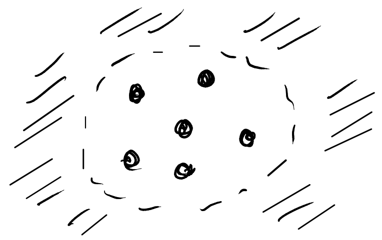
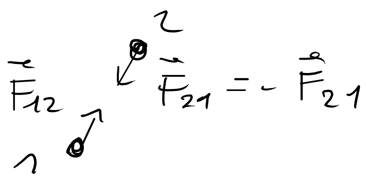
Moto di un sistema di particelle



N particelle



II Newton : $\sum \vec F_i = m_i \vec a_i$ $\qquad i = 1, \dots, N$

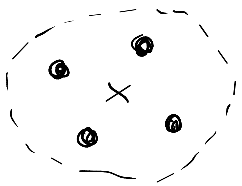
$$\underbrace{\sum_{i=1}^{N} \left(\sum \vec F_i \right)} = \sum_{i=1}^{N} m_i \frac{d\vec v_i}{dt} = \frac{d}{dt}\Big(\underbrace{\sum_{i=1}^{N} m_i \vec v_i}_{\vec p} \Big)$$

$$\underbrace{\sum \vec F_{int}}_{= \vec 0} + \sum \vec F_{est} = \frac{d\vec p}{dt}$$

$= \vec 0$ III Newton

$\vec p$ del sistema

$$\boxed{\sum \vec F_{est} = \frac{d\vec p}{dt}}$$

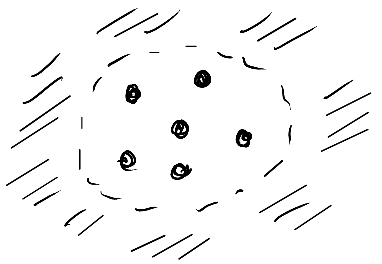


massa totale : $M = \sum_{i=1}^{N} m_i$ $\qquad$ centro di massa : $\vec r_{CM} \equiv \frac{1}{M} \sum_{i=1}^{N} m_i \vec r_i$

$$\vec p = \sum_{i=1}^{N} m_i \vec v_i = M \cdot \frac{1}{M} \sum_{i=1}^{N} m_i \vec v_i = M \left[\frac{d}{dt} \left(\frac{1}{M} \sum_{i=1}^{N} m_i \vec r_i \right) \right] = M \vec v_{CM} = M \frac{d\vec r_{CM}}{dt}$$

$$\sum \vec{F}_{est} = \frac{d}{dt}\left(\sum_{i=1}^{N} m_i \vec{v}_i \right) = M \frac{d}{dt}\left[\frac{1}{M} \sum_{i=1}^{N} m_i \vec{v}_i \right] = M \frac{d\vec{v}_{CM}}{dt} = M \frac{d^2 \vec{r}_{CM}}{dt^2}$$

Leggi di conservazione



1) **Sistema isolato** (non interagisce con l'ambiente)

$\Rightarrow \sum \vec{F}_{est} = \vec{0}$

a) forze interne conservative $\Rightarrow \Delta E = 0 \quad (E_i = E_f)$

b) III Newton $\sum \vec{F}_{int} = \vec{0} \Rightarrow \Delta \vec{p} = \vec{0} \quad (\vec{p}_i = \vec{p}_f)$

$\uparrow$
sistema

2) **sistema non isolato** ma $\sum \vec{F}_{est} = \vec{0}$ vedi 1)

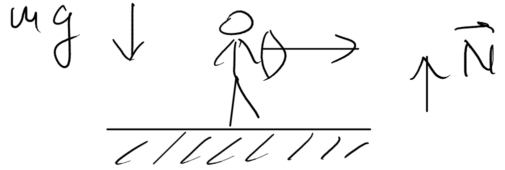
3) Sistema non isolato ma $\sum \vec{F}_{est} \neq \vec{0}$

a) forze esterne cost nel tempo e conservative $\Rightarrow \Delta E = 0$

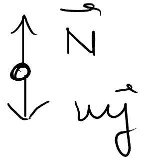
b) componente α di $\vec{p}$ si conserva se la componente α di $\sum \vec{F}_{est}$ è nulla
$(\alpha = x, y, z)$ $\Rightarrow$ $\Delta p_\alpha = 0$

Es.: arciere

sistema = { arciere, arco, freccia }



$\sum \vec{F}_{est} = \vec{0}$



$\Rightarrow \Delta \vec{p} = \vec{0} \Rightarrow \vec{p}_i = \vec{p}_f$

$\longrightarrow \vec{e}_x$

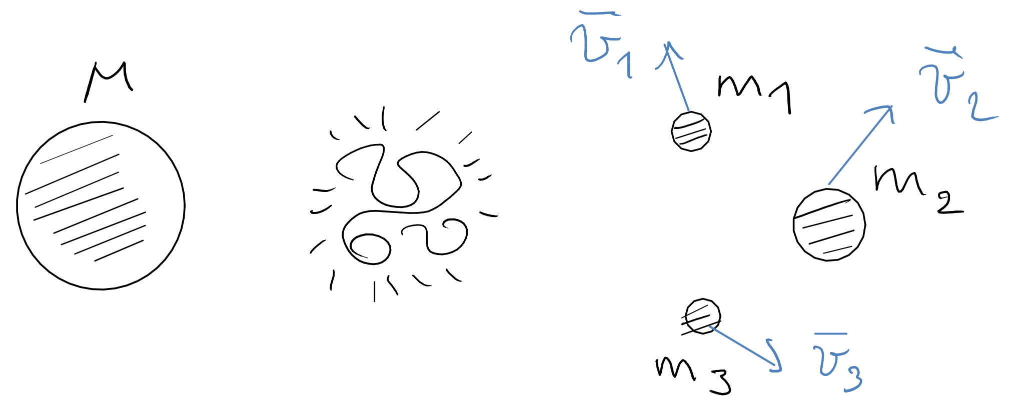
$\begin{cases} m_1 & \text{arciere + arco} \\ m_2 & \text{freccia} \end{cases}$

$$\vec{0} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\vec{v}_{1f} = - \underbrace{\frac{m_2}{m_1}}_{>0} \vec{v}_{2f} = - \frac{0.03\,kg}{60\,kg} \cdot 85 \frac{m}{s} \vec{e}_x$$

$$= -0.042 \frac{m}{s} \vec{e}_x \quad \square$$

Disintegrazione, autopropulsione



iniziale finale

$$\vec{p}_i = \vec{p}_f$$

$$\vec{0} = \sum_{i=1}^{N} m_i \vec{v}_i$$

Urti

iniziale finale

$$\vec{p}_i = \vec{p}_f$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_1 \vec{v}_{2f}$$

Sistema isolato $\Rightarrow \sum \vec{F}_{est} = \vec{0} \Rightarrow \Delta \vec{p} = \vec{0}$ conservazione q. moto

↗ forze interne conservative $\Rightarrow \Delta E = 0$ —//— energia meccanica

↘ forze interne non conservative $\Rightarrow \Delta E < 0$ NO conservazione

Stato iniziale e finale : $E_{pi} = 0$, $E_{pf} = 0$

↗ urti elastici : $\Delta E_c = 0$ conservazione energia cinetica

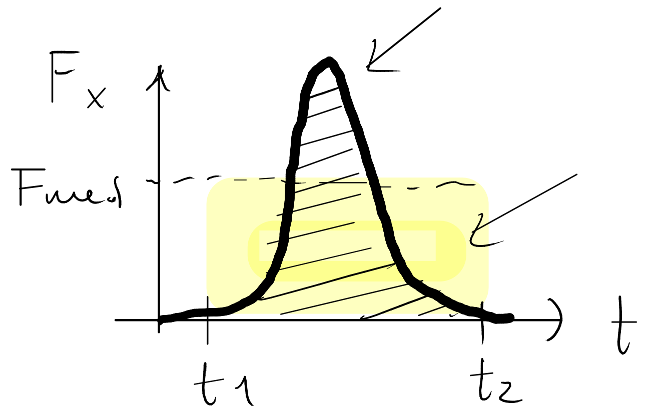
↘ urti anelastico : $\Delta E_c < 0$

$$\Delta E_c = W[\Sigma \vec{F}] \rightarrow dE_c = \delta W[\Sigma \vec{F}] = (\Sigma \vec{F}) \cdot d\vec{r}$$ teor. en. cinetica

$$\Delta \vec{p} = \vec{I}[\Sigma \vec{F}] \rightarrow d\vec{p} = \delta \vec{I}[\Sigma \vec{F}] = (\Sigma \vec{F})\, dt$$

↑ impulso elementare

SI : $|\vec{I}| \rightarrow N \cdot s$



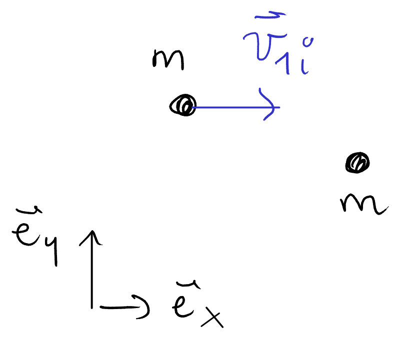
$$I_x = \int_{t_1}^{t_2} F_x\, dt = F_{med}\, \Delta t$$

↑ $\Delta t = t_2 - t_1$

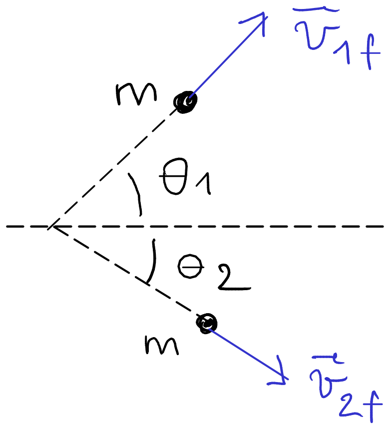
$$\Delta \vec{p} = \vec{I}$$

Es. : airbag

Es.: urto tra 2 protoni



iniziale



finale

isolato

$|\vec{v}_{1i}| = 3.5 \times 10^5$ m/s $\qquad \vec{v}_{2i} = \vec{0}$

$|\vec{v}_{1f}| = 2.8 \times 10^5$ m/s

$\theta_1 = 37°$

$\Rightarrow \theta_2 = ?\ ; \quad |\vec{v}_{2f}| = ? \qquad$ Elastico ?

Isolato $\Rightarrow$ conservazione q. moto : $\vec{p}_i = \vec{p}_f$

$$\cancel{m}\vec{v}_{1i} = \cancel{m}\vec{v}_{1f} + \cancel{m}\vec{v}_{2f}$$

Base cartesiana $\{\vec{e}_x, \vec{e}_y\}$ $\qquad v_{1f} \equiv |\vec{v}_{1f}| \qquad v_{2f} \equiv |\vec{v}_{2f}|$

$$v_{1i}\,\vec{e}_x = v_{1f}\cos\theta_1\,\vec{e}_x + v_{1f}\sin\theta_1\,\vec{e}_y + v_{2f}\cos\theta_2\,\vec{e}_x - v_{2f}\sin\theta_2\,\vec{e}_y$$

$$\begin{cases} v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 \\ 0 = v_{1f}\sin\theta_1 - v_{2f}\sin\theta_2 \end{cases} \Rightarrow \quad v_{2f} = \frac{\sin\theta_1}{\sin\theta_2}\, v_{1f}$$

$$v_{1i} = v_{1f}\cos\theta_1 + \frac{\sin\theta_1}{\tan\theta_2} v_{1f} = v_{1f}\left(\cos\theta_1 + \frac{\sin\theta_1}{\tan\theta_2}\right)$$

$$(v_{1i} - v_{1f}\cos\theta_1) = \frac{\sin\theta_1}{\tan\theta_2} v_{1f}$$

$$\tan\theta_2 = \frac{\sin\theta_1 \cdot v_{1f}}{v_{1i} - v_{1f}\cos\theta_1} \Rightarrow \theta_2 = \arctan\left(\frac{\sin\theta_1 \, v_{1f}}{v_{1i} - v_{1f}\cos\theta_1}\right)$$

$$= \dots = 53^\circ$$

$$v_{2f} = 2.1 \times 10^5 \ \frac{m}{s}$$

$$E_{ci} = \frac{1}{2} m v_{1i}^2$$

$$E_{cf} = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} m v_{2f}^2$$