On the rheology of cats

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In this letter I highlight some of the recent developments around the rheology of *Felis catus*, with potential applications for other species of the felidae family. In the linear rheology regime many factors can enter the determination of the characteristic time of cats: from surface effects to yield stress. In the nonlinear rheology regime flow instabilities can emerge. Nonetheless, the flow rate, which is the usual dimensional control parameter, can be hard to compute because cats are active rheological materials.

$\pi\alpha\nu\tau\alpha\ \rho\epsilon\iota$! Everything flows! This famous aphorism used to characterize Heraclitus' thought is also the motto of *rheology*. "Everything flows and nothing abides; everything gives way and nothing stays fixed." a recipe for insubordination actually from Simplicius and Plato. Everything flows? Well, it depends on the definition of a *flow*; if sufficiently general, there is no doubt that there are no exceptions to the rule! What is a flow? What is a fluid? As pointed out from the start by Reiner, the essential value of rheology is to recognize that states of matter are a matter of time(s). The first time, is a *time of observation* T. What is true today may not be true tomorrow. Time over time, one day 49, the next 50.

Historically, the popular distinction between states of matter has been made based on qualitative differences in bulk properties. Solid is the state in which matter maintains a fixed volume and shape; liquid is the state in which matter maintains a fixed volume but adapts to the shape of its container; and gas is the state in which matter expands to occupy whatever volume is available. Following these common sense definitions, a meta-study untitled "Cats are liquids" was recently published on `boredpanda.com`. I propose here to check if the panda's claim that the cats are liquid is solid, by using the tools of modern rheology.

First of all, 'maintains', 'adapts' or 'expands' are verbs. They describe actions unfolding with a characteristic time scale τ, which we will call *relaxation time*. From T and τ we can define the *Deborah number* as:

$$De \equiv \frac{\tau}{T} \tag{1}$$

Usually T is just the duration of the experiment, but for oscillatory flows it is the inverse of the frequency (and thus De is analogous to a Strouhal number). The relaxation time τ can have a variety of origins. When one seeks the difference between gas and liquid, 'relaxing' will mean 'expanding' and so τ will be linked to the characteristic rate of expansion of the material. The expansion

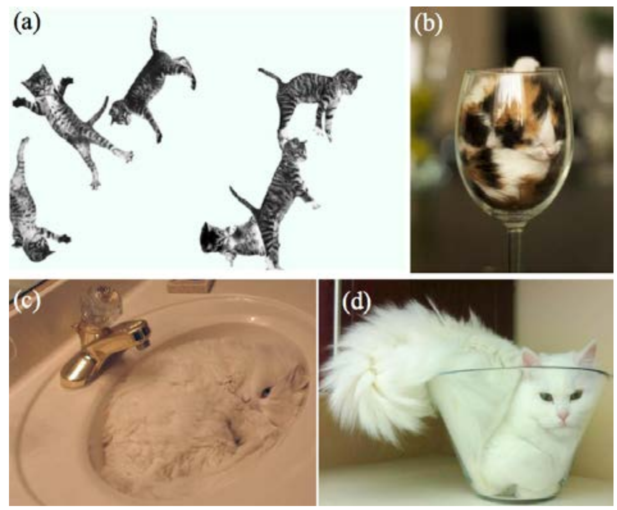

FIG. 1: (a) A cat appears as a solid material with a consistent shape rotating and bouncing, like Silly Putty on short time scales. We have $De \gg 1$ because the time of observation is under a second. (b) At longer time scales, a cat flows and fills an empty wine glass. In this case we have $De \ll 1$. In both cases, even if the samples are different, we can estimate the relaxation time to be in the range $\tau = 1$ s to 1 min. (c-d) For older cats, we can also introduce a characteristic time of expansion and distinguish between liquid (c) and gaseous (d) feline states. [(a) Courtesy of http://catbounce.com, (b) http://www.dweebist.com/2009/07/kitten-in-wine-glass/, (c) http://imgur.com/gallery/UuNSR, (d) http://imgur.com/s7JtV]

is a type of flow. In this case, we will say that we have a gas if $De \ll 1$. When one seeks the difference between liquid and solid, 'relaxing' will mean 'adapting' and so τ will be linked to the characteristic rate of adaptation of the shape of the material to its container. The adaptation of the shape of the material is a type of flow. In this case, we will say that we have a liquid if $De \ll 1$. Solids 'maintain' their shape and volume, *i.e.* they do not flow. But solids can be deformed under stress. Note finally that any flow is intrinsically made of deformations.

As illustrated in Fig. 1a, for $De \gg 1$ a cat appears

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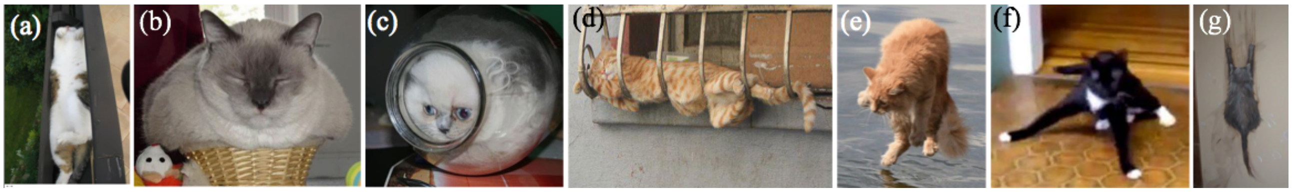

FIG. 2: (a) Extensional rheology of a cat before capillary break-up. (b) Cat on a superfelidaphobic substrate showing a high contact angle. (c) Tilted jar experiment showing the yield stress of a kitten. (d) Spreading of a cat on a very rough substrate. (e) Low affinity between cats and water surfaces. (f) Sliding cat on smooth floor. (g) Adhesion of a cat on a vertical wall. [Courtesy of (a) facebook.com, (c) http://metro.co.uk/2011/02/18/ksyusha-the-kitten-is-cat-in-a-jar-639735/ , (d) http://www.theyfailed.com/cats-sleep-anywhere/, (f) http://www.mirror.co.uk/news/world-news/youtube-watch-hilarious-viral-of-two-882779, (g) http://amazinghandpaintedmurals.com/picture_gallery_-_page_3]

solid, whereas for $De \ll 1$ it seems liquid. From these preliminary experiments, knowing T we can estimate the relaxation time to be in the range $\tau = 1$ s to 1 min, for normal cases of *Felis catus*. Note that the samples used in Fig. 1a-b are relatively young. Older cats may have a shorter relaxation time, and thus become liquid more easily than agitated kittens, for which τ can reach values as high as a few hours. The assumption of incompressibility may also fail for older cats, which can acquire gaseous properties like in Fig. 1c-d. In this letter, we will tend to ignore this thixotropic behavior. There's an old saying in investing: even a dead cat will bounce if it is dropped from high enough. Where, of course, the dead cat bounce refers to a short-term recovery in a declining trend.

Overall, the Deborah number is the dimensionless expression of the concept of linear viscoelasticity. The greater the Deborah number, the more elastic/solid the material; the smaller the Deborah number, the more viscous/fluid it is. Thus, rheology suggests only two states of matter: solids that deform; and fluids that flow. Both gases and liquids flow, they are fluids, the first compressible, the other incompressible. In general, both the fluid-like and the solid-like properties of a material can be complex, in the sense that the solid part may not be purely elastic, and the fluid part may not be purely viscous. For simple incompressible and athermal molecular fluids, the relaxation time will simply be the viscous dissipation time $\tau = \delta^2/\nu$, where δ is the thickness of the momentum boundary layer and ν is the kinematic viscosity. For more complex fluids, τ can have a large range of origins, which often require chemistry and/or biology to be well understood.

In the first part of this letter I wish to highlight the potential factors that have to be taken into account in computing the value of τ for cats. Fig. 2a shows the capillary bridge formed during extensional rheometry of *Felis catus*. First, in the introduction, we assumed τ to be a scalar, but it can have a higher dimensionality. Usually the time scale is considered as a contribution to viscosity, which in the most general case is a tensor of rank 2. For simple incompressible fluids symmetry considerations reduce this tensor to a scalar. The extensional viscosity is simply 3 times the shear viscosity. For complex fluids, the extensional viscosity can be orders of magnitude different, usually larger than the shear viscosity for polymeric materials. For cats, the determination of the Trouton ratio is complicated but the situation seems opposite. In the absence of reliable extensional rheology data, we can only point to the fact that when cats are deformed along their principal axis, they tend to relax more easily, suggesting that the extensional time is smaller than the shear time. Transient strain-hardening can nonetheless occur. Second, because, flows of cats are usually free surface flows, the surface tension between the cat and its surrounding medium can be important and even dominant in the rheology, especially in CATBER (Capillary thinning and breakup extensional rheometer) experiments. The catpillary number becomes important $\tau = f(Ca)$, with $Ca \equiv \eta U/\gamma_{LV}$, where η is the shear viscosity, U is a characteristic flow velocity and γ_{LV} is the surface tension (not to be confused with the deformation). Let us recall that even water droplets bouncing on hydrophobic substrates can behave elastically, with a response time $\tau = \sqrt{\rho R_0^3/\gamma}$, where ρ is the density and R_0 the size of the drops. When the fluid is complex, the situation can be even more entangled.

The wetting and general tribology of cats has not progressed enough to give a definitive answer to the capillary dependence of the feline relaxation time. Fig. 2b gives an example of a lotus effect of *Felis catus*, suggesting that the substrate is superfelidaphobic. This behavior is usually distinguished from the yield stress that cats can also display, as shown in Fig. 2c, where the kitten cannot flow because it is below its yield stress, like ketchup in its bottle. It is still unclear what physical and chemical properties generate superfelidaphobicity, but a Cassie-Baxter-like model seems plausible. Here, the roughness of the cat's fur would be as determinant as the roughness of the substrate, but probably with somewhat opposite effects. Indeed, cats are often found to spread on rough substrates as seen in Fig. 2d, but they have low affinity for substrates that smooth their fur, like water in Fig. 2e. Significant wall slip and shear localization can also be involved in some experiments, like shown in Fig. 2f, where there is a very significant relative velocity between the substrate and the cat. Counter-intuitively, gravity seems

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(Cats, continued from page 17)

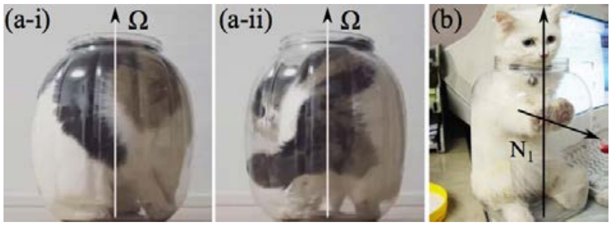

FIG. 3: (a) A cat spontaneously rotates in a cylindrical jar. (b) Normal forces and Weissenberg effect in a young sample of *Felis catus*. [Courtesy of (a) http://guremike.jp/, (b) http://buzzlamp.com/10-weird-places-cats-get-stuck-in/]

to enhance adhesiveness, as shown in Fig. 2g.

In the last part of this letter, I wish to discuss the possibility of flow instabilities in *Felis catus*. Linear viscoelasticity conceptualizes the fact that if its Deborah number is small a material is flowing. The physics of flow instabilities warns us that, as the characteristic rate of deformation $\dot{\gamma}$ increases, non trivial secondary flows emerge and eventually become chaotic. Here, the important dimensionless number will be the Reynolds-Weissenberg number (a sort of Péclet number):

$$Rw \equiv \tau\dot{\gamma} \qquad (2)$$

The limit $Rw \ll 1$ defines the *laminar base flow*. Nontrivial *secondary flows* will usually appear around $Rw \sim 1$. Finally, the flow will be *turbulent* if $Rw \gg 1$. For simple fluids, the relaxation time is the viscous dissipation time, the driving force of instability is inertia and the dimensionless number is just the usual Reynolds number $Rw = Re$. For more complex fluids in creeping flow ($Re = 0$) recent progress on instabilities in viscoelastic polymers and micelles solutions suggests that the relevant dimensionless number is the Weissenberg number alone, *i.e.* $Rw = Wi$ if $Re = 0$. In this case elastic turbulence can be achieved without inertia. We speak of viscoelastic flow instabilities.

When taken in its philosophical form, "panta rhei" is *the theory of motion*: the belief that everything is dynamic and that the state of rest is illusory. But for centuries, this ontology was superseded by Aristotle's viewpoint. He posited that in the absence of an external motive power all objects would come to rest and that moving objects only continue to move so long as there is a power inducing them to do so. Modern physics started when Galileo and his followers put an end to Aristotle's dogma by showing that, unless acted upon by a net unbalanced force, an object will maintain a constant velocity. This was key to the realization that motion is relative and preceded by the more fundamental concept of *frame of reference*, *e.g.* the train moves *with respect to* the frame of the platform, but the platform moves *with respect* to the frame of the train. Note that even if rheologists have taken Heraclitus' doctrine as their motto, they depart from his thoughts by a paradoxical but useful conception of motion or flow, alternatively faithful to Aristotle or Galileo.

Simple fluids like water are "passive", they continue to move or deform so long as there is a power inducing them to do so. In this case, the typical flow rate $\dot{\gamma}$ is simply imposed by the operator and Rw is a natural control parameter. For cats, assuming we have a well-defined relaxation time τ, computing Rw is still challenging because defining $\dot{\gamma}$ can be difficult since cats are "active" materials. They have their own motive power. Like other biologically active materials (acto-myosin gels, bacterial swimmers, epithelium, packs, flocks, schools, *etc.*), they can exhibit spontaneous rotation as shown in Fig. 3a.

Despite these difficulties, the question remains: are cats prone to flow instabilities when Rw increases? In a cylindrical flow geometry, instabilities in the purely inertial case (i.e. $Rw = Re$) and in the purely elastic case (i.e. $Rw = Wi$) lead to vortex flows. In the inertial case, the centrifugal force drives this instability and is also responsible for the deformation of the top free surface, which climbs up the outer walls of the cylinder. In the purely elastic case, the mechanism is opposite: centripetal normal forces ("hoop stresses") drive the instability and are also responsible for the Weissenberg effect, where the fluid climbs at the center of the free surface. In general, both inertial and elastic effects can occur. In flows of *Felis catus*, significant normal forces can occur and they seem to be able to drive a Weissenberg-type effect, as shown in Fig. 3b.

In conclusion, much more work remains ahead, but cats are proving to be a rich model system for rheological research, both in the linear and nonlinear regimes. Standing questions include the potential implications of the rheology of cats on their righting reflex, and whether the nonlinear self-sustaining mechanism for turbulence in pipe is applicable to streaks of tigers. Very recent experiments from Japan also suggest that we should not see cats as isolated fluid systems, but as able to transfer and absorb stresses from their environment. Indeed, in Japan, they have cat cafes, where stressed out customers can pet kitties and purr their worries away.

Acknowledgments

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