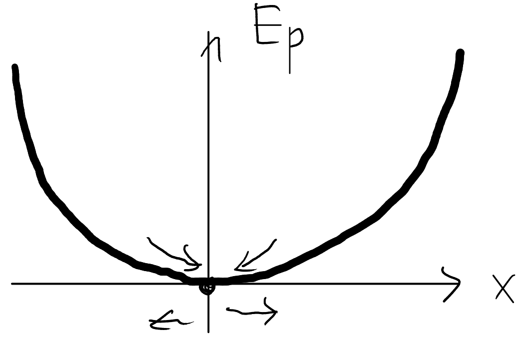
Condizione di equilibrio meccanico

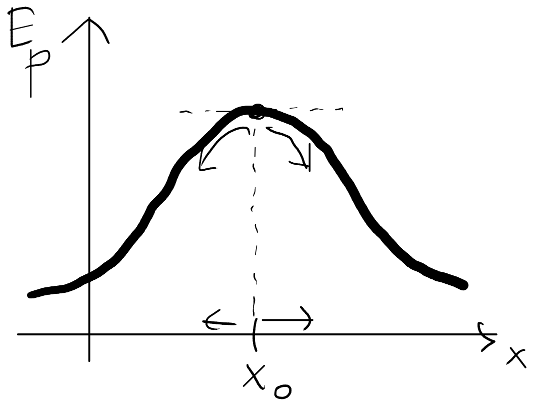
$\Sigma \vec{F} = \vec{0}$ eq. meccanico

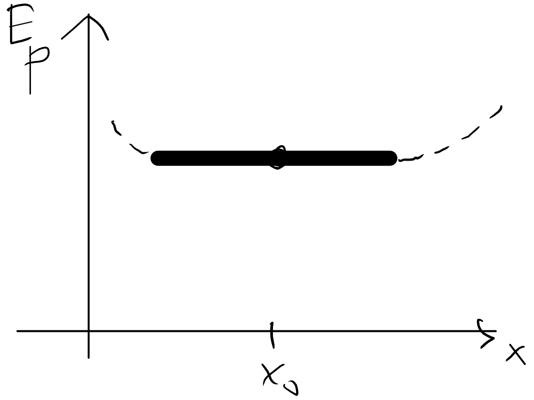
$\vec{v} = \vec{0}$ eq. statico

$F_x = -\dfrac{dE_p}{dx}$

es: molla ideale $E_p = \frac{1}{2} k x^2$ (+cost)







$x = 0$: $\dfrac{dE_p}{dx} = 0 \Rightarrow$ equilibrio

$x > 0$: $F_x < 0$

$x < 0$: $F_x > 0$ } stabile

$\dfrac{d^2E_p}{dx^2} > 0$

$x = x_0$: equilibrio

$x - x_0 > 0$: $F_x > 0$

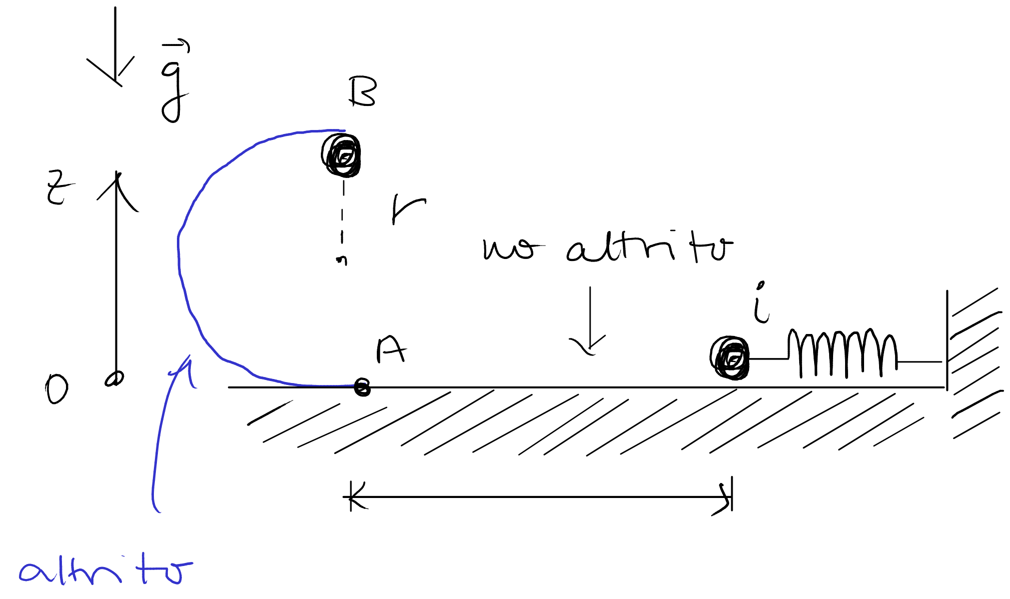
$x - x_0 < 0$: $F_x < 0$ } instabile

$\dfrac{d^2E_p}{dx^2} < 0$

indifferente

$\dfrac{d^2E_p}{dx^2} = 0$

Es. blocco su guida circolare



$m = 0.5\ kg$

$v_A \equiv |\vec{v}_A| = 12\ m/s$

$r = 1m$

$|\vec{F}_a| = 7\ N$

$K = 450\ N/m$

1) determina Δx

2) determina $v_B \equiv |\vec{v}_B|$

3) riesce ad arrivare in B aderendo alla guida?

1) Forze che agiscono sul blocco e sulla molla sono conservative e indipendenti dal tempo
$\Rightarrow$ conservazione dell'energia meccanica per il sistema {blocco, molla}

$$E_{ci} + E_{pi} = E_{cA} + E_{pA}$$

$$0 + \frac{1}{2} K \Delta x^2 = \frac{1}{2} m v_A^2 + 0$$

$$\Delta x = \sqrt{\frac{m v_A^2}{K}} = \sqrt{\frac{m}{K}}\, v_A = \sqrt{\frac{0.5\ kg}{450\ N/m}} \cdot 12\ \frac{m}{s} = 0.4\ m$$

2) Teorema energia meccanica : $\Delta E = W[\Sigma \vec{F}_{nc}]$ → blocco

$$\underbrace{(E_{cB} + E_{pB})}_{E_B} - \underbrace{(E_{cA} + E_{pA})}_{E_A} = \int_A^B (\Sigma \vec{F}_{nc}) \cdot d\vec{r} \qquad |\vec{F}_a|$$

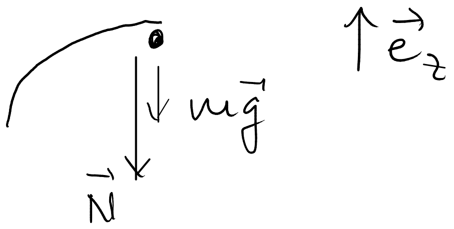
$$\frac{1}{2}\cancel{m} v_B^2 + \cancel{m} g 2r - \frac{1}{2}\cancel{m} v_A^2 - 0 = \int_A^B \vec{F}_a \cdot d\vec{r} = -\int_A^B |\vec{F}_a|\, dr = -\frac{|\vec{F}_a| \pi r}{m}$$

$$\frac{1}{2} v_B^2 = \frac{1}{2} v_A^2 - 2gr - \frac{|\vec{F}_a| \pi r}{m}$$

$$v_B^2 = v_A^2 - 4gr - \frac{2\pi |\vec{F}_a| r}{m}$$

$$v_B = \sqrt{v_A^2 - 4gr - \frac{2\pi |\vec{F}_a| r}{m}} = \dots = 4.1 \text{ m/s}$$

3)



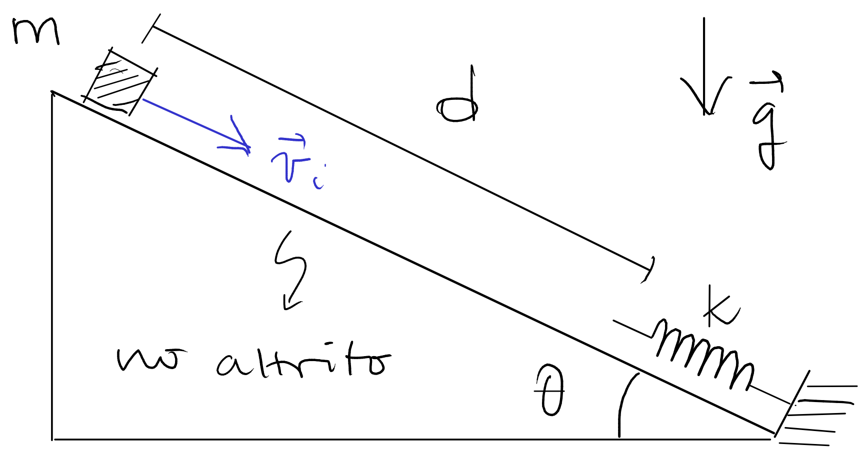
II Newton: $\vec{N} + m\vec{g} = m\vec{a}_c$

$$-|\vec{N}|\vec{e}_z - mg\vec{e}_z = -m\frac{v_B^2}{r}\vec{e}_z$$

$$\frac{|\vec{N}|}{m} = \cancel{m}\frac{v_B^2}{r} - \cancel{m}g \geq 0 \quad \Rightarrow \quad v_B^2 \geq gr = 9.81 \frac{m}{s^2} \times 1 m = 9.81 \frac{m^2}{s^2}$$

$$v_B \geq \sqrt{9.81} \frac{m}{s} = 3.13 \frac{m}{s} \quad \boxed{✓}$$

Es.: blocco + molla su piano inclinato



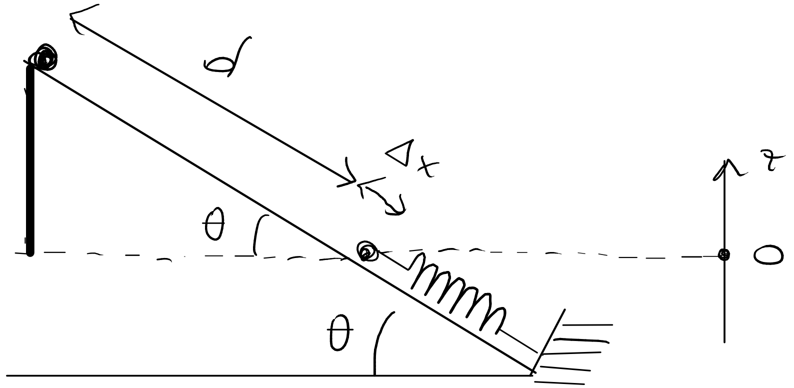
$|\vec{v}_i|$ velocità iniziale ; molla ideale (k).

Determina la massima compressione $\Delta x > 0$ della molla

Sistema {blocco, molla} in presenza di forze conservative (costanti nel tempo)

$\Rightarrow$ conservazione energia meccanica

Blocco = particella



$$E_{ci} + E_{pi} = E_{cf} + E_{pf} \qquad mgz = E_p$$

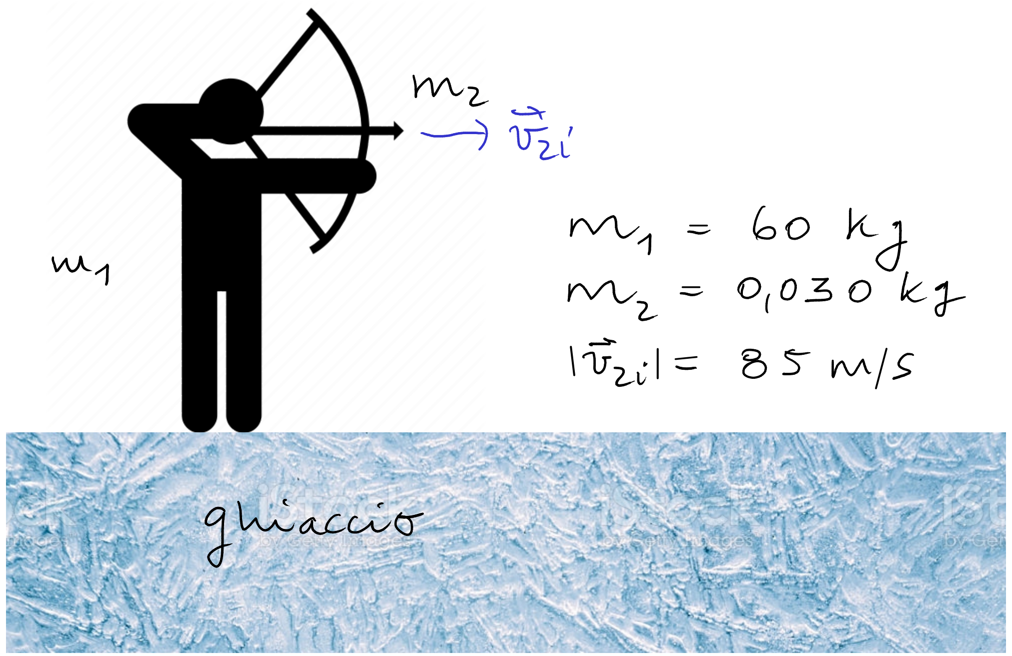
$$\frac{1}{2} m |\vec{v}_i|^2 + mg(d + \Delta x)\sin\theta = 0 + \frac{1}{2} k \Delta x^2$$

$$\frac{1}{2} k \Delta x^2 - mg\sin\theta\, \Delta x - \left(\frac{1}{2} m |\vec{v}_i|^2 + mgd\sin\theta\right) = 0$$

$$ax^2 + bx + c = 0 \qquad x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta x = \frac{mg\sin\theta \pm \sqrt{(mg\sin\theta)^2 + 2k\left(\frac{1}{2} m |\vec{v}_i|^2 + mgd\sin\theta\right)}}{k} > 0$$

QUANTITÀ DI MOTO



$m_1 = 60$ kg
$m_2 = 0{,}030$ kg
$|\vec{v}_{2i}| = 85$ m/s

sistema = { arciere, arco, freccia }

$m_1 \to$ arciere + arco
$m_2 \to$ freccia

II Newton: $\sum \vec{F}_1 = m_1 \vec{a}_1$

$\sum \vec{F}_2 = m_2 \vec{a}_2$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = \sum_{i=1}^{2} \left(\sum \vec{F}_i \right) = \sum \vec{F}_{int} + \sum \vec{F}_{est}$$
$$= \vec{0} + \vec{0}$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{0}$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = \vec{0}$$

$$\frac{d}{dt}(m_1 \vec{v}_1) + \frac{d}{dt}(m_2 \vec{v}_2) = \vec{0}$$

$$\frac{d}{dt}\left(m_1 \vec{v}_1 + m_2 \vec{v}_2 \right) = \vec{0}$$

Quantità di moto di una particella

$$\vec{p} = m\vec{v}$$
$\uparrow$
vettore

$[\vec{p}] = [m][|\vec{v}|] = \frac{ML}{T}$ SI: $\frac{kg\,m}{s}$

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$$E_{c1} = E_{c2} \qquad \cancel{\frac{1}{2}} m_1 |\vec{v}_1|^2 = \cancel{\frac{1}{2}} m_2 |\vec{v}_2|^2 \qquad |\vec{p}| = m |\vec{v}|$$

$$|\vec{p}_1| \equiv p_1 \qquad m_1 |\vec{v}_1| \cdot |\vec{v}_1| = m_2 |\vec{v}_2| \cdot |\vec{v}_2|$$

$$|\vec{p}_2| \equiv p_2 \qquad p_1 \cdot |\vec{v}_1| = p_2 \cdot |\vec{v}_2|$$

$$p_1 = \frac{|\vec{v}_2|}{|\vec{v}_1|} p_2 \quad \Rightarrow \quad 4 \quad V$$