$$\frac{1}{\rho}\frac{d\rho}{dt} = -\nabla\cdot\bar{u} \qquad \rho = \rho_0 + \rho'(x,y,z,t)$$

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \bar{u}\cdot\nabla\rho$$

$$\frac{\partial\rho}{\partial t} + \bar{u}\cdot\nabla\rho = -\rho\,\nabla\cdot\bar{u}$$

$$\underbrace{\frac{\partial\rho'}{\partial t} + \bar{u}\cdot\nabla\rho'}_{\frac{d\rho'}{dt}} = -(\rho_0 + \cancel{\rho'})\,\nabla\cdot\bar{u} = -\rho_0\,\nabla\cdot\bar{u}$$

$$\rho = \rho_0(1 - \alpha\Delta T + \beta\cancel{\Delta S})$$

$$\rho = \rho_0 - \underbrace{\alpha\rho_0\Delta T}_{\rho'} \qquad \alpha \sim 10^{-4}$$

$$\tau_{ij} = \mu(\partial_i u_j + \partial_j u_i)$$

$$x: \quad \dots \frac{\partial \tau^{xx}}{\partial x} + \frac{\partial \tau^{xy}}{\partial y} + \frac{\partial \tau^{xz}}{\partial z}$$

$$\dots \frac{\partial}{\partial x}\left(\mu(\underline{\partial_x u} + \partial_x u)\right) + \frac{\partial}{\partial y}\left(\mu(\partial_x v + \underline{\partial_y u})\right)$$

$$+ \frac{\partial}{\partial z}\left(\mu(\partial_x w + \underline{\partial_z u})\right)$$

$\nabla \cdot \vec{u} = 0$

$$\dots \mu\left[\partial_x^2 u + \partial_y^2 u + \partial_z^2 u + \cancel{\partial_x(\partial_x u + \partial_y v + \partial_z w)}\right)$$

$$\dots \mu \nabla^2 u$$

$$\underline{p_0}(t) = P_0 - \rho_0 g z \qquad \boxed{p = p_0 + p' \qquad \rho = \rho_0 + \rho'}$$

$$\rho\left(\frac{dw}{dt} - f_* u\right) = -\frac{\partial p}{\partial z} + \mu \nabla^2 w - \rho g$$

$$(\rho_0 + \cancel{\rho'})(\quad '' \quad) = -\frac{\partial}{\partial z}(p_0 + p') + \mu \nabla^2 w - (\rho_0 + \rho') g$$

$$'' \quad = -\underbrace{\frac{dp_0}{dz}} - \frac{\partial p'}{\partial z} + \mu \nabla^2 w - \rho_0 g - \rho' g$$

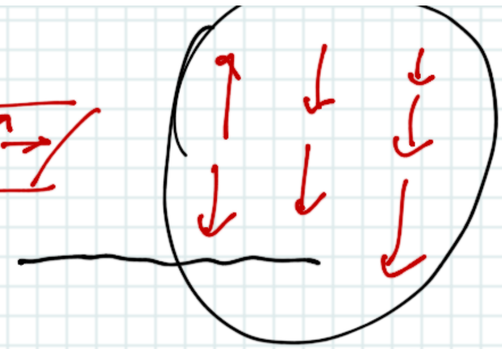
$$\frac{\cancel{\rho_0}(\quad '' \quad)}{\cancel{\rho_0}} = \cancel{\rho_0 g} - \frac{\partial p'}{\partial z} + \mu \nabla^2 w \cancel{- \rho_0 g} - \rho' g$$

$$= -\frac{1}{\rho_0}\partial_z p' + \nu \nabla^2 w - \frac{\rho' g}{\rho_0}$$

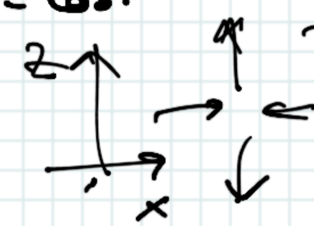
$\nabla \cdot \bar{u} = 0$

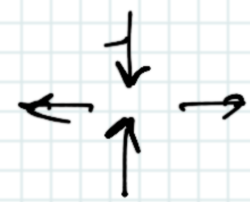
$$\underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}} + \overbrace{\frac{\partial w}{\partial t}} = 0$$

$$\frac{U}{L} \quad \frac{U}{L} \quad \frac{W}{H}$$



$\frac{W}{H} \gg \frac{U}{L}$ $\partial_z w = 0$ $w = \text{cost}$

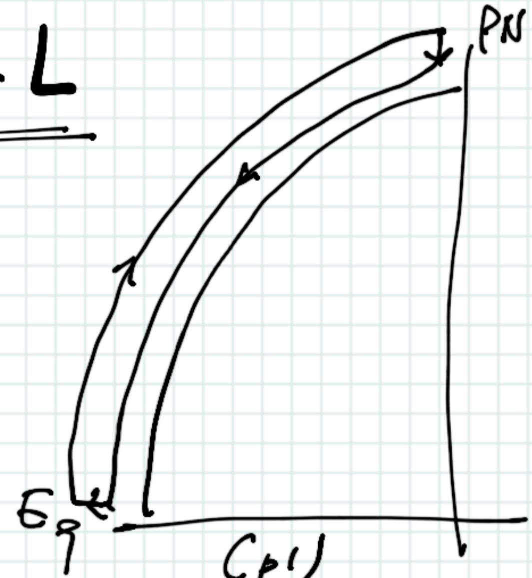




$\frac{W}{H} \ll \frac{U}{L}$ $\partial_x u + \partial_y v = 0$

$\frac{W}{H} \sim \frac{U}{L}$ $\partial_x u + \partial_y v + \partial_z w = 0$ $\frac{W}{H} \lesssim \frac{U}{L}$

$T \gtrsim \frac{1}{\Omega} \qquad \frac{U}{L} \lesssim \Omega \qquad \underline{\underline{H \ll L}}$



$$W \lesssim \frac{U}{L} H \quad \Rightarrow \quad \underline{\underline{W \ll U}}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + f_* w - f v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu \nabla^2 u$$

$$\frac{U}{T} \quad \frac{U^2}{L} \quad \frac{U^2}{L} \quad \frac{WU}{H} \qquad \cancel{\Omega W} \quad \Omega U \qquad \frac{P}{\rho_0 L} \quad \cancel{\frac{\nu U}{L^2}} \quad \frac{\nu U}{H^2}$$

$$\frac{\partial u}{\partial t} + \overline{u} \cdot \nabla u - f v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu \left[\frac{\partial^2 u}{\partial z^2}\right] \qquad (v)$$

$f = 2\Omega \sin\varphi$.

$f_* = 2\Omega \cos\varphi$ ..

$\uparrow$

$\nu_T \quad \nu_E \gg \nu$

$K_T \quad \nu_E \gg K$

$A_H \gg A_V$

$10^4 \, m^2/s \quad 10^{-4} \, m^2/s$

$A_H \frac{U}{L^2} \sim A_V \frac{U}{H^2}$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial t} - f_* u = -\frac{1}{\rho_0}\frac{\partial p}{\partial t} - g\frac{\rho}{\rho_0}$$

$$\cancel{\frac{W}{T}} \quad \cancel{\frac{UW}{L}} \quad \cancel{\frac{UW}{L}} \quad \cancel{\frac{W^2}{H}} \quad \cancel{\Omega U} \qquad + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$$= \frac{P}{\rho_0 H} \quad \frac{g\Delta\rho}{\rho_0} \qquad \cancel{\frac{\nu W}{L^2}} \quad \cancel{\frac{\nu W}{L^2}} \quad \cancel{\frac{\nu W}{H^2}}$$

$T \gtrsim \frac{1}{\Omega}$

$\frac{U}{L} \lesssim \Omega$

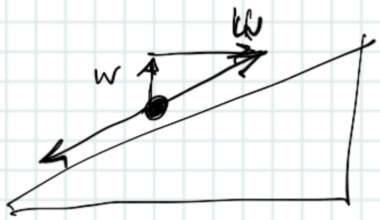
$H \ll L$

$W \ll U$

$U \gg W \qquad \Omega U \gg \Omega W \gtrsim \frac{W}{T}$

$\frac{UW}{L} \lesssim \Omega W \ll \Omega U$

$W \lesssim \frac{H}{L} U \qquad \frac{W^2}{H} \lesssim \frac{WU}{L} \ll \Omega U$



Tab. 3.1

$$\frac{\Omega U}{g\Delta\rho/\rho_0} \ll 1 \quad (<1)$$

$$\sim 10^{-3} \quad 10^{-2}$$

OC ATM

$L \gg H \qquad \underline{\underline{\frac{\nu W}{H^2}}} \gg \frac{\nu W}{L^2}$

$$\boxed{0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{g\rho}{\rho_0}}$$

$$\frac{\nu W}{H^2} \lesssim \Omega U \ll \frac{g\Delta\rho}{\rho_0}$$

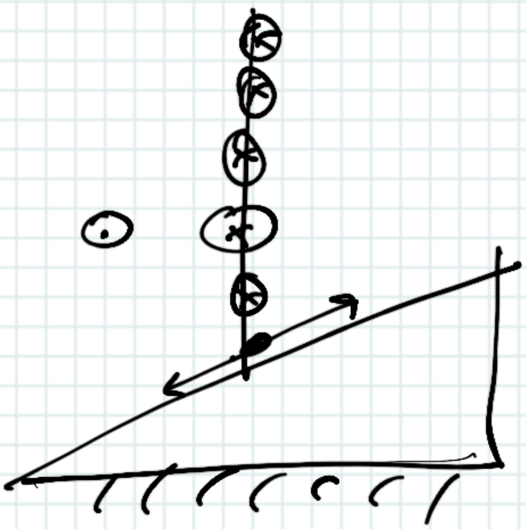
$$\Updownarrow$$

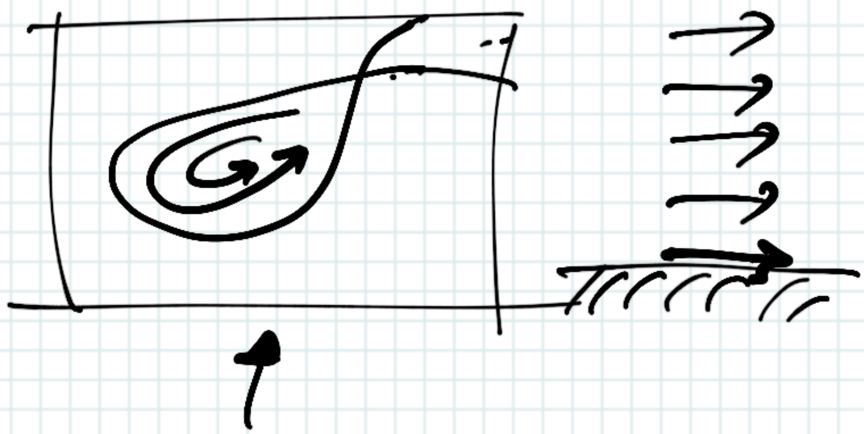
$$\frac{dp_0}{dz} = -g\underline{\underline{\rho_0}}$$

$\rho_0 \gg \rho'$

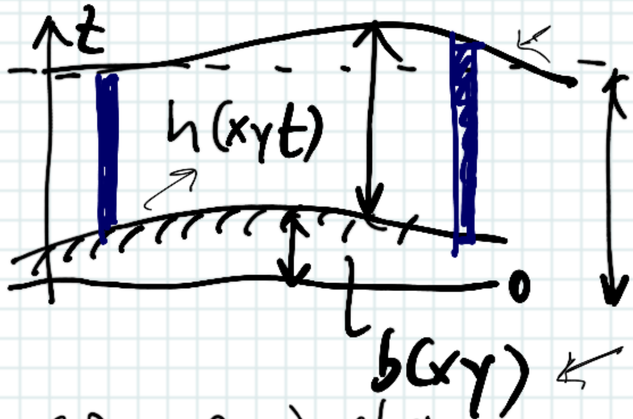
$$\frac{d\rho}{dt} = K \nabla^2 \rho = K \left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2} \right)$$

$$\cancel{\frac{\Delta \rho}{L^2}} \quad \cancel{\frac{\Delta \rho}{L^2}} \quad \frac{\Delta \rho}{H^2}$$





$\exists\, u_H(z=0) \neq 0$



$$\updownarrow \quad \eta = b + h - H$$

$$\int_b^{b+h} \underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}} + \frac{\partial w}{\partial z}\, dz = 0$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \int_b^{b+h} dz + \int_b^{b+h} \frac{\partial w}{\partial z}\, dz = 0$$

$$\left[\partial_x u + \partial_y v\right] (\cancel{b} + h - \cancel{b}) + w\big|_{b+h} - w\big|_b = 0 \qquad (*)$$

$$w(z = b+h) = \frac{dz}{dt}\Big|_{b+h} = \frac{\partial z}{\partial t}\Big|_{b+h} + u\frac{\partial z}{\partial x}\Big|_{b+h} + v\frac{\partial z}{\partial y}\Big|_{b+h} + w\frac{\partial z}{\partial z}\Big|_{b+h}$$

$$= \frac{\partial h}{\partial t} + u\frac{\partial}{\partial x}(b+h) + v\frac{\partial}{\partial y}(b+h)$$

$$w(z=b) = u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y}$$

$$h(\partial_x u + \partial_y v) + \partial_t h + \cancel{u\partial_x b} + \cancel{v\partial_y b} + u\partial_x h + v\partial_y h \quad (*)$$
$$- \cancel{u\partial_x b} - \cancel{v\partial_y b} = 0$$

$$\underline{\partial_t h} + h\,\partial_x u + h\,\partial_y v + u\,\underline{\partial_x h} + v\,\underline{\partial_y h} = 0$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0$$

$$\frac{dh}{dt} + h\,\nabla\cdot\bar{u}_H = 0 \qquad \boxed{\frac{1}{h}\frac{dh}{dt} = -\nabla\cdot\bar{u}_H}$$

DIV:
$\frac{1}{h}\frac{dh}{dt} < 0$

CON:
$\frac{1}{h}\frac{dh}{dt} > 0$

$$p = p_0(z) + p'(xyzt) \qquad p_0(z) = P_0 - \rho_0 g z$$

$$p' = p - p_0(z) = p - P_0 + \rho_0 g z$$

$$\underline{z = b+h} \qquad P_0 = p \quad \Rightarrow \quad p' = \rho_0 g z = \rho_0 g (b+h)$$

$$p' = p'(xy\underset{\substack{\uparrow \\ t}}{\,}) = \rho_0 g \left(b(xy) + h(xyt) \right) \qquad \partial_z p' = 0$$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial x} = -\frac{1}{\cancel{\rho_0}}\frac{\partial}{\partial x}\cancel{\rho_0} g(b+h) = -g\,\partial_x(b+h)$$

$$-\frac{1}{\rho_0}\frac{\partial p}{\partial y} = -\frac{1}{\cancel{\rho_0}}\frac{\partial}{\partial y}\cancel{\rho_0} g(b+h) = -g\,\partial_y(b+h)$$

$$\partial_t u + u\,\partial_x u + v\,\partial_y u - f v = -g\,\partial_x h \qquad (1)$$

$$\partial_t v + u\,\partial_x v + v\,\partial_y v + f u = -g\,\partial_y h \qquad (2)$$

$$-\frac{\partial}{\partial y}(1) + \frac{\partial}{\partial x}(2) =$$

$$-\Big[\partial_{yt} u + \partial_y(u\,\partial_x u) + \partial_y(v\,\partial_y u) - \partial_y(f v) = -g\,\partial_{xy} h\Big]$$

$$\Big[\partial_{xt} v + \partial_x(u\,\partial_x v) + \partial_x(v\,\partial_y v) + \partial_x(f u) = -g\,\partial_{xy} h\Big]$$

$$\frac{d}{dt}\left(f + \partial_x v - \partial_y u\right) + \left(\partial_x u + \partial_y v\right)\left(f + \partial_x v - \partial_y u\right) = 0$$

$\uparrow$

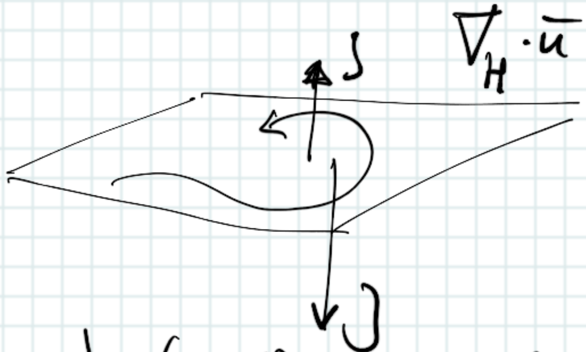
$(u\,\partial_x f)$

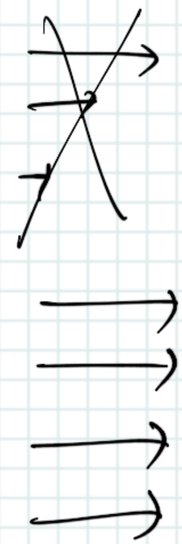
$\zeta := \partial_x v - \partial_y u$ VORTICITÀ RELATIVA $= \nabla \times \bar{u}\big|_z$

f: V. AMBIENTALE

$$\frac{d}{dt}(f+\zeta) + \underbrace{(\partial_x u + \partial_y v)}_{\nabla_H \cdot \bar{u}}(f+\zeta) = 0 \qquad \zeta \parallel \hat{z}$$

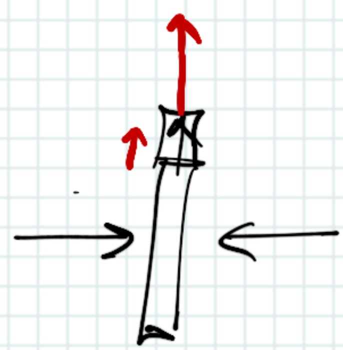
$$(\nabla \cdot \bar{u}_H) \qquad \partial_z \begin{pmatrix} u \\ v \end{pmatrix} = 0$$





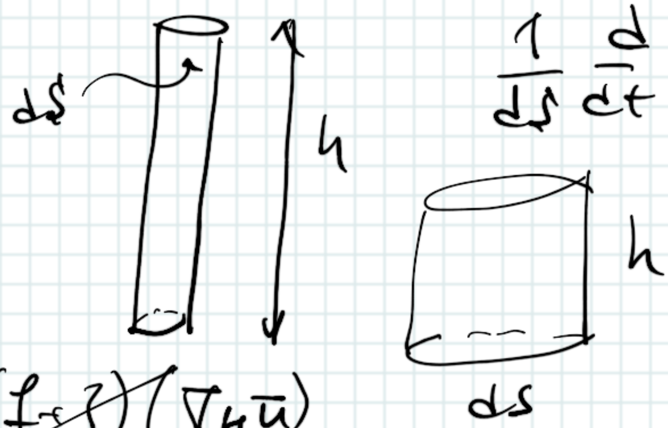
$$\frac{1}{f+\zeta}\frac{d}{dt}(f+\zeta) = -\nabla_H \cdot \bar{u}$$

$$\frac{1}{h}\frac{dh}{dt} = -\nabla_H \cdot \bar{u}$$



$$\frac{d}{dt}(h\,d\Sigma)=0$$

$$\frac{1}{d\Sigma}\frac{d}{dt}d\Sigma = \nabla_H \bar{u}$$

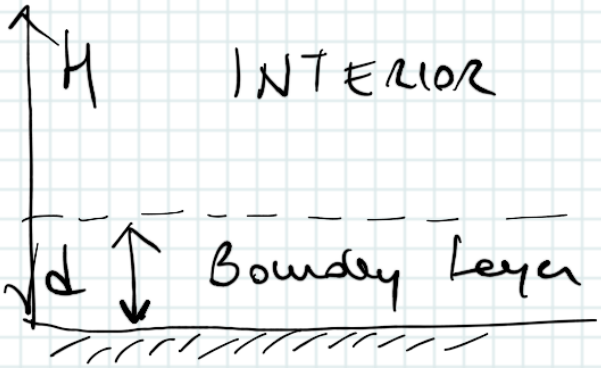


$$d\Sigma\,\frac{d}{dt}(f+\zeta) + d\Sigma\,\cancel{(f+\zeta)}(\nabla_H \bar{u})$$

$$+\ (f+\zeta)\frac{d}{dt}d\Sigma + \cancel{(f+\zeta)}(\nabla_H u)\,d\Sigma = 0$$

$$\frac{d}{dt}\Big[\underbrace{(f+\zeta)\,d\Sigma}_{\text{flusso di vorticità}}\Big]=0$$

Th. KELVIN



$\dot{E}k \ll 1$

$Ek \sim 1$

$$Ek = \frac{\nu_E}{\Omega H^2}$$

$$Ek \sim 1 \Rightarrow Ek = \frac{\nu_E}{\Omega d^2} \sim 1 \Rightarrow d \sim \sqrt{\frac{\nu_E}{\Omega}}$$

$\nu_E \sim 10^{-2} m^2/s$

$$d \sim \sqrt{\frac{10^{-2}}{10^{-4}}} \sim 10 m$$

$\Omega \sim 10^{-4} s^{-1}$

$H \sim 10^3 m$

$$Ek \sim \frac{10^{-2}}{10^{-4}(10^3)^2} = 10^{-4} \ll 1$$

$d \ll H$

$\delta \propto \sqrt{x}$

$\Omega \to 0 \quad d \to \infty$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \int_0^\infty \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dt = -\frac{d}{2} \left(\cancel{\frac{\partial \bar{u}}{\partial x}} + \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} + \cancel{\frac{\partial \bar{v}}{\partial y}} \right)$$

$$= -\frac{d}{2} \bar{\zeta}$$

$$U = -\frac{d}{2} (\bar{u} + \bar{v})$$

$$V = \frac{d}{2} (\bar{u} - \bar{v})$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$

$$\partial_z w = -(\partial_x u + \partial_y v) \qquad \int_0^\infty dt$$

$$W|_\infty - W|_0 = -\int_0^\infty \partial_x u + \partial_y v \, dt$$

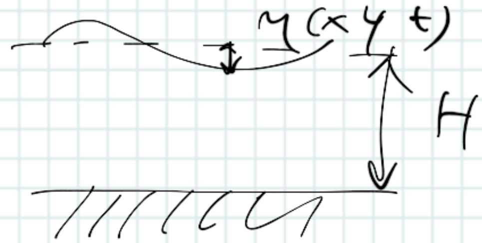
$$\bar{W} = \frac{d}{2} \bar{\zeta}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} > 0$$

$$\bar{\zeta} < 0$$

$H(x,y) = H = \text{cst}$

$\boxed{\eta \sim \Delta H}$



$\partial_t h + \partial_x (hu) + \partial_y (hv) = 0$

$C = \frac{L}{T} \gg U$

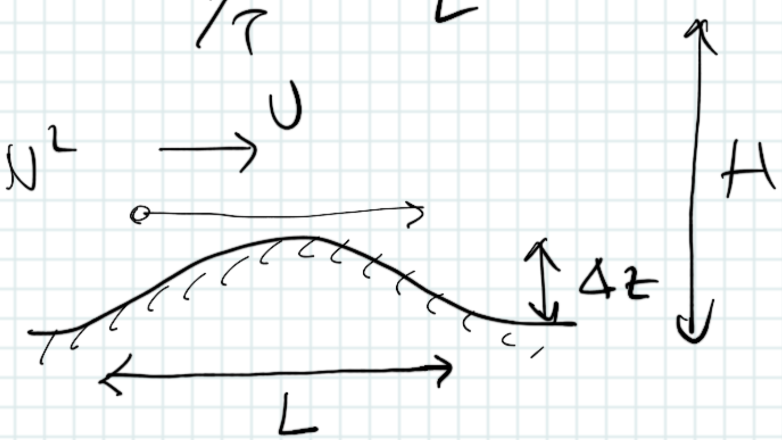
$h = H + \eta(xyt)$

$\partial_t \eta + \partial_x ((H+\eta) u) + \partial_y ((H+\eta) v) = 0$ $\qquad \partial_{x,y} H = 0$

$\partial_t \eta + (u \partial_x \eta + \cancel{v \partial_y \eta}) + H (\partial_x u + \partial_y) + \eta (\cancel{\partial_x u + \partial_y v}) = 0$

$\frac{\Delta H}{T} \qquad \frac{U \Delta H}{L} \qquad \frac{HU}{L} \qquad \frac{\Delta H\, U}{L}$

$\frac{\Delta H U / L}{\Delta H / T} \sim \frac{U}{L} T \ll 1$ $\qquad C = \frac{L}{T} \gg U$



$\bar{\rho}(z)$

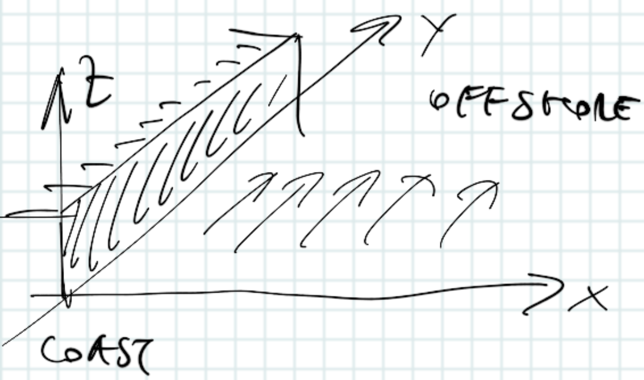
$T = L/U$

$\Delta z = WT = WL$

$$\partial_t \eta + H(\partial_x u + \partial_y v) = 0$$

$$\underbrace{\partial_x u + \partial_y v = 0}_{f\text{-plane}}$$

$$\frac{\Delta H}{T} \sim \frac{HU}{L} \qquad \frac{\Delta H}{H} \sim \frac{UT}{L} \ll 1 \qquad \Delta H \ll H$$



$x = 0 \quad u = 0 \quad v \neq 0$

$\forall x \quad u = 0$

$$\left.\begin{aligned} &\cancel{\partial_t u} - fv = -g\partial_x \eta \Leftarrow \\ &\partial_t v + \cancel{fu} = -g\partial_y \eta \\ &\partial_t \eta + H\partial_y v = 0 \end{aligned}\right\} \begin{matrix} \partial_t \\ g\partial_y \end{matrix}$$

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial y^2} \qquad c := \sqrt{gH}$$

$g = 9.81 m/s^2$

$H = 10^3 m$

SURFACE GRAVITY WAVES

$= \sqrt{10^4} = 10^2 m/s$

$v = V_1 + V_2$

$= V_1(x, y+ct) + V_2(x, y-ct)$

$V_1 = V_{10}(y+ct)\, e^{-x/R}$ //

$V_2 = V_{20}(y-ct)\, e^{+x/R}$

$$R := \frac{\sqrt{gH}}{f} = \frac{c}{f} = \frac{cT}{2\pi}$$

ROSSBY RADIUS OF DEFORMATION

$f = 2\pi/T$

$x \gg R \qquad \lim_{x \to \infty} V_2 = \infty$

$\omega = 0$

$$\omega = \sqrt{f^2 + gH(k_x^2 + k_y^2)}$$

$\frac{m}{s} \cdot \frac{1}{m}$

$$\frac{\omega}{f} = \sqrt{1 + \frac{gH}{f^2}(k_x^2 + k_y^2)}$$

$\frac{gHk}{f} \gg 1 \quad gHk \gg f$

$\frac{\omega}{f} = \frac{\sqrt{gH}}{f} \, k_y$

$\to f = 0 \quad \omega = \sqrt{gH}\, k$

$= ck$

HF $\quad k^2 \gg \frac{f^2}{gH} = 1/R^2 \Rightarrow \lambda \ll R \Rightarrow \omega \sim k\sqrt{gH}$

LF $\quad k^2 \ll \frac{f^2}{gH} = 1/R^2 \Rightarrow \lambda \gg R \Rightarrow \omega \sim f$

$\boxed{f \leq \omega \leq k\sqrt{gH}}$

$$c(k) = \frac{\omega}{k} = \frac{\sqrt{f^2 + gHk^2}}{k} \qquad \text{DISP}$$

$$c = \sqrt{gH} \qquad \omega = \sqrt{gHk^2} \qquad \text{NON. DISP}$$

TSUNAMI $\quad c = \sqrt{gH} \quad \omega \gg f$

$$\overset{S}{\partial_t} u - (\overset{L}{f_0} + \overset{S}{\beta_0 y})v = -g\overset{L}{\partial_x}\eta \qquad Ro_T \sim 1$$

$$\partial_t v + (f + \beta_0 y)u = -g\partial_y \eta \qquad Ro_T \ll 1$$

$$\underset{S}{\partial_t}\eta + H(\overset{L}{\partial_x} u + \partial_y v) = 0 \qquad \omega \ll f$$

$$\rho(z)\, V \frac{d^2h}{dt^2} = g\left[\rho(z+h) - \rho(z)\right] V \qquad h(t) = ?$$

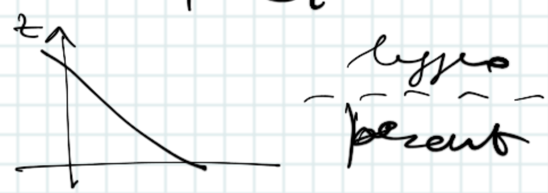
$$\boxed{\begin{array}{l}\rho = \rho_0 + \rho' \\ \rho' \ll \rho_0\end{array}}$$

$$\rho(z) V \frac{d^2h}{dt^2} = \rho_0 V \frac{d^2h}{dt^2}$$

$$\rho(z+h) \doteq \rho(z) + \frac{d\rho}{dz} h$$

$$\rho_0 \cancel{V} \ddot{h} = g \frac{d\rho}{dz} h \cancel{V} \quad \Rightarrow \quad \ddot{h} - \frac{g}{\rho_0}\frac{d\rho}{dz} h = 0$$

$$-\frac{g}{\rho_0}\frac{d\rho}{dz} > 0 \qquad \frac{d\rho}{dz} < 0$$

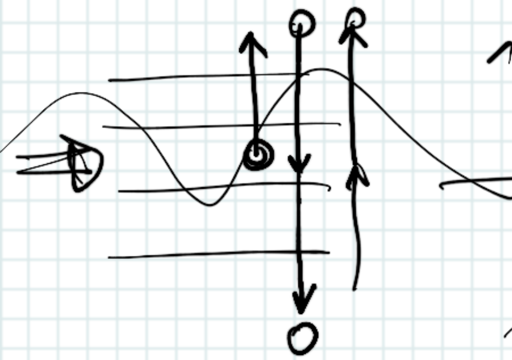


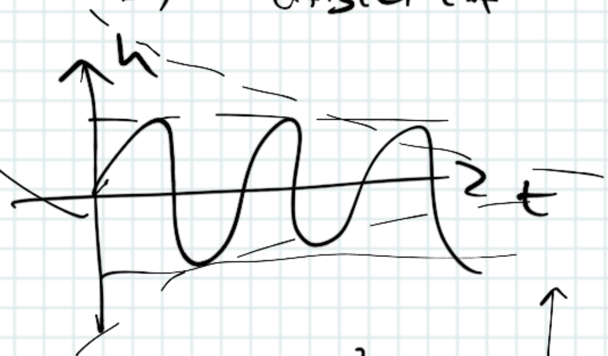
$$\frac{\cancel{m}}{s^2} \cdot \frac{\cancel{m^3}}{\cancel{kg}} \frac{\cancel{kg}}{\cancel{m^3}} \frac{1}{\cancel{m}} \qquad [N^2] = s^{-2}$$

BRUNT-VAISALA

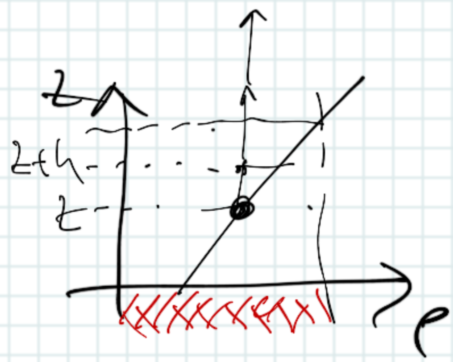
$\ddot{h} + N^2 h = 0$ $\qquad h(t) = h_0 e^{iNt}$ $\qquad$ sin / cos

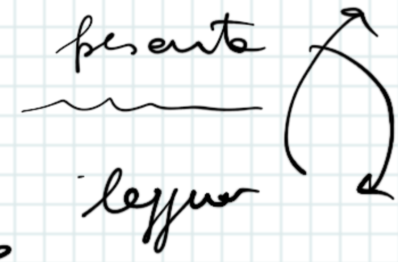
OSCILLAZIONI $\Rightarrow$ STABILITA'





$-\frac{g}{\rho_0}\frac{d\rho}{dz} < 0 \qquad \frac{d\rho}{dz} > 0$





$\ddot{h} - D^2 h = 0 \qquad D^2 = \frac{g}{\rho_0}\frac{d\rho}{dz} \qquad h(t) = h_0 e^{Dt}$

INSTABILITA'

$T = L/U$ ×

$\Delta t = WT = WL/U$ ×

$\Delta\rho = \left|\frac{d\bar{\rho}}{dt}\right|\Delta t = \frac{\rho_0 N^2}{g}\frac{WL}{U}$ ×

$N^2 = -\frac{d\bar{\rho}}{dt}\frac{g}{\rho_0}$

$0 = -\partial_z p - \rho g$

$p \sim \rho g h$

$P = gH\Delta\rho = \cancel{g}H \cdot \frac{\rho_0 N^2}{\cancel{g}}\frac{WL}{U}$ ×

$u\,\partial_x u \sim \frac{1}{\rho_0}\partial_x p$

$\frac{U^2}{L} \sim \frac{P}{\rho_0 L}$

$U^2 \sim \frac{P}{\rho_0} = \frac{H\cancel{\rho_0}N^2}{\cancel{\rho_0}}\frac{WL}{U} = N^2 H\frac{WL}{U}$

$$U^2 = N^2 H L \frac{W}{U} \quad \Rightarrow \quad \frac{W}{H} = \frac{U^2}{N^2H^2} \cdot \frac{U}{L}$$

$$\frac{W/H}{U/L} = \frac{\text{conv./div. vert.}}{\text{div./conv. horiz.}} = \frac{U^2}{N^2H^2}$$

$$\Delta t = WT = W\frac{L}{U} \quad \Rightarrow \quad \frac{\Delta t}{H} = \frac{W/H}{U/L} = \frac{U^2}{N^2H^2}$$

$U < NH$ $\quad (\Delta t < H) \quad$ $W/H < U/L$

$\underline{\partial_z w} < \partial_x u + \partial_y v$

$\partial_z w \sim Fr^2 \, \partial_x u$

$U \ll NH$

$\partial_x u \sim \partial_y v$

$$\underset{\text{FROUDE}}{Fr} = \frac{U}{NH} \qquad Ro = \frac{U}{\Omega L}$$

$Ro \ll 1$

$Fr \ll 1$

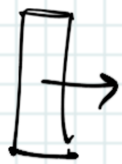
$Ro \ll 1 \qquad \partial_x u + \partial_y v = 0 \qquad \partial_z w = 0$

$$\partial_x u + \partial_y v \sim Ro \frac{U}{L} \sim \frac{W}{H} \Rightarrow Ro = \frac{W/H}{U/L}$$

$$\partial_x u + \partial_y v = -\partial_z W$$

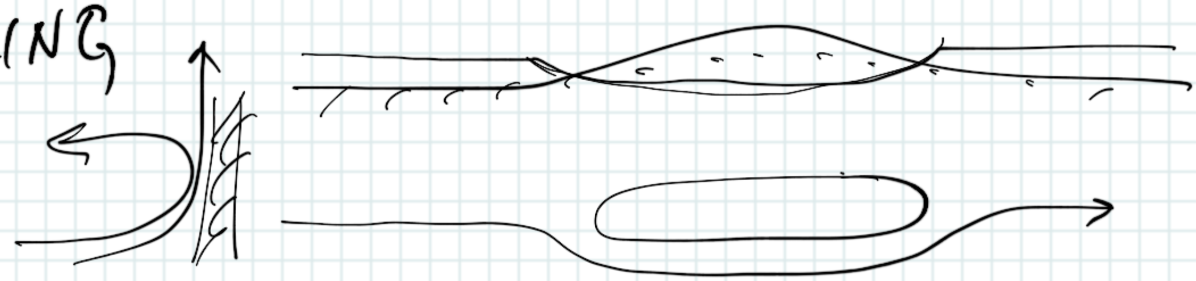
$$\partial_z W \sim Ro\,(\partial_x u + \partial_y v)$$

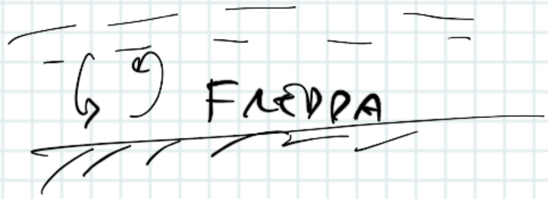
Taylor column



Horiz.

BLOCKING





$Fr \ll 1$

$$\frac{W/H}{U/L} = \frac{\cancel{P}U}{\cancel{\rho_0}N^2H\cancel{L}} \cdot \frac{\cancel{L}}{H} \frac{\Omega \cancel{\rho_0} L}{\cancel{P}} \frac{U}{U} = \frac{U^2}{N^2H^2}\frac{\Omega L}{U} = \frac{Fr^2}{Ro}$$

$$P = \rho_0 \frac{N^2HLW}{U} \qquad \Omega U = \frac{P}{\rho_0 L}$$

$$\partial_z W \sim \frac{Fr^2}{Ro}(\partial_x u + \partial_y v)$$

$\partial_z w \lesssim \partial_x u + \partial_y v$ $\qquad W/H \lesssim U/L$ $\qquad Fr \lesssim 1$

$$\frac{W/H}{U/L} = \frac{Fr^2}{Ro} \Rightarrow Fr^2 \lesssim Ro \qquad \frac{U^2}{N^2H^2} \lesssim \frac{U}{\Omega L}$$

$$\boxed{\frac{U}{NH} \lesssim \frac{NH}{\Omega L}} \qquad U \lesssim \frac{N^2H^2}{\Omega L} \qquad L \lesssim \frac{N^2H^2}{\Omega U}$$

$$H \gtrsim \frac{\sqrt{U\Omega L}}{N}$$

OC ~ 50 km

ATM ~ 500 km

$H \ll L$

$\Omega \ll N$

$$Bu = \left(\frac{NH}{\Omega L}\right)^2 = \left(\frac{Ro}{Fr}\right)^2 \qquad Bu \sim 1 \Longrightarrow L = \frac{NH}{\Omega}$$

BURGER

$$\frac{W/H}{U/L} = \frac{Fr^2}{R_o},\ Fr^2,\ R_i$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial u'}{\partial t} + (\bar{u}+u')\frac{\partial(\cancel{\bar{u}}+u')}{\partial x} + w'\frac{\partial(\bar{u}+u')}{\partial z} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

$$\partial_t u' + \bar{u}\,\partial_x u' + \cancel{u'\partial_x u'} + w'\frac{d\bar{u}}{dz} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

$$\frac{\partial w'}{\partial t} + \bar{u}\frac{\partial w'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\rho' g}{\rho_0}$$

$u = \bar{u} + u'$ $\quad \bar{u}(z)$

$w = w'$

$p = \bar{p} + p'$ $\quad \bar{p}(z)$

$\rho = \bar{\rho} + \rho'$ $\quad \bar{\rho}(z)$

$u', w, p', \rho'\ (x\,z\,t)$

$$\frac{d\bar{p}}{dz} = -g\bar{\rho}(z)$$