Exercise 1.

Consider a complex variable z, and compute the following Gaussian integral

$$I = \int_{\mathbb{C}} dz dz^* \, e^{-z^* a z + J^* z + z^* J} \,, \tag{1}$$

where $a > 0$.
Hint. Consider $z = x + iy$ and recall that the measure of integration changes as $dzdz^* \to dxdy$.

Extend this to d-dimensional complex space, with A_{ij} an Hermitian positive-definite matrix, computing:

$$I = \int_{\mathbb{C}} (\Pi_i dz_i dz_i^*) \, e^{-z_i^* A_{ij} z_j + J_i^* z_i + z_i^* J_i} \,, \tag{2}$$

Exercise 2.

Consider the quantum field theory for a complex scalar field, described by the Lagrangian

$$\mathscr{L} = \partial_\mu \phi^*(x))\partial^\mu \phi(x) - m^2 \phi^*(x)\phi(x) \,. \tag{3}$$

Solve the theory by computing the generating functional

$$Z[J, J^*] = \int \mathscr{D}\phi \mathscr{D}\phi^* e^{i\int d^4x \left(\partial_\mu \phi^*)\partial^\mu \phi - m^2 \phi^* \phi + J^* \phi + \phi^* J\right)} \,. \tag{4}$$

From the result obtained, compute the two-point Green's functions

$$\langle 0 | T[\phi(x)\phi^*(y)] | 0 \rangle \,, \tag{5}$$

and

$$\langle 0 | T[\phi(x)\phi(y)] | 0 \rangle \,. \tag{6}$$