline{M}$ e $\underline{\tau}$ $\qquad \underline{M} = \underline{e}_2 \qquad \underline{\tau} = \underline{e}_1$

Allora : $-\underline{F} + N\,\underline{e}_2 + T\,\underline{e}_1 = \underline{0}$

$$\underline{F} = F \,\text{vers}\,(\underline{x}_B - \underline{x}_A) = F \,\frac{(\underline{e}_1 + 2\,\underline{e}_2)}{\sqrt{5}}$$

$\underline{x}_A = \underline{0}$

$\underline{x}_B = \ell\,\underline{e}_1 + 2\ell\,\underline{e}_2 \qquad \dfrac{\cancel{\ell}\,\underline{e}_1 + 2\cancel{\ell}\,\underline{e}_2}{\sqrt{\cancel{\ell^2} + 4\cancel{\ell^2}}}$

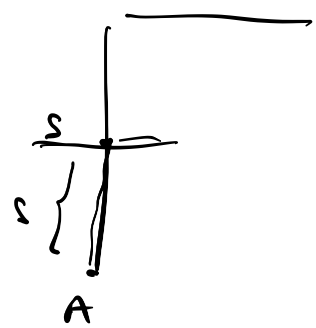
$$-\frac{F}{\sqrt{5}}(\underline{e}_1 + 2\underline{e}_2) + N\,\underline{e}_2 + T\,\underline{e}_1 = \underline{0}$$

$$\Rightarrow \quad N = \frac{2}{\sqrt{5}}F \;, \quad T = \frac{1}{\sqrt{5}}F$$

$$M_{f(s)} + (\underline{x}_A - \underline{x}_s) \wedge (-\underline{F}) \cdot \underline{e}_3 = 0$$

$\underline{x}_A = \underline{0}$

$\underline{x}_s = s\,\underline{e}_2$



$$M_f + \left(s\,\underline{e}_2 \wedge \frac{F}{\sqrt{5}}(\underline{e}_1 + 2\underline{e}_2)\right) \cdot \underline{e}_3 = 0$$

$$M_f + s\frac{F}{\sqrt{5}}\underbrace{(\underline{e}_2 \wedge \underline{e}_1)}_{-\underline{e}_3} \cdot \underline{e}_3 = 0$$

$\rightarrow \quad M_f = \dfrac{F}{\sqrt{5}}\, s$

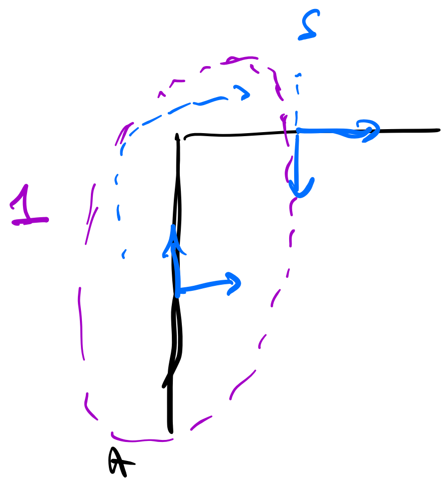
$$\rightarrow \begin{cases} N = \dfrac{2}{\sqrt{5}} F \\ T = \dfrac{1}{\sqrt{5}} F \\ M_f = \dfrac{F}{\sqrt{5}}\, s \end{cases}$$

$$\boxed{\frac{d}{ds} M_{f(s)} = T_{(s)}}$$

$\rightarrow \dfrac{d}{ds} M_f = \dfrac{F}{\sqrt{5}} = T$

Per $2\ell < s < 3\ell$

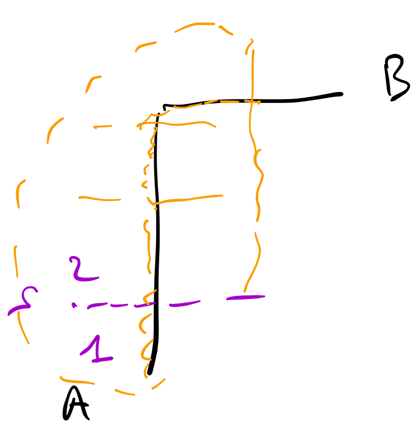
s va da 0
alla lunghezza
totale 3ℓ

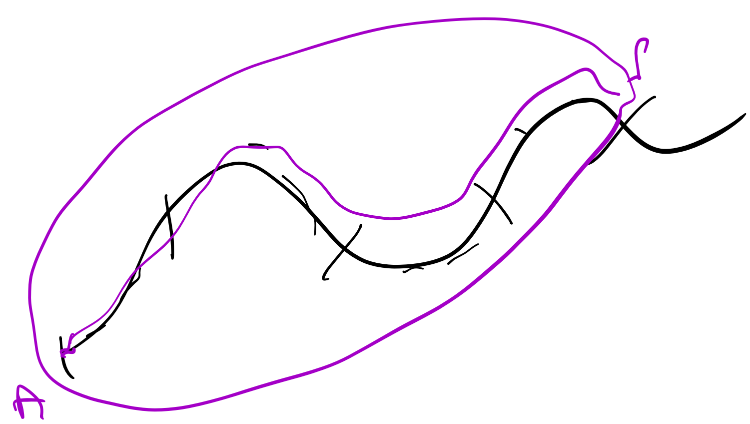


$\underline{n} = \underline{e}_1$

$\underline{\tau} = \underline{e}_1 \wedge \underline{e}_3 = -\underline{e}_2$

$-\underline{F} + N\,\underline{n} + T\,\underline{\tau} = \underline{0}$





$$-\underline{F} + N\underline{n} + T\underline{\tau} = \underline{0}$$

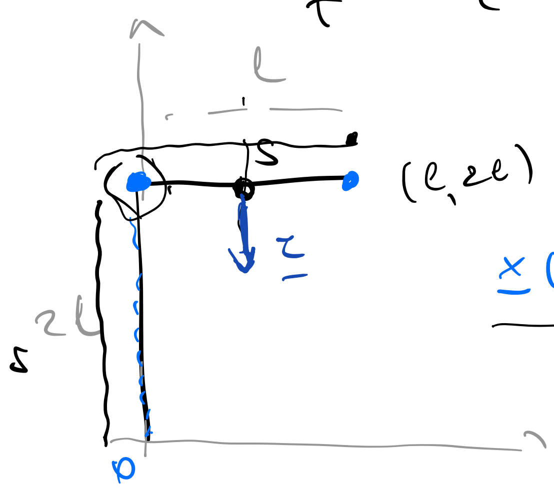
$$\rightarrow \quad -\underline{F} + N\underline{e}_1 - T\underline{e}_2 = \underline{0}$$

$$-\frac{F}{\sqrt{5}}(\underline{e}_1 + 2\underline{e}_2) + N\underline{e}_1 - T\underline{e}_2 = \underline{0}$$

$$N = \frac{F}{\sqrt{5}}, \quad \boxed{T = -\frac{2}{\sqrt{5}}F}$$

Per i momenti:

$$M_f + \left[(\underline{x}_A - \underline{x}_s) \wedge (-\underline{F})\right]\underline{e}_3 = 0$$



$$\underline{x}(s) = (s - 2\ell)\underline{e}_1 + 2\ell\,\underline{e}_2$$

s misura la lunghezza
va da 0 a 3ℓ

s è tale che $\underline{x}(2\ell) = 2\ell\,\underline{e}_2$

$$\underline{x}(3\ell) = \ell\,\underline{e}_1 + 2\ell\,\underline{e}_2$$

$\underline{x}_B$

$$M_f + \left[(1-2\ell)\,\underline{e}_1 + 2\ell\,\underline{e}_2\right] \wedge \frac{F}{\sqrt{5}}\left(\underline{e}_1 + 2\,\underline{e}_2\right)\cdot \underline{e}_3 = 0$$

$$M_f + \left((1-2\ell)\,\underline{e}_1 \wedge \frac{F}{\sqrt{5}}\,2\,\underline{e}_2\right)\cdot \underline{e}_3 +$$

$$+\left(2\ell\,\underline{e}_2 \wedge \frac{F}{\sqrt{5}}\,\underline{e}_1\right)\cdot \underline{e}_3 =$$

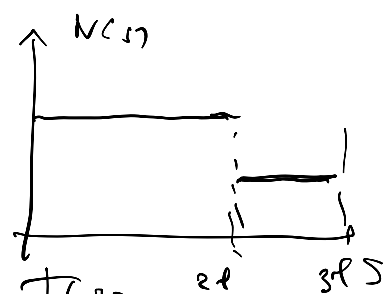
$$M_f + \left[2(1-2\ell)\frac{F}{\sqrt{5}} - 2\ell\frac{F}{\sqrt{5}}\right] = 0$$

$$M_f = -\frac{2}{\sqrt{5}}F s + \frac{6}{\sqrt{5}}F\ell$$

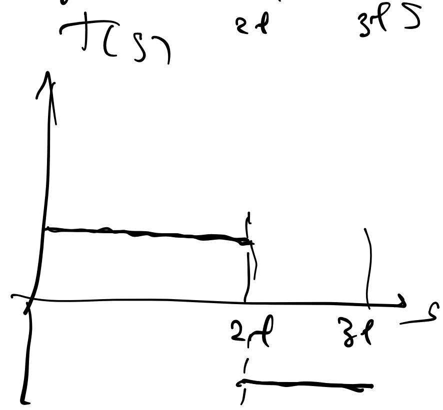
$$\frac{dM_f}{ds} \overset{?}{=} T = -\frac{2}{\sqrt{5}}F$$

Riassumendo

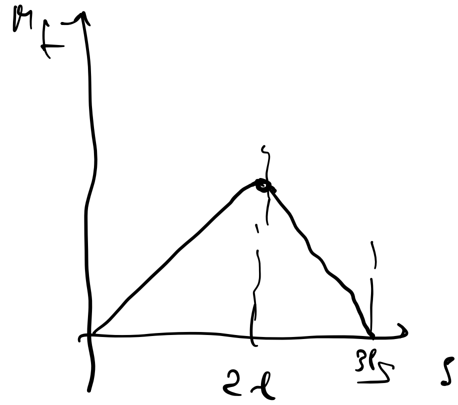
$$N(s) = \begin{cases} \dfrac{2}{\sqrt{5}}F & 0 < s < 2\ell \\[2ex] \dfrac{1}{\sqrt{5}}F & 2\ell < s < 3\ell \end{cases}$$



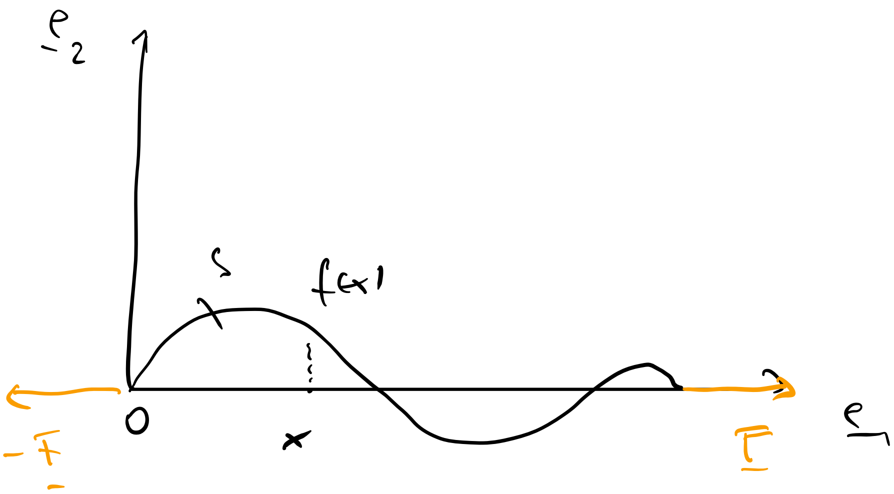
$$T(s) = \begin{cases} \dfrac{1}{\sqrt{5}}F & 0 < s < 2\ell \\[2ex] -\dfrac{2}{\sqrt{5}}F & 2\ell < s < 3\ell \end{cases}$$



$$M_f(s) = \begin{cases} \dfrac{F}{\sqrt{5}}\, s & 0 < s < 2\ell \\[2ex] \dfrac{2}{\sqrt{5}}\, F(3\ell - s) & 2\ell < s < 3\ell \end{cases}$$



Esercizio



$\underline{F} = F\,\underline{e}_1$

$\{x, y \;:\; x \in [0, L]$

$y = f(x)\}$

$x\,\underline{e}_1 + f(x)\,\underline{e}_2$

Vogliamo $\underline{n}$ e $\underline{\tau}$.

$$\underline{n} = \frac{\underline{e}_1 + f'\,\underline{e}_2}{\sqrt{1 + (f')^2}}$$

$$\underline{\tau} = \underline{n} \wedge \underline{e}_3 = \frac{\underline{e}_1 + f'\,\underline{e}_2}{\sqrt{1 + (f')^2}} \wedge \underline{e}_3$$

$$= \frac{f'\,\underline{e}_1 - \underline{e}_2}{\sqrt{1 + (f')^2}}$$

$\underline{\mu} \cdot \underline{\tau} = 0$

$$\begin{cases} -\underline{F} + N\,\underline{\mu} + T\,\underline{\tau} = \underline{0} \\ M_f(x) + (\underline{x}_0 - \underline{x}_s) \wedge (-\underline{F}) \cdot \underline{e}_3 = 0 \end{cases}$$

$$\begin{cases} N(x) = \underline{F} \cdot \underline{\mu} = \dfrac{F}{\sqrt{1 + (f')^2}} \\ T(x) = \underline{F} \cdot \underline{\tau} = \dfrac{F f'}{\sqrt{1 + (f')^2}} \end{cases}$$

$$M_f(x) = (\underline{x}_0 - \underline{x}_s) \wedge \underline{F} \cdot \underline{e}_3$$

$$= -(x\,\underline{e}_1 + f(x)\,\underline{e}_2) \wedge F\,\underline{e}_1 \cdot \underline{e}_3$$

$$= f(x)\,F$$

Notiamo: x non è la lunghezza dell'arco s. $\dfrac{dM_f}{dx} \neq T(x)$

$$\frac{d}{ds} M_f(x(s)) = \frac{dM_f}{dx}\,\frac{dx}{ds} = T(x(s))$$

$$s(x) = \int_0^{s} \dots$$

$$\underline{v}(\tau) = \frac{d}{d\tau}\underline{z}(\tau) = \frac{d}{d\tau}\left(\tau\,\underline{e}_1 + f(\tau)\,\underline{e}_2\right)$$

$$= \underline{e}_1 + f'\,\underline{e}_2$$

$$s(x) = \int_0^{x} v(\tau)\,d\tau = \int_0^{x}\sqrt{1+(f')^2}\,d\tau$$

$$v(\tau) = \|\underline{v}(\tau)\| = \sqrt{1+(f')^2}$$

$$\frac{ds}{dx} = \sqrt{1+(f')^2} \Rightarrow \frac{dx}{ds} = \frac{1}{\sqrt{1+(f')^2}}$$

$$\frac{d}{ds} M_f(x(s)) = \frac{dM_f}{dx}\,\frac{dx}{ds} = \frac{1}{\sqrt{1+(f')^2}}\,\frac{dM_f}{dx} \dots$$

$$M_f(x) = f(x)\,F$$

$$\frac{dM_f}{dx} = f'(x)\,F$$

$$\longrightarrow \quad \frac{d}{ds} M_f(x) = \frac{1}{\sqrt{1+(f')^2}} \, f' F \stackrel{?}{=} T(x(s))$$

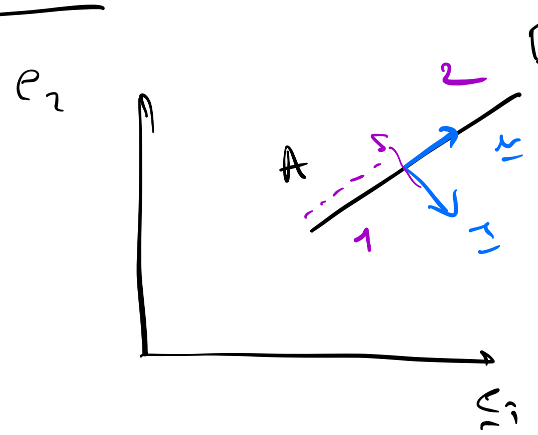
$$T(x) = \frac{F f'}{\sqrt{1+(f')^2}}$$

$$\frac{d M_f}{ds} = T(s)$$

Aste

aste piane

- forze distribuite
- carichi concentrati



La parte 1 è in equilibrio sotto l'azione di:

1) Carichi concentrati: $\underline{R}_1^c$, $\underline{M}_1^c(s)$ (momento rispetto ad s)

2) Forze distribuite con forza specifica $\underline{f}(s)$

$$\underline{R}_1^{d} = \int_0^s \underline{f}(\xi)\, d\xi$$

$$\underline{M}_1^{d}(s) = \int_0^s (\xi - s)\, \underline{u} \wedge \underline{f}(\xi)\, d\xi$$

$$(\underline{x}_\tau - \underline{x}_s) \wedge \underline{f}(\tau)\, d\tau$$

3) Sforzi interni applicati in S

$$\begin{cases} N(s)\, \underline{u} + T(s)\, \underline{\tau} \\ M_t(s)\, \underline{e}_3 \end{cases}$$