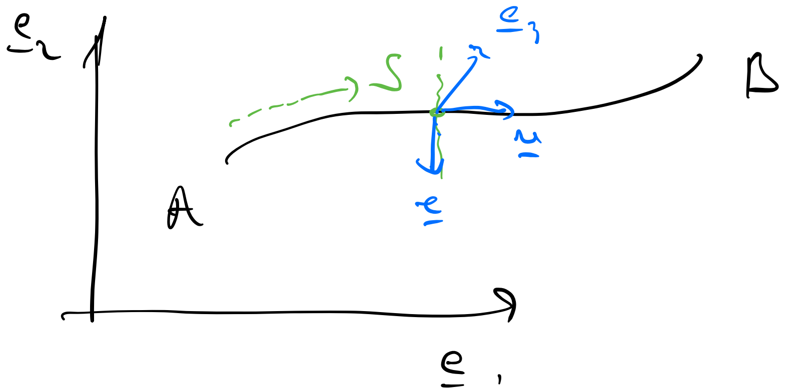
MECCANICA RAZIONALE

Sforzi interni ad un rigido



$\underline{u}$ tangente

$N(s)$, $T(s)$, $M_f(s)$

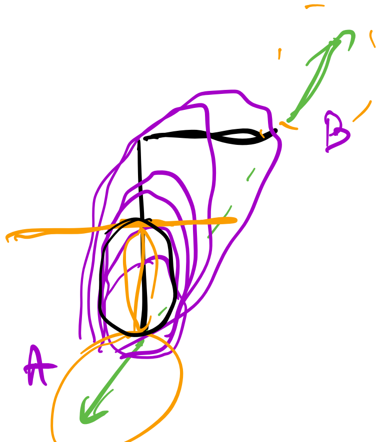
normale, Taglio, momento flettente

Relazioni differenziali

$$\begin{cases} \dfrac{d}{ds}\left(N\underline{u} + T \right) = -f(s) \\[2ex] \dfrac{d\underline{M}_f}{ds} = \underline{T}(s) \wedge \underline{u}(s) \end{cases}$$

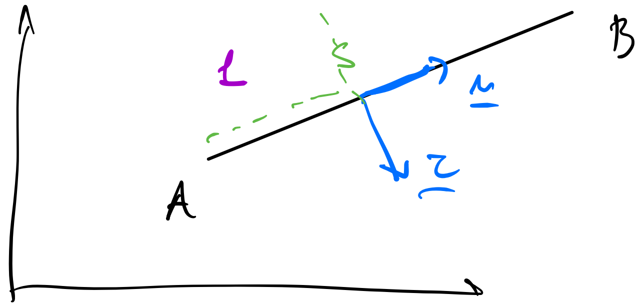
$\rightarrow$ carichi concentrati: N, T discontinui

$M_f \rightarrow$ può essere non derivabile



$\rightarrow \quad -\underline{F} + N\underline{u} + T\underline{z} = 0 \quad \leftarrow$

Aste



- forze concentrate → $\underline{R}^c_1$, $\underline{M}^c_1(s)$
- forze distribuite

$$\underline{R}^d_1 = \int_0^1 \underline{f}(\xi)\,d\xi \qquad \left(\sum_p \underline{f}_p\right)$$

$\rho(s)\,\underline{g}$

$$\underline{M}^d_1 = \int_0^1 (\xi - 1)\,\underline{M} \wedge \underline{f}(\xi)\,d\xi$$

$$\left(\sum_p (\underline{x}_{p_\xi} - \underline{x}_s) \wedge \underline{f}_{p}(x_1)_\xi\right)$$

- $N(s)\,\underline{t} + T(s)\,\underline{n}$

  $M_f(s)\,\underline{e}_3$

Volendo →

$$\frac{d}{ds}\left(N_{(s)}\,\underline{n} + T_{(s)}\,\underline{t}\right) = \frac{dN}{ds}\,\underline{n} + \frac{dT}{ds}\,\underline{t} = -\underline{f}(s)$$

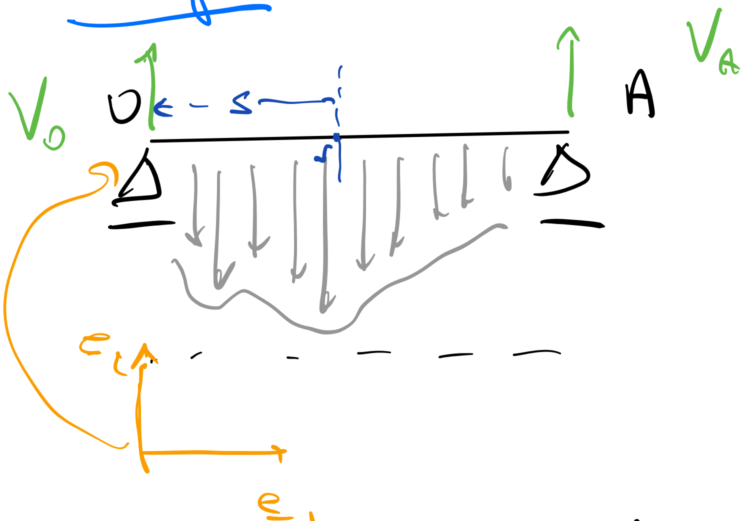
$$\frac{dN(s)}{ds} = -\underline{f}(s)\cdot\underline{n}$$

$$\frac{dT(s)}{ds} = -\underline{f}(s)\cdot\underline{t}$$

$$\frac{d}{ds} M_f(s) = T(s) \qquad \left[\underline{t} = \underline{n} \wedge \underline{e}_3\right]$$

$$M_f(s) - M_f(0) = \int_0^s T(\xi)\, d\xi$$

Esempio



soggetta a una forza specifica

$$\underline{f}(s) = \rho(s)\,\underline{g}$$

$$\overline{OA} = L$$

Determinare gli sforzi interni in funzione di s misurata a partire da O

•) Reazioni vincolari : ECS a tutto il sistema.

$$\left[\, x_G = \frac{\sum_p x_p m_p}{M} \right]$$

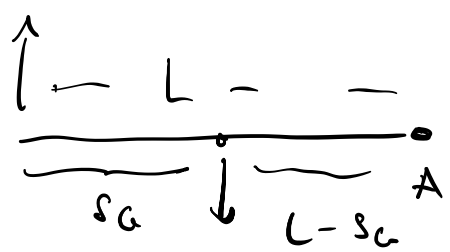
$$M = \int_0^{L} \rho(\xi)\, d\xi$$

$$\longrightarrow \quad s_G = \frac{1}{M} \int_0^{L} \xi\, \rho(\xi)\, d\xi$$

$$\frac{1}{M} \sum_p x_p m_p$$

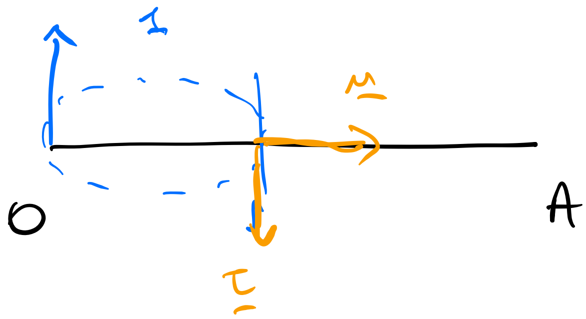
ECS : $\quad V_O + V_A - Mg = 0$

$$\underline{M}^{e}(A) = \underline{0} \longrightarrow V_O L = Mg\,(L - s_G)$$



• Calcolo degli sforzi interni.

$$N\,\underline{n} + T\,\underline{t} - V_O\,\underline{t} + m(s)\,g\,\underline{t} = \underline{0}$$



$\underline{n} = \text{vers}(\underline{x}_A - \underline{x}_O)$

$\longrightarrow$

$\underline{\tau} = \underline{n} \wedge \underline{e}_3$

$= \text{vers}\ \underline{g}$

$$m(s) = \int_0^s \rho(\xi)\, d\xi$$

$$\begin{cases} N = 0 \\ T = V_0 - m(s)\, g \end{cases}$$

$M_f \rightarrow$ 2 modi :

1) eq di bloccaggio con polo S

$$0 = \begin{matrix}\text{momento} \\ \text{di} \\ V_0\end{matrix} + \int_0^s (\xi - s)\, \underline{n} \wedge \underline{f}(\xi)\, d\xi + M_f(s)$$

$$(\underline{x}_0 - \underline{x}_s) \wedge \underline{V}_0$$

$$\tau$$

2) usiamo $\dfrac{d}{ds} M_f(s) = T(s)$

$$M_f(s) - \underbrace{M_f(0)}_{=\,0} = \int_0^s T(\xi)\, d\xi$$

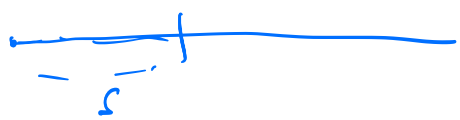
Notiamo anche $T = V_0 - m(s)\, g$

$$\Rightarrow \quad \frac{dT}{ds} = -\rho(s)\, g \qquad \left(\frac{dR}{ds} = -f \right)$$

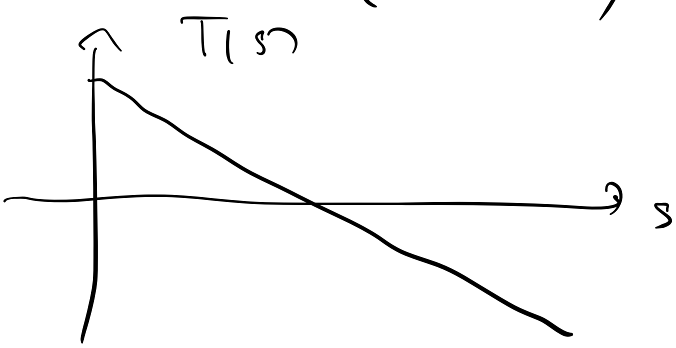
1) ρ = costante = $\dfrac{M}{L}$

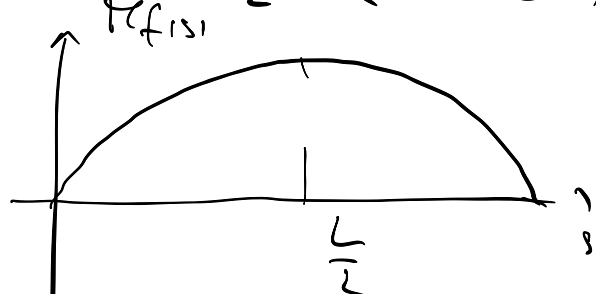
$$V_0 = Mg\left(1 - \frac{s_G}{L}\right) = \frac{Mg}{2} = V_A$$

$$\int_0^s \rho(\xi)\, d\xi = \int_0^s \frac{M}{L}\, d\xi = \frac{M}{L}\int_0^s d\xi = \frac{M}{L}\, s$$



$$T(s) = Mg\left(\frac{1}{2} - \frac{s}{L}\right) \qquad M_f(s) = \frac{Mg}{2}\, s \left(1 - \frac{s}{L}\right)$$





a) $\rho(\xi) = 6\,\frac{M}{L}\,\frac{\xi}{L}\left(1 - \frac{\xi}{L}\right)$

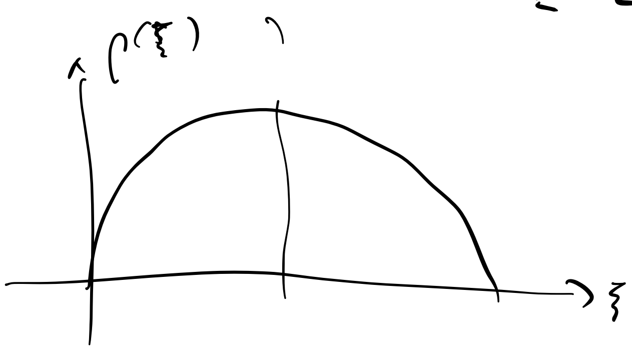
Massa Totale $= \int_0^L \rho(\xi)\,d\xi = 6\,\frac{M}{L^2}\,\frac{L^2}{2} - \frac{6M}{L^3}\,\frac{L^3}{3}$

$= 3M - 2M = M$

$\int_0^L \xi^2 = \frac{\xi^3}{3}\Big|_0^L = \frac{L^3}{3}$

$x_G = \frac{1}{M}\int_0^L \xi\,\rho(\xi)\,d\xi = \frac{1}{M}\left(6\,\frac{M}{L^2}\,\frac{L^3}{3} - 6\,\frac{M}{L^3}\,\frac{L^4}{4}\right)$

$= 2L - \frac{3}{2}L = \frac{L}{2}$



$V_0 = V_A = \frac{Mg}{2}$

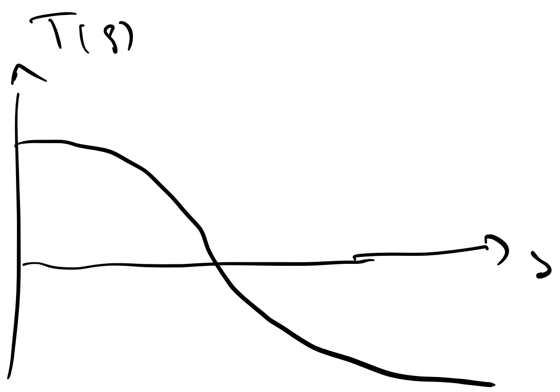
$\rightarrow$ uguale al caso asta omogenea!

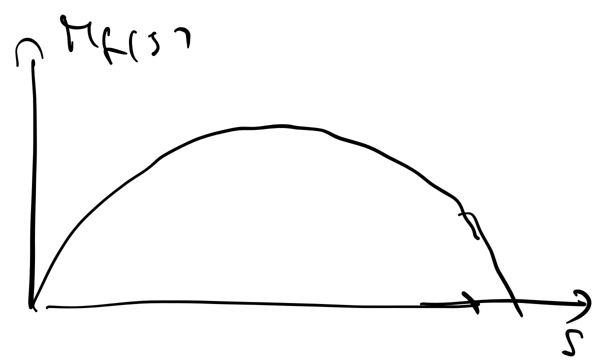
$m(s) = \int_0^s \rho(\xi)\,d\xi = \int_0^s 6\,\frac{M}{L}\,\frac{\xi}{L}\left(1 - \frac{\xi}{L}\right)d\xi$

$$= 3M \frac{s^2}{L^2}\left(1 - \frac{2}{3}\frac{s}{L}\right)$$

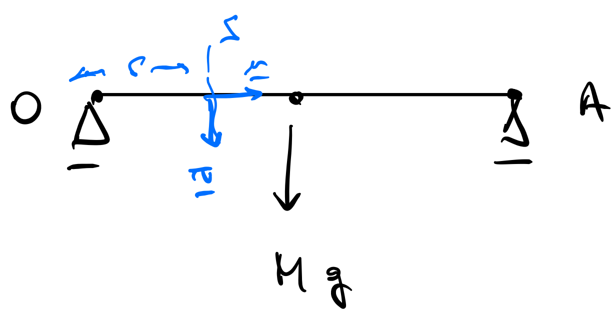
$$T(s) = \frac{Mg}{2} - m(s)g = Mg\left(\frac{1}{2} - 3\frac{s^2}{L^2} + 2\frac{s^3}{L^3}\right)$$

$$\underline{M_f(s)} = MgL\left(\frac{1}{2}\frac{s}{L} - \frac{s^3}{L^3} + \frac{1}{2}\frac{s^4}{L^4}\right)$$





<u>Esempio</u>

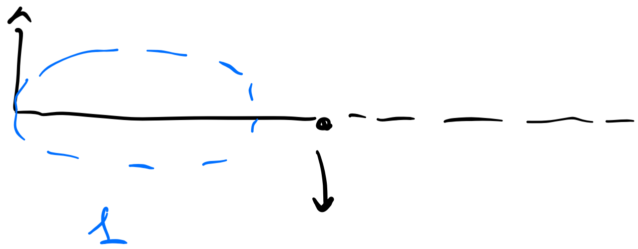


Carico concentrato in $\frac{L}{2}$ e pari a $M\underline{g}$

ECS : $V_0 = V_A = \frac{Mg}{2}$

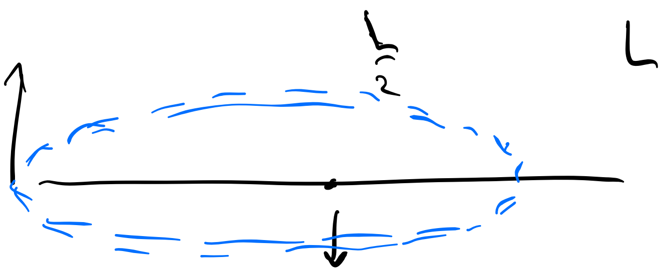
Sforzi interni

Se $s \in [0, \frac{L}{2})$



$$N \underline{n} + T \underline{\tau} - V_0 \underline{\tau} = \underline{0}$$

$$\begin{cases} N(s) = 0 \\ T(s) = \frac{Mg}{2} \end{cases} \qquad 0 \leq s < \frac{L}{2}$$



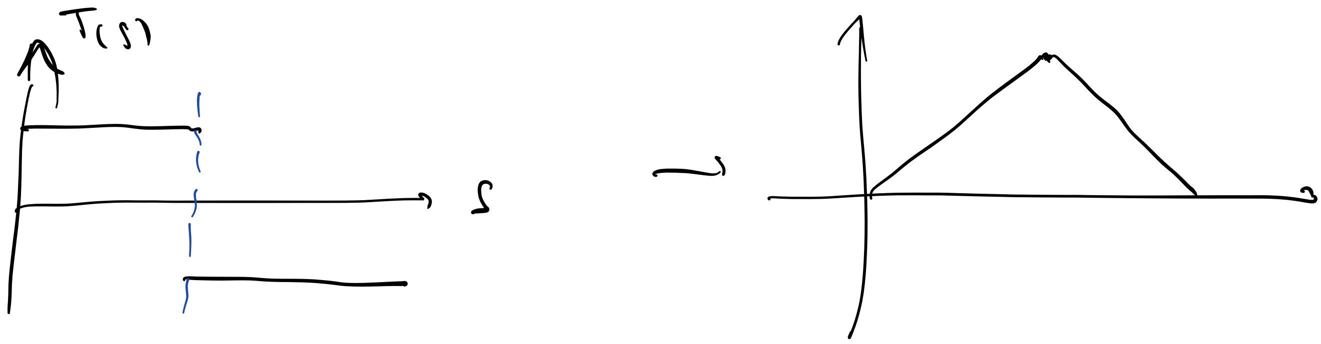
$s \in (\frac{L}{2}, L]$

$$N \underline{n} + T \underline{\tau} - V_0 \underline{\tau} + M g \underline{\tau} = \underline{0}$$

$$\begin{cases} N(s) = 0 \\ T(s) = \frac{Mg}{2} - Mg = -\frac{Mg}{2} \end{cases}$$

Quindi

$$N(s) = 0 \qquad T(s) = \begin{cases} \frac{Mg}{2} & 0 \leq s < \frac{L}{2} \\ -\frac{Mg}{2} & \frac{L}{2} < s \leq L \end{cases}$$



$$M_f(s) = \int_0^s T(\xi)\,d\xi \qquad \left(M_f(0) = 0 \right)$$

$$M_f(s) = \begin{cases} \frac{Mg}{2}\, s & 0 \le s < \frac{L}{2} \\ \frac{Mg}{2}\frac{L}{2} - \frac{Mg}{2}\left(s - \frac{L}{2}\right) & \frac{L}{2} < s \le L \end{cases}$$

$$\int_0^s T(\xi)\,d\xi = \int_0^{\frac{L}{2}} \frac{Mg}{2} + \int_{\frac{L}{2}}^{s} \left(-\frac{Mg}{2} \right)$$

$$T = \begin{cases} \frac{Mg}{2} & 0 \le s < \frac{L}{2} \\ -\frac{Mg}{2} & \frac{L}{2} < s \le L \end{cases}$$

$$-\frac{Mg}{2}\left(\xi \Big|_{\frac{L}{2}}^{s} \right)$$

$$-\frac{Mg}{2}\left(s - \frac{L}{2} \right)$$

CONTINUI DEFORMABILI

Modello matematico:

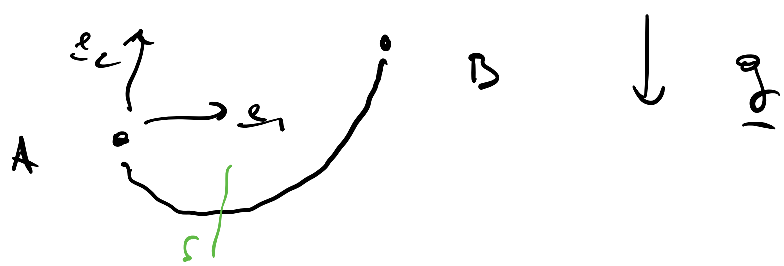
- geometria → continuo unidimensionale (come una curva)
- forze agenti → come per il corpo rigido
- struttura meccanica del materiale: equazioni costitutive del materiale.

↳ bilancio delle forze per ogni punto del continuo.

(metodi variazionali)

<u>Esempio</u>

Catena



$\rho(s)\,\underline{g}$, lunghezza fissata $= L$

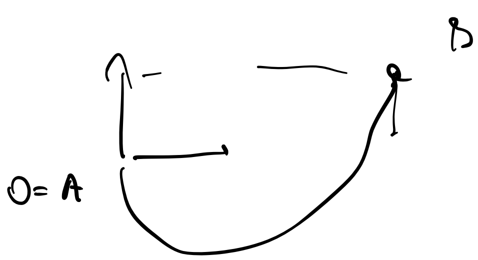
Parametro d'arco s : inestensibile

Configurazione generica = curva regolare di estremi A e B e lunghezza L

$$\begin{cases} x = x(s) \\ y = y(s) \end{cases} \qquad s \in [0, L]$$

$x(0) = x_A = 0$

$x(L) = x_B$



$y(0) = y_0 = 0$

$y(L) = y_B$

$$\begin{cases} \dfrac{d}{ds}\left(N(s)\,\underline{t}(s) + \underline{T}(s) \right) = -\rho\,\underline{g} \\[2ex] \dfrac{d}{ds}\,\underline{M}_f = \underline{T}(s) \wedge \underline{t}(s) \end{cases}$$

$$\underline{t}(s) = \frac{dx}{ds}\,\underline{e}_1 + \frac{dy}{ds}\,\underline{e}_2$$

Equazioni costitutive

$$\boxed{\underline{T} = \underline{0} \qquad \underline{M}_f = \underline{0} \qquad N > 0}$$

$$\frac{d}{ds}\left[\underbrace{N\left(\frac{dx}{ds}e_1 + \frac{dy}{ds}e_2\right)}_{\underline{u}}\right] = -\rho\,\underline{g}$$

cerchiamo $x = x(s)$, $y = y(s)$

$N = N(s) > 0$

$$\begin{cases} \dfrac{d}{ds}\left(N\dfrac{dx}{ds}\right) = 0 \\[2ex] \dfrac{d}{ds}\left(N\dfrac{dy}{ds}\right) = \rho g \end{cases}, \quad \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1$$

$$L, \qquad N(\;)\,\frac{d\underline{x}}{ds}\Bigg|_{0,\;L} \longrightarrow \text{reatti vincolari}$$

Catenarie :

$$y(x) = b + a\cosh\left(\frac{x - x_0}{a}\right)$$

con a, b, x_0 certi parametri

Sono determinati da

$$y(0) = 0$$

$$y(x_B) = y_B$$

$$L = \int_0^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

lunghezza della catena

$$\sqrt{1 + (f')^2}$$

Corpo rigido

↓

coordinate libere (2D, 3D)

↓

sistemi olonomi

↓

PLV

$\xrightarrow[\text{temporale}]{\text{evoluzione}}$

$$\underline{q} = \underline{q}(t)$$

Equazioni Cardinali della Statica $\longrightarrow$ Equazioni Cardinali della Dinamica.

$\underline{F} = \underline{0}$ $\longrightarrow$ $\underline{F} = m\underline{a}$

$m \dfrac{d^2x}{dt^2}$

PLV $\longrightarrow$ Equazioni di Lagrange

$\sum_i Q_i \, \delta q_i$

(conservative o no) (conservative o no)

I

↳ metodi di approssimazione