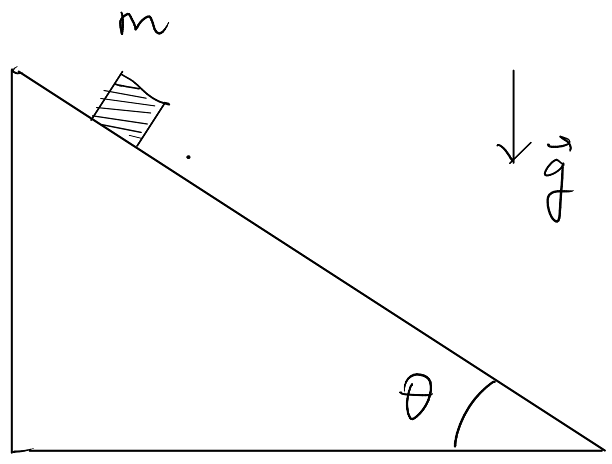
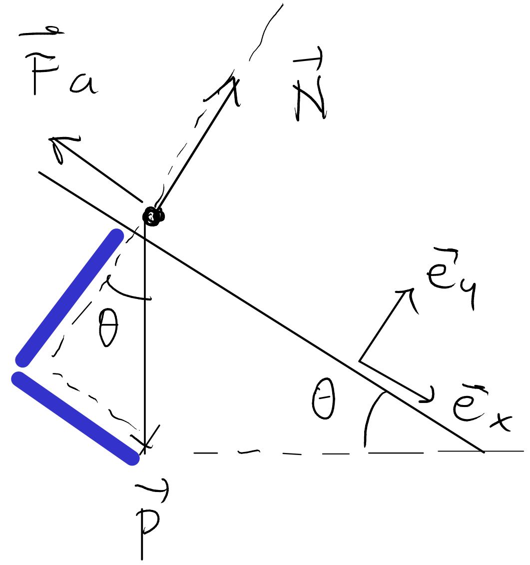
Es.: misura di μ_s e μ_d



piano inclinato

$\longrightarrow$



II Newton:

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{N} + \vec{F}_a + \vec{P} = m\vec{a}$$

Caso statico: $\vec{a} = \vec{0}$

$$\vec{N} + \vec{F}_a + \vec{P} = \vec{0}$$

Componenti cartesiane:

$$|\vec{N}|\,\vec{e}_y - |\vec{F}_a|\,\vec{e}_x + mg\sin\theta\,\vec{e}_x - mg\cos\theta\,\vec{e}_y = \vec{0}$$

$$\begin{cases} mg\sin\theta - |\vec{F}_a| = 0 \\ |\vec{N}| - mg\cos\theta = 0 \end{cases} \quad \begin{array}{l} \Rightarrow |\vec{F}_a| = \cancel{mg}\sin\theta \le \mu_s |\vec{N}| = \mu_s \cancel{mg}\cos\theta \\ \Rightarrow |\vec{N}| = mg\cos\theta \end{array}$$

$$\tan\theta \le \mu_s \rightarrow \tan\theta_{max} = \mu_s$$

Caso dinamico: $\vec{N} + \vec{F}_a + \vec{P} = m\vec{a} = m a_x \vec{e}_x$

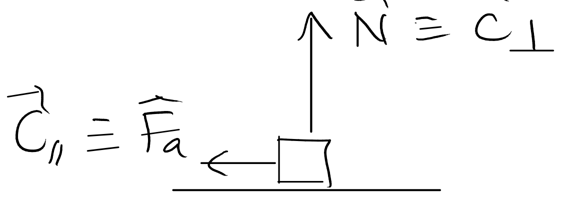
$$\begin{cases} mg\sin\theta - |\vec{F}_a| = m a_x \\ |\vec{N}| = mg\cos\theta \end{cases} \Rightarrow \begin{cases} mg\sin\theta - \mu_d |\vec{N}| = m a_x \\ |\vec{N}| = mg\cos\theta \end{cases}$$

$$\cancel{m} a_x = \cancel{m} g\sin\theta - \mu_d \cancel{m} g\cos\theta \Rightarrow a_x = g(\sin\theta - \mu_d\cos\theta)$$

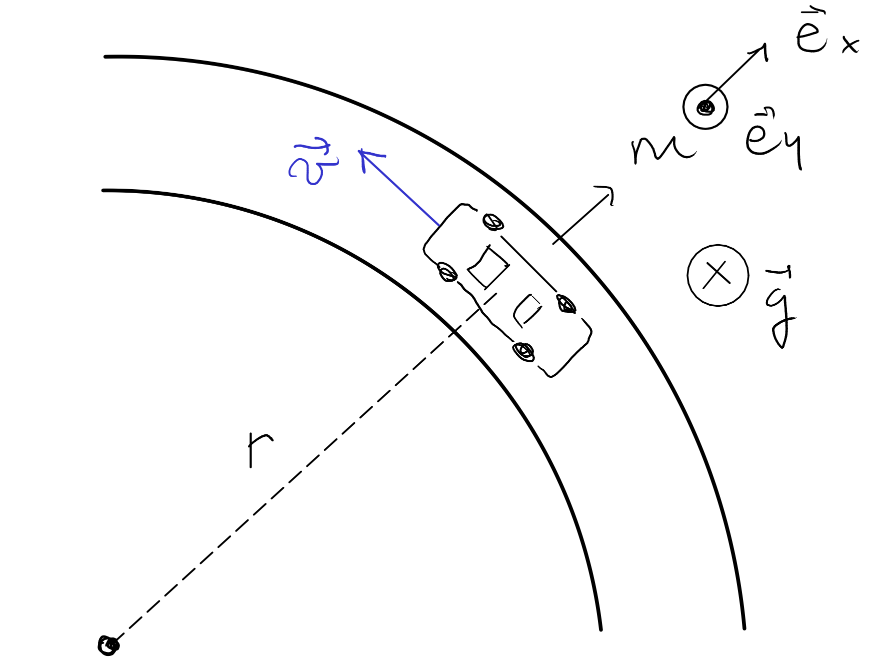
$$|\vec{v}| = \text{cost} \Rightarrow \mu_d = \tan\theta$$

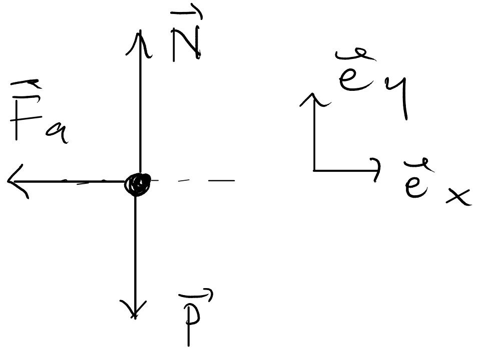
<u>unificazione delle forze macroscopiche</u>

forza elastica, tensione $\Big\}$ elasticità $\Big\{$ reazione normale; forza attrito statico; — " — dinamico



Es.: aderenza in curva





II Newton: $\sum \vec{F} = m\vec{a}$

$$\vec{N} + \vec{F}_a + \vec{P} = m\vec{a}$$

Moto circolare uniforme: $\vec{a} = \vec{a}_c$

$$a_c \equiv |\vec{a}_c| = \frac{|\vec{v}|^2}{r}$$

Caso statico ⚠

$$|\vec{F}_a| \leq \mu_s |\vec{N}|$$

↑ coeff. attrito statico

Componenti cartesiane

$$\underline{|\vec{N}|\,\vec{e}_y} - |\vec{F}_a|\,\vec{e}_x \underline{- mg\,\vec{e}_y} = -m a_c\,\vec{e}_x$$

$$\begin{cases} m a_c = |\vec{F}_a| \leq \mu_s |\vec{N}| = \mu_s m g \\ |\vec{N}| = mg \end{cases}$$

$$\cancel{m} a_c \leq \mu_s \cancel{m} g$$

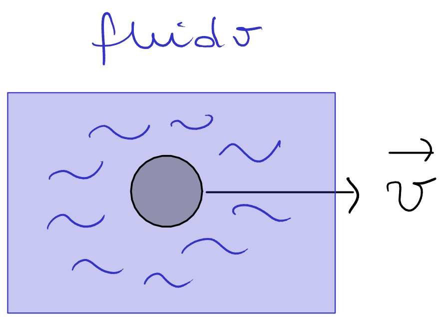
$$\frac{|\vec{v}|^2}{r} \leq \mu_s g \Rightarrow |\vec{v}| \leq \sqrt{\mu_s r g} \quad \square$$

$$\sqrt{1 \times 30\,\mathrm{m} \times 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}}$$

$$\approx 17\,\mathrm{m/s} = 60\,\frac{\mathrm{km}}{\mathrm{h}}$$

6. Attrito viscoso

Moto di un corpo in un fluido



"Bassa" velocità

$$\boxed{\vec{F}_v = -\mu_v \vec{v}}$$

coeff. attrito viscoso

$|\vec{F}_v| \sim |\vec{v}|$

SI: $\frac{Ns}{m}$

"Alta" velocità

$|\vec{F}_v| \sim |\vec{v}|^2$

$\vec{F}_v \longleftarrow \bullet \qquad \longrightarrow \vec{e}_x$

II Newton: $\sum \vec{F} = m\vec{a}$

$$m \frac{d\vec{v}}{dt} = -\mu_v \vec{v}$$

Componente cartesiana:

$$m \frac{dv_x}{dt} \vec{e}_x = -\mu_v v_x \vec{e}_x \qquad v \equiv v_x$$

$$\boxed{\frac{dv}{dt} = -\frac{\mu_v}{m} v}$$

$$\left[\frac{d^2x}{dt^2} = \text{cost} \rightarrow x(t), \quad \frac{dv}{dt} = \text{cost} \rightarrow v(t) \right]$$

Eq. differenziale per $v(t)$ ordinaria

$$\frac{dv}{dt} = v \quad \rightarrow \quad v(t) = \exp(t) \quad \rightarrow \quad v(t) = A\exp(t) \qquad A = \text{cost}$$

Cerco soluzione di tipo: $v(t) = A\exp(Bt)$ $\quad A, B$ costanti

$$B\,\cancel{A\exp(Bt)} = -\frac{\mu_v}{m}\,\cancel{A\exp(Bt)} \quad \Rightarrow \quad B = -\frac{\mu_v}{m}$$

$$\left[\begin{aligned} y &= A\exp(Bx) \\ \log y &= \log A + \log[\exp(Bx)] \\ &= \log A + Bx \end{aligned}\right]$$

$$v(t) = A\exp\left(-\frac{\mu_v}{m}t\right) \quad \text{è soluzione}$$

Separazione delle variabili

$$\frac{dv}{dt} = -\frac{\mu_v}{m}v$$

$$\frac{dv}{v} = -\frac{\mu_v}{m}dt$$

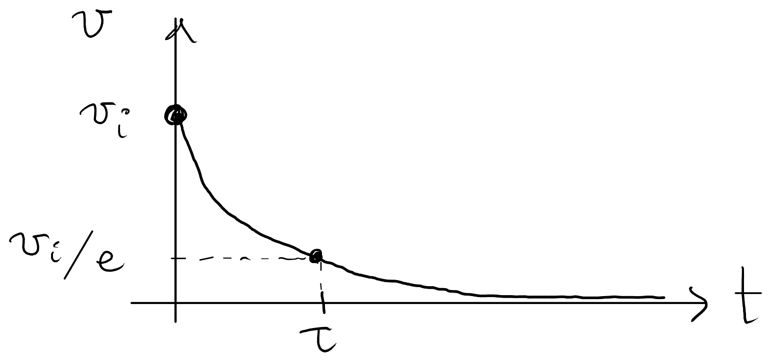
$$\int_{v_i}^{v_f}\frac{dv}{v} = -\frac{\mu_v}{m}\int_{t_i}^{t_f}dt$$

$$\ln v_f - \ln v_i = -\frac{\mu_v}{m}(t_f - t_i)$$

$$\ln(v_f/v_i) = -\frac{\mu_v}{m}(t_f - t_i)$$

$$v_f = v_i\exp\left[-\frac{\mu_v}{m}(t_f - t_i)\right]$$

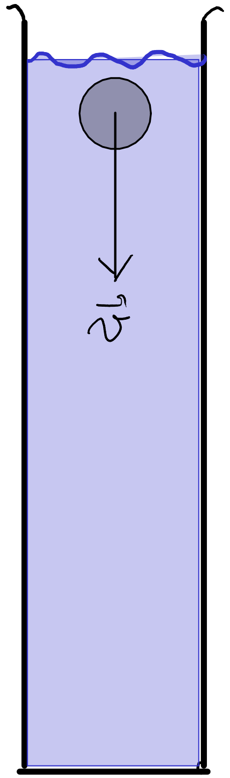
$$v = v_i\exp\left[-\frac{\mu_v}{m}(t - t_i)\right] \qquad t_i = 0$$

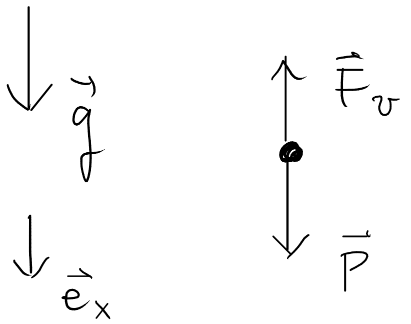


$$v = v_i\exp\left(-\frac{\mu_v}{m}t\right)$$

$$\tau \equiv \frac{m}{\mu_v} \quad \text{tempo di rilassamento}$$

Es.: velocità limite di una biglia in un fluido viscoso





II Newton: $\vec{P} + \vec{F}_v = m\vec{a}$

$$mg\vec{e}_x - \mu_v v \vec{e}_x = m a_x \vec{e}_x$$

$$mg - \mu_v v = m a_x = m\frac{dv}{dt}$$

Velocità limite: $v = \text{cost}$ se $t \to \infty \Rightarrow a_x = 0$

$$mg - \mu_v v_\infty = 0 \Rightarrow v_\infty = \frac{mg}{\mu_v}$$

v_∞: velocità limite