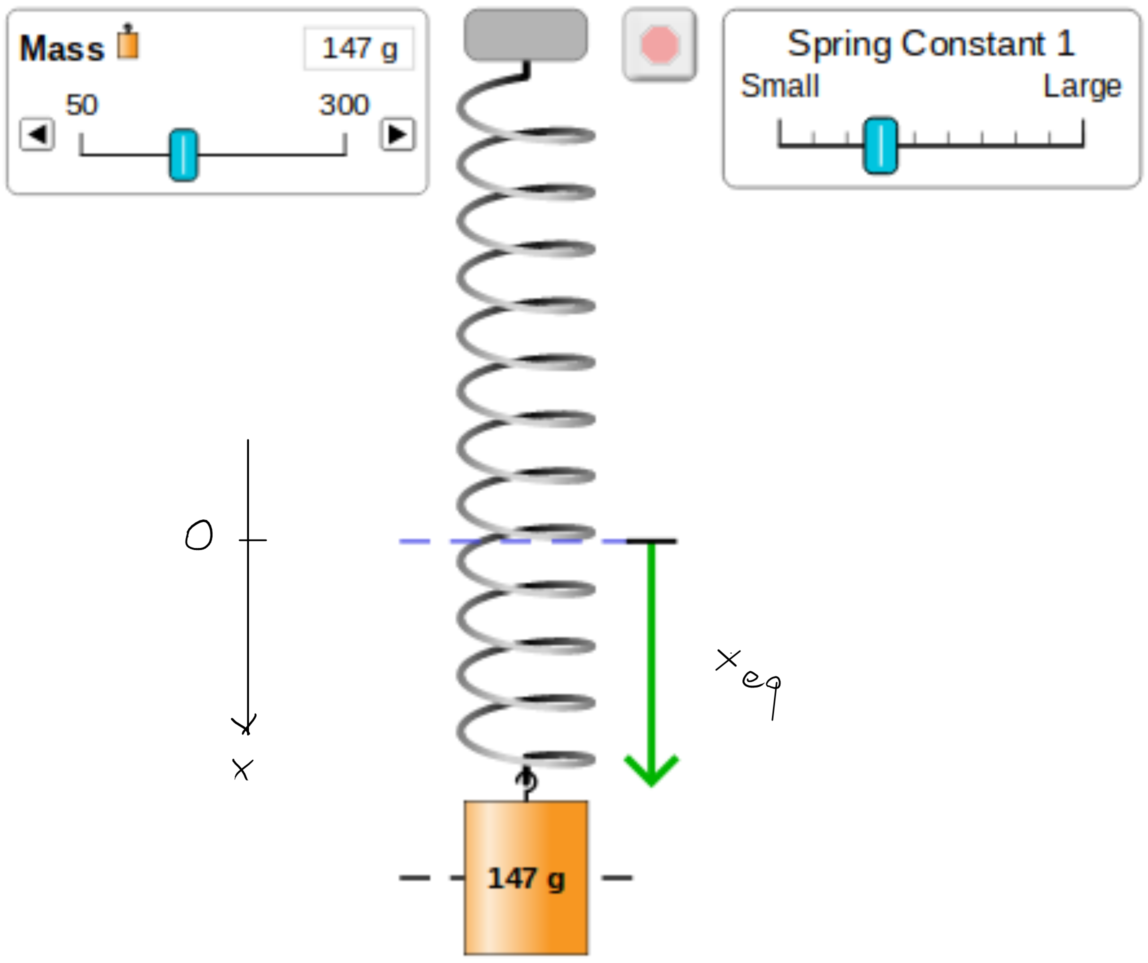
OSCILLAZIONI



costante elastica K ?

$$mg = K x_{eq}$$

$$K = \frac{mg}{x_{eq}}$$

Analisi dimensionale : $\tau \sim \sqrt{\frac{m}{K}}$

$$m,\ K,\ l \rightarrow \tau$$

$$[\tau] = [m]^{\alpha} \cdot [K]^{\beta} [l]^{\gamma}$$

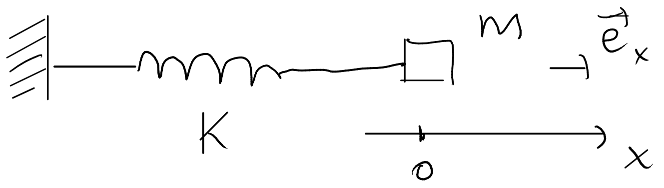
$$T = M^{\alpha} \cdot \left(\frac{M}{T^2}\right)^{\beta} \cdot L^{\gamma}$$

$$T^1 = M^{\alpha+\beta} \cdot T^{-2\beta} \cdot L^{\gamma}$$

$$\begin{cases} \alpha+\beta = 0 \\ -2\beta = 1 \\ \gamma = 0 \end{cases} \Rightarrow \beta = -\frac{1}{2} \Rightarrow \alpha = \frac{1}{2}$$

$$\tau \sim m^{1/2} \cdot K^{-1/2} \sim \left(\frac{m}{K}\right)^{\frac{1}{2}}$$

Oscillatore armonico



II Newton: $m\vec{a} = \sum \vec{F}$

$m\vec{a} = \vec{F}_{el}$ $\qquad m \dfrac{d^2x}{dt^2}\vec{e}_x = -Kx\vec{e}_x$

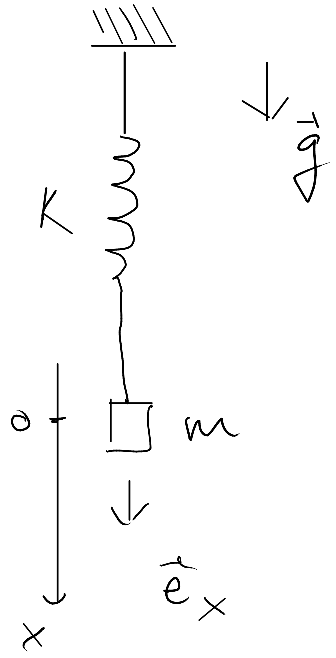
$\Rightarrow \quad m\dfrac{d^2x}{dt^2} = -Kx$

$\dfrac{d^2x}{dt^2} = -\dfrac{K}{m}x$ eq. del moto

$\dfrac{d^2x}{dt^2} = F$ (cost) moto unif. accelerato

$\dfrac{d^2x}{dt^2} = -\dfrac{\mu_d}{m}\dfrac{dx}{dt} \to \dfrac{dv_x}{dt} = -\dfrac{\mu_d}{m}v_x$ moto smorzato (attrito viscoso)

$\dfrac{d^2x}{dt^2} = -\dfrac{K}{m}x$ moto armonico / oscillazioni armoniche



II Newton: $m\vec{a} = \sum \vec{F}$

$$m\frac{d^2x}{dt^2}\,\vec{e}_x = -Kx\,\vec{e}_x + mg\,\vec{e}_x$$

$$\frac{d^2x}{dt^2} = -\frac{K}{m}x + g$$

Posizione di equilibrio: $x_{eq} = \dfrac{mg}{K}$

$$X \equiv x - x_{eq} \Rightarrow x = X + x_{eq}$$

$$\frac{d^2X}{dt^2} = -\frac{K}{m}X - \frac{K}{m}x_{eq} + g = -\frac{K}{m}X \cancel{-g} + \cancel{g} = -\frac{K}{m}X \Rightarrow \frac{d^2X}{dt^2} = -\frac{K}{m}X$$

Soluzione generale

$$\frac{d^2x}{dt^2} = -x$$

$\downarrow$

$$\begin{cases} x(t) = \sin t \\ x(t) = \cos t \end{cases}$$

$$x(t) = A\cos t + B\sin t \rightarrow x(t) = A\cos(t+\phi)$$

condizioni iniziali

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$$\frac{d^2x}{dt^2} = -\frac{K}{m}x \qquad x(t) = A\cos(\omega t + \phi) \qquad \omega^2 = \frac{K}{m}$$

$$\frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 \underbrace{A\cos(\omega t + \phi)}_{x}$$

Soluzione:

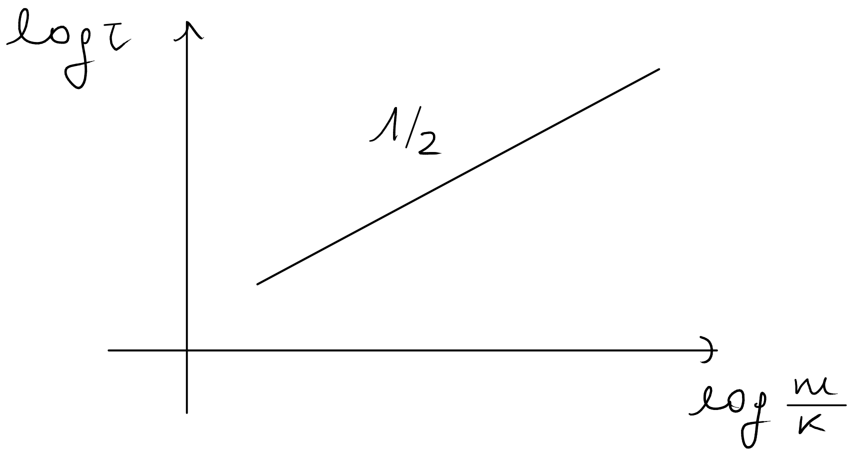
$$x(t) = A\cos\left(\sqrt{\frac{K}{m}}\,t + \phi\right)$$

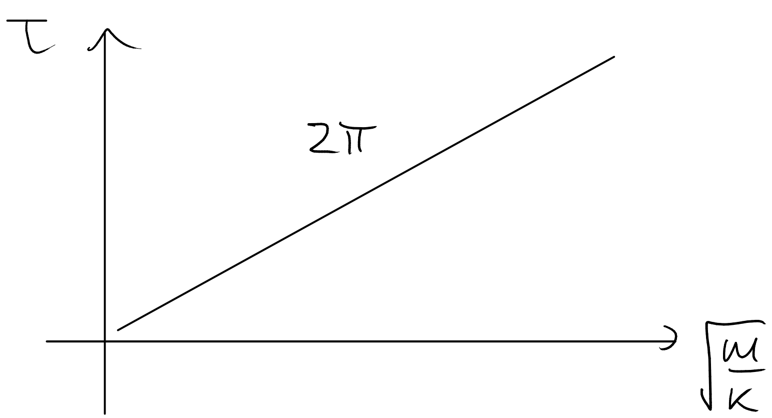
Periodo:

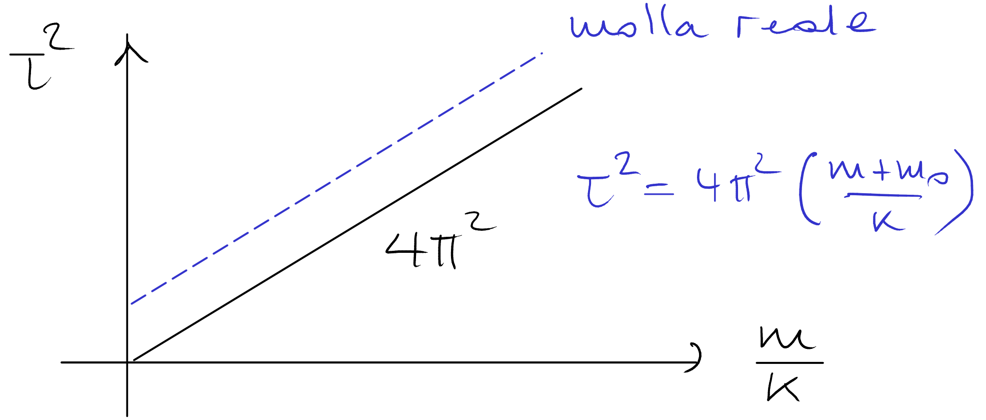
$$x(t+\tau) = x(t) \rightarrow A\cos\left(\sqrt{\frac{K}{m}}\,t + \sqrt{\frac{K}{m}}\,\tau + \phi\right) = A\cos\left(\sqrt{\frac{K}{m}}\,t + \phi\right)$$

$$\cancel{\sqrt{\frac{K}{m}}\,t} + \sqrt{\frac{K}{m}}\,\tau + \cancel{\phi} = \cancel{\sqrt{\frac{K}{m}}\,t} + \cancel{\phi} + 2\pi \Rightarrow \sqrt{\frac{K}{m}}\,\tau = 2\pi \rightarrow \tau = 2\pi\sqrt{\frac{m}{K}}$$

$$\omega\tau = 2\pi \rightarrow \omega = \frac{2\pi}{\tau}$$ frequenza angolare $\qquad f = \frac{1}{\tau}$ frequenza

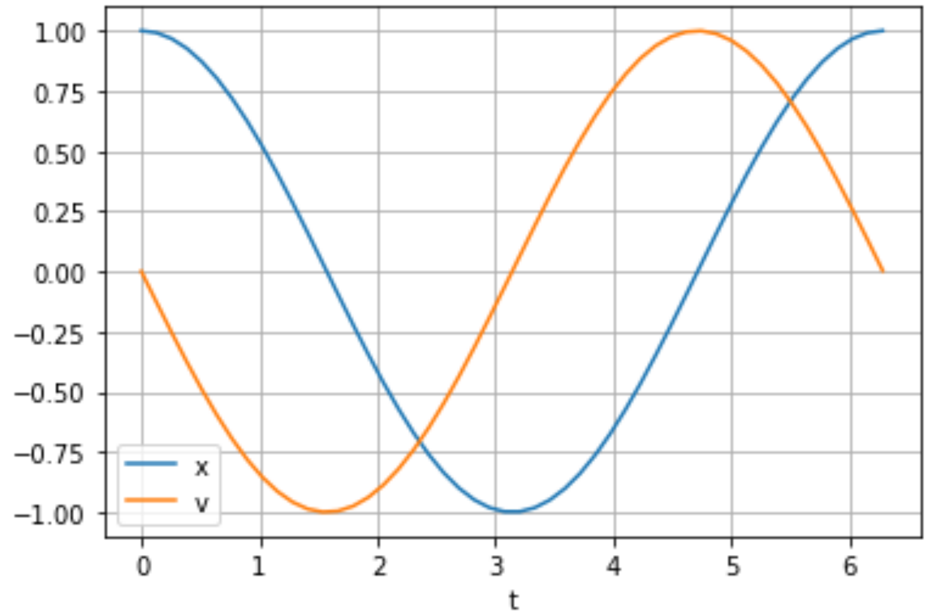






Condizioni iniziali

$$\begin{cases} x_i = x(0) \\ v_{xi} = v(0) = 0 \end{cases} \qquad t_i = 0 \qquad\qquad x_i = 1$$

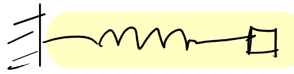


$$\begin{cases} x(0) = A\cos(\phi) = x_i & 1) \\ \frac{dx}{dx}(0) = -\omega A \sin(\phi) = 0 & 2) \end{cases}$$

$2) \Rightarrow \phi = 0$

$1)+2) \Rightarrow A = x_i \qquad \Rightarrow \quad x(t) = x_i \cos\left(\sqrt{\frac{K}{m}}\, t\right)$

Energia meccanica

Sistema : $\{$ corpo , molla $\}$ 

$$E = E_c + E_p = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2$$

$$= \frac{1}{2} m \underbrace{A^2 \omega^2 \sin^2(\omega t + \phi)}_{v_x^2} + \frac{1}{2} k \underbrace{A^2 \cos^2(\omega t + \phi)}_{x^2}$$

$$= \frac{1}{2} K A^2 \left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right] = \frac{1}{2} k A^2$$