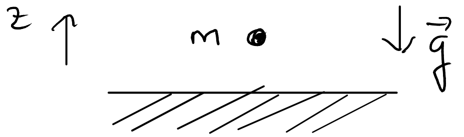
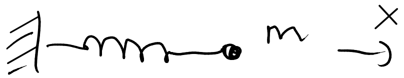
Es.

1) Peso



$$W_{AB} = \int_A^B \vec F \cdot d\vec r = -mg(z_B - z_A)$$

2) Molla ideale



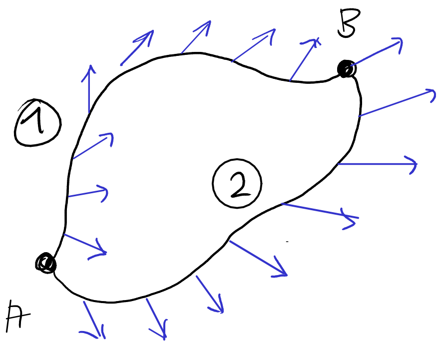
$$W_{AB} = \int_A^B \vec F \cdot d\vec r = -\frac{1}{2} k (x_B^2 - x_A^2)$$

non dipendono dal percorso

Teor. energia cinetica

$$W_{if}[\Sigma \vec F] = \Delta E_c$$

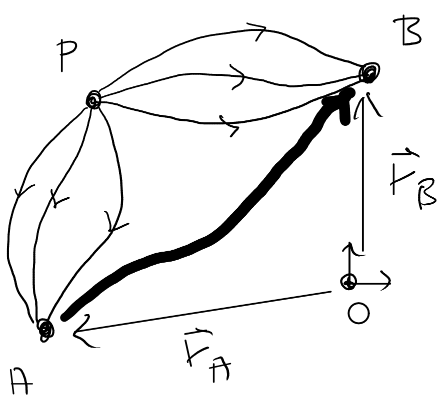
$$\int_i^f (\Sigma \vec F)\cdot d\vec r = \Delta E_c$$



Campo di forze non dipende dal tempo

$\forall A, B$ $\forall$ percorso

W_{AB} non dipende dal percorso $\Rightarrow$ campo di forze CONSERVATIVO



P di riferimento

$$W_{PB} = \int_P^B \vec F \cdot d\vec r \equiv -E_p(\vec r_B)$$

$$W_{PA} = \int_P^A \vec F \cdot d\vec r \equiv -E_p(\vec r_A)$$

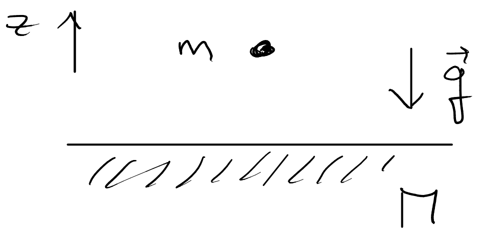
Energia potenziale SI: J

$$\Delta E_p = -W_{AB}$$

$$W_{AB} = W_{AP} + W_{PB} = -W_{PA} + W_{PB} = E_p(\vec r_A) - E_p(\vec r_B) = -\Delta E_p$$

Esempi di energie potenziali

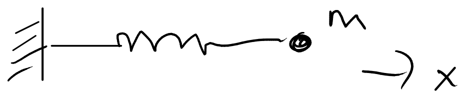
1) Gravitazione terrestre



$$\begin{cases} \Delta E_p = -W_{AB} = -\int_A^B \vec{F}\cdot d\vec{r} = mg(z_B - z_A) \\ E_{pB} - E_{pA} = \Delta E_p \end{cases} \Rightarrow \boxed{E_p = mgz} + \text{cost}$$

$\{m, M\}$ sistema

2) Molla ideale

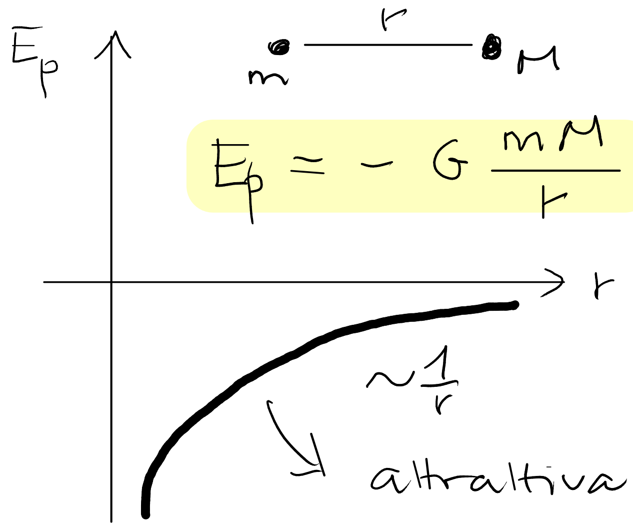


$$\Delta E_p = -W_{AB} = -\int_A^B \vec{F}\cdot d\vec{r} = \frac{1}{2}K(x_B^2 - x_A^2) \Rightarrow \boxed{E_p = \frac{1}{2}kx^2} + \text{cost}$$

{molla, corpi}

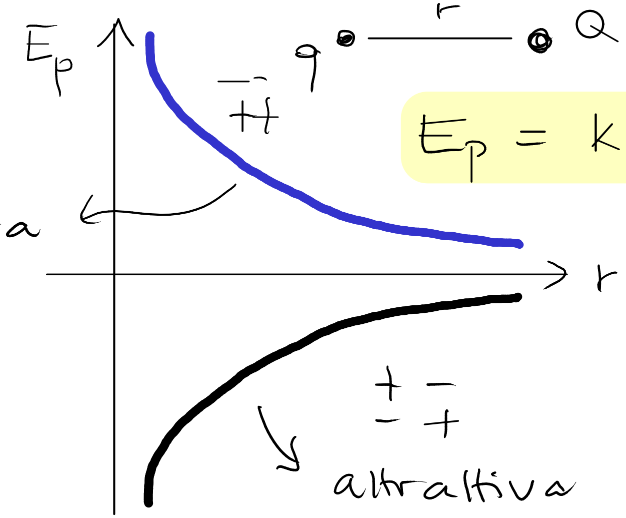
3) Gravitazione universale

$$\boxed{E_p = -G\frac{mM}{r}}$$



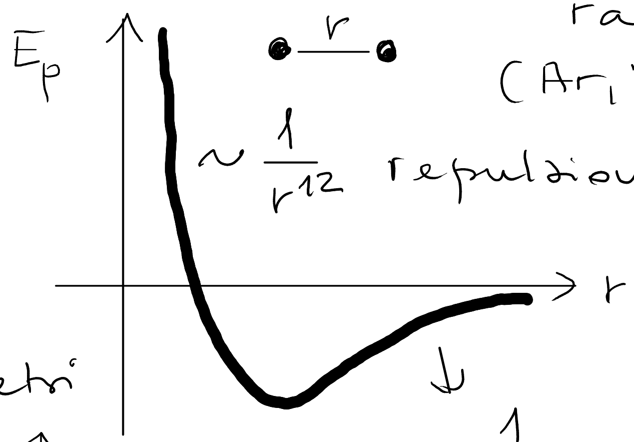
4) Elettrostatica

$$\boxed{E_p = k_e\frac{qQ}{r}}$$



5) Interatomica

tra gas rari (Ar, Ne, Xe)



parametri

$$\boxed{E_p = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]}$$

Lennard-Jones

Relazione tra energia potenziale e forza

$$\Delta E_p = -\int_A^B \vec{F}\cdot d\vec{r} \xrightarrow[1d]{} \Delta E_p = -\int_{x_0}^{x} F_{x'}\,dx' = E_p(x) - E_p(x_0)$$

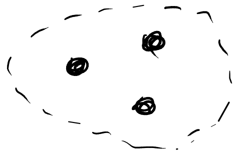
$$E_p(x) = E_p(x_0) - \int_{x_0}^{x} F_{x'}\,dx'$$

$$\frac{dE_p}{dx} = -\frac{d}{dx}\left(\int_{x_0}^{x} F_{x'}\,dx'\right) = -F_x$$

$$\Rightarrow \quad \boxed{F_x = -\frac{dE_p}{dx}}$$

$$\Rightarrow \quad F_y = -\frac{dE_p}{dy} \quad \rightarrow \quad \vec{F} = -\underbrace{\vec{\nabla}}_{\text{gradiente}} E_p$$

Conservazione dell'energia meccanica



$$\begin{cases} W[\Sigma \vec{F}] = \Delta E_c & \text{teor. energia cinetica} \\ W[\Sigma \vec{F}] = -\Delta E_p & \text{forze conservative (non dipendenti dal tempo)} \end{cases}$$

$$\Delta E_c = -\Delta E_p \rightarrow \Delta E_c + \Delta E_p = 0 \rightarrow (E_{cf} - E_{ci}) + (E_{pf} - E_{pi}) = 0$$

$$(E_{cf} + E_{pf}) - (E_{ci} + E_{pi}) = 0$$

$E_c + E_p \equiv E$ energia meccanica

$$E_f - E_i = 0 \Leftrightarrow E_f = E_i \Leftrightarrow \Delta E = 0$$

legge di conservazione dell'energia meccanica

$$\Sigma \vec{F} = \Sigma \vec{F}_c + \Sigma \vec{F}_{nc}$$

$\Sigma \vec{F}_c$ ↑ cons. — $\Sigma \vec{F}_{nc}$ ↑ non cons.

- [V] peso
- [V] gravitazione
- [V] elettrostatica
- [V] elastica

- [X] attrito dinamico
- [X] attrito viscoso

↓

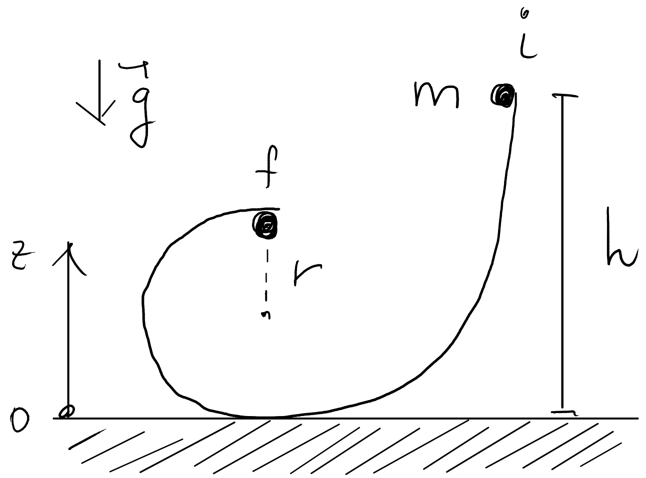
non conservative

$$W[\Sigma \vec{F}_c] + W[\Sigma \vec{F}_{nc}] = \Delta E_c$$

$$\Delta E_c + \Delta E_p = \Delta E = W[\Sigma \vec{F}_{nc}]$$

Esempio : skateboard

velocità iniziale $\vec{v}_i = \vec{0}$, no altrito $\Rightarrow$ traiettoria circolare fino a f $\Rightarrow$ $h = ?$



Conservazione energia meccanica

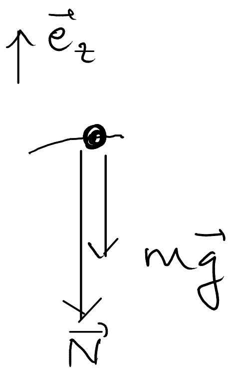
$$E_i = E_f$$

$$E_{ci} + E_{pi} = E_{cf} + E_{pf}$$

$$0 + mgh = \frac{1}{2} m |\vec{v}_f|^2 + mg2r$$

$$|\vec{v}_f|^2 = 2(gh - g2r) \Rightarrow |\vec{v}_f| = \sqrt{2g(h-2r)}$$

II Newton : $\sum \vec{F} = m\vec{a}$



$$m\vec{g} + \vec{N} = m\vec{a}_c$$

$$-mg\,\vec{e}_z - |\vec{N}|\,\vec{e}_z = -m\frac{|\vec{v}_f|^2}{r}\vec{e}_z \Rightarrow mg + |\vec{N}| = \frac{|\vec{v}_f|^2}{r}$$

$$|\vec{N}| = m\frac{|\vec{v}_f|^2}{r} - mg \geq 0 \quad \text{(condizione di aderenza)}$$

$$\frac{2\cancel{g}h}{r} - 4\cancel{g} - \cancel{g} \geq 0 \qquad \frac{2h}{r} - 5 \geq 0 \qquad h \geq \frac{5}{2}r = \frac{5}{4}(2r) \quad \square$$