Oscillatore armonico come modello per diversi fenomeni fisici

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

$x(t) = A \sin(\omega t) + B \cos(\omega t)$ $\qquad$ $x(t) = A \cos(\omega t + \phi)$

Funzione $f(t)$ è periodica se $\exists\, T \in \mathbb{R}$ tale che $f(t+T) = f(t)$ $\forall t$

$$\sin(\omega(t+\tau)) = \sin(\omega t)$$

$$\cancel{\omega t} + \omega\tau = \cancel{\omega t} + 2\pi$$

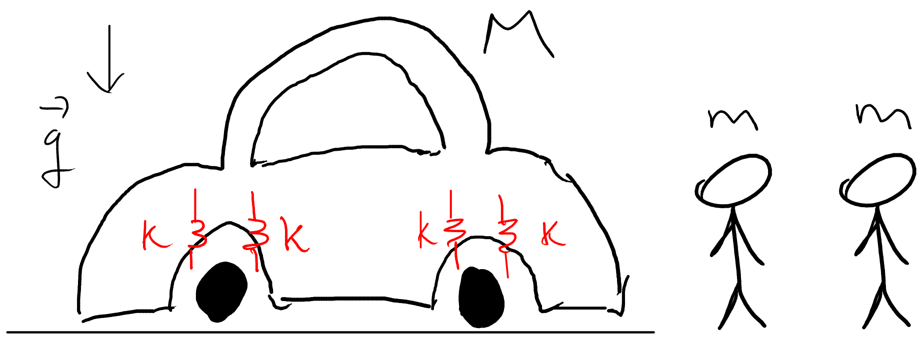
$$\omega\tau = 2\pi \leftarrow$$

$\tau = \frac{2\pi}{\omega}$ periodo

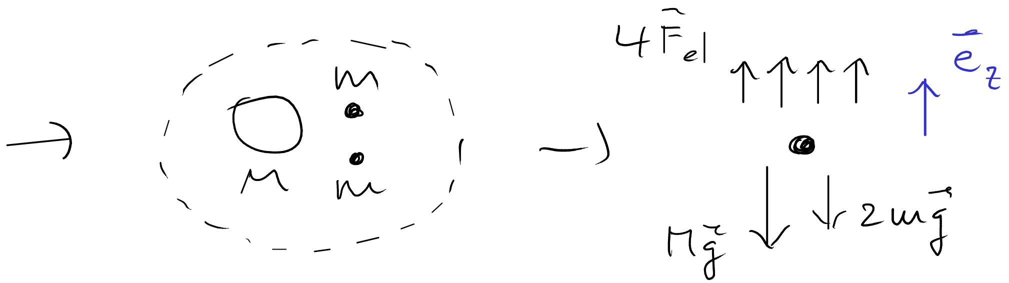
$\omega = \frac{2\pi}{\tau}$ frequenza angolare / pulsazione

$f = \frac{1}{\tau}$ frequenza

Es: oscillazioni di un automobile



Sistema: $\{M, m, m\}$



- Sistema
- Forze
- II Newton
- base
- eq. moto → osc.

$M = 1300$ kg

$m = 80$ kg

$k = 20000$ N/m

f = frequenza delle oscillazioni verticali = ?

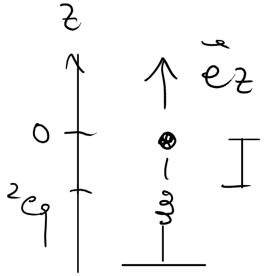
— Forza elastica: $4\vec F_{el}$

— Peso: $M\vec g$, $2m\vec g$

II Newton: $\sum \vec F = m\vec a$

$$4\vec F_{el} + (M+2m)\vec g = m\vec a$$

Equilibrio: $\sum \vec F = \vec 0$



$$-4kz_{eq}\vec e_z - (M+2m)g\vec e_z = 0 \Rightarrow 4kz_{eq} = -(M+2m)g$$

$$z_{eq} = -\frac{M+2m}{4k}g < 0 \quad ✓$$

$$(M+2m)\frac{d^2 z}{dt^2} = -4Kz - (M+2m)g \qquad Z \equiv z - z_{eq} \qquad z = z_{eq} + Z$$

$$(M+2m)\frac{d^2 Z}{dt^2} = -4KZ + \cancel{4K\frac{M+2m}{4K}g - (M+2m)g} = -4KZ$$

$$\frac{d^2 Z}{dt^2} = -\frac{4K}{M+2m} Z \qquad \omega^2 = \frac{4K}{M+2m}$$

SI: $1\,Hz = 1\,s^{-1}$

$$\omega \tau = 2\pi$$

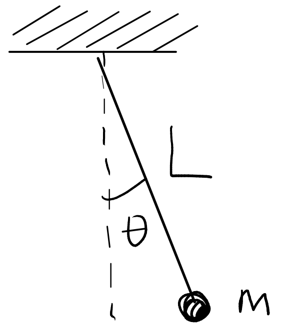
$$\frac{\omega}{f} = 2\pi \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4K}{M+2m}} = \dots = 1.18\ s^{-1}$$

$$= 1.18\ Hz \quad \square$$

$\nearrow K \quad \nearrow f$

$\nearrow M \quad \searrow f$

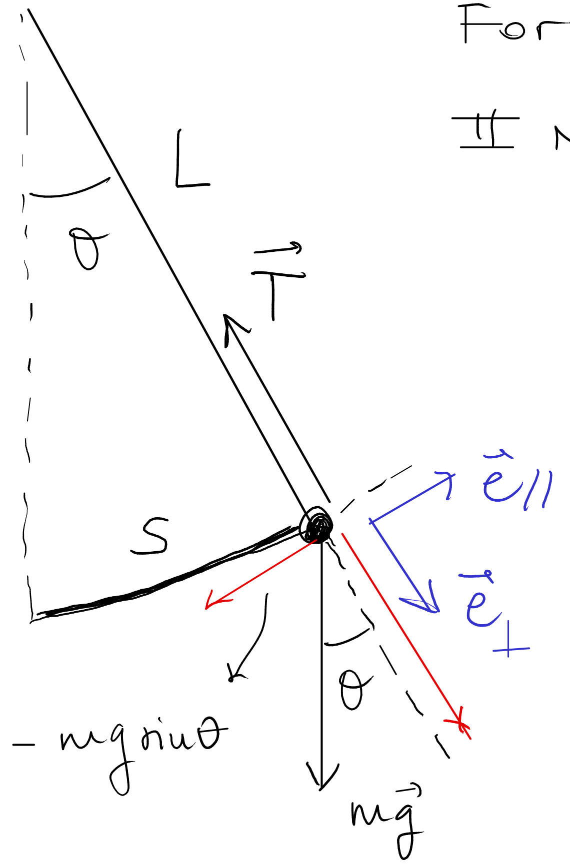
Pendolo semplice



no attrito

corpo puntiforme ≡ particella

filo ideale → inestensibile, senza massa



Sistema = particella

Forze: peso, tensione

II Newton: $\Sigma \vec{F} = m\vec{a}$

$m\vec{g} + \vec{T} = m\vec{a} \quad \rightarrow \quad \vec{a} = a_{\parallel}\, \vec{e}_{\parallel}$

componente lungo $\vec{e}_{\parallel}$:

$$- \cancel{m} g \sin\theta = \cancel{m} a_{\parallel} = \cancel{m} \frac{d^2 s}{dt^2} \qquad \theta = \frac{s}{L} \Rightarrow s = \theta L$$

$$\frac{d^2}{dt^2}(\theta L) = - g \sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = - \frac{g}{L} \sin\theta}$$

Piccole oscillazioni $|\theta| \ll 1$ rad Se $\theta \lesssim 10°$ $\Rightarrow$ $\sin\theta \approx \theta$ errore $\leq 1\%$

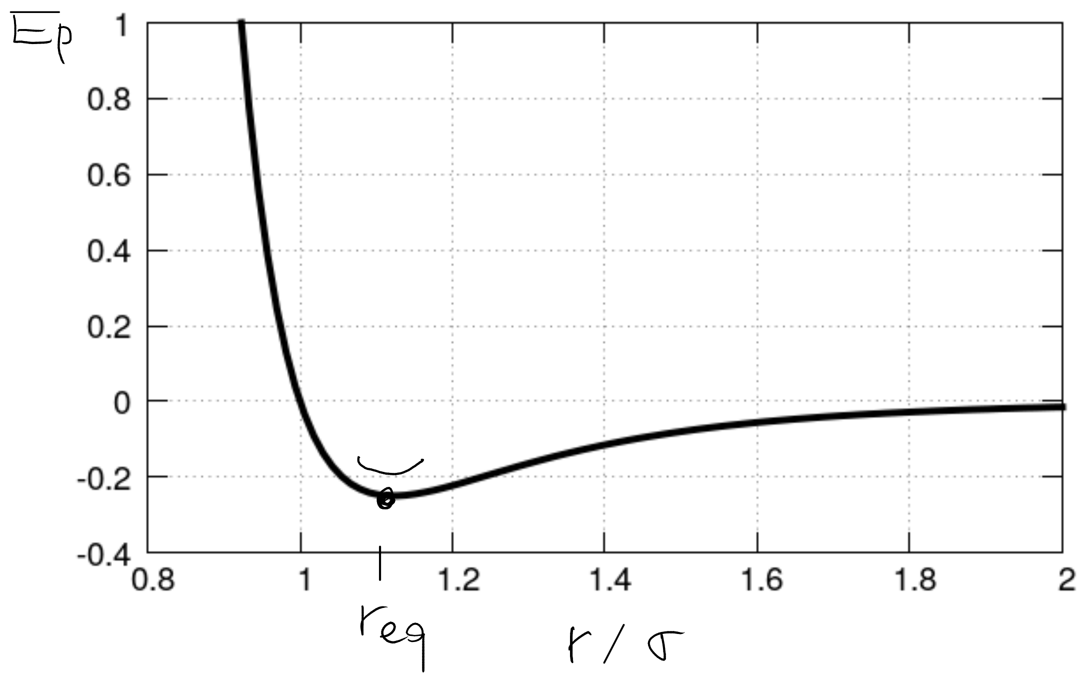
$\sin\theta \approx \theta$ Taylor I ordine

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \qquad \omega^2 = \frac{g}{L} \rightarrow \text{oscillatore armonico}$$

$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

Periodo: $\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$ $\qquad \tau \sim \sqrt{\frac{L}{g}}$ analisi dimensionale

Interazioni tra atomi neutri ! gas rari Ar, Xe, Ne (He)



$\bigcirc \quad \bigcirc$
$\overset{\longleftrightarrow}{r}$

Energia potenziale d'interazione

$$E_p(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

Lennard - Jones

Minimo $\rightarrow$ equilibrio meccanico

$$\frac{dE_p}{dr} = 4\epsilon\left[-12\frac{\sigma^{12}}{r^{13}} + 6\frac{\sigma^6}{r^7}\right]$$

$$\frac{dE_p}{dr}(r_{eq}) = 0 \Rightarrow \cancel{12}^{2}\,\frac{\sigma^{\cancel{12}\,6}}{r_{eq}^{\cancel{13}\,6}} = \cancel{6}\,\frac{\cancel{\sigma^6}}{r_{eq}^{\cancel{7}}}$$

$$r_{eq}^6 = 2\sigma^6 \Rightarrow r_{eq} = \sqrt[6]{2}\,\sigma \approx 1.12\,\sigma$$

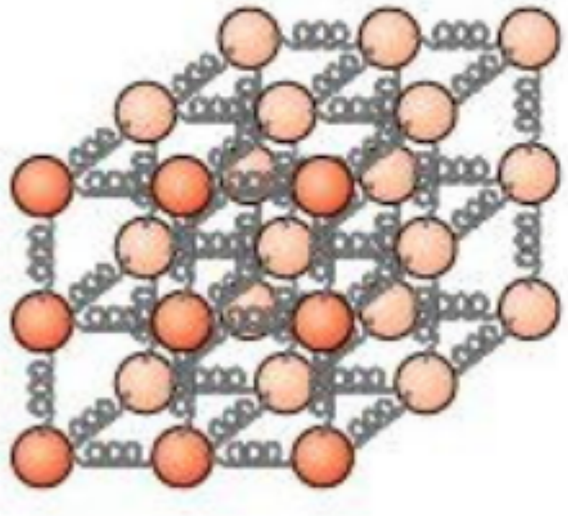
Taylor II ordine nell'intorno di r_{eq}

$$E_p(r) \approx E_p(r_{eq}) + \frac{1}{2}\frac{d^2E_p}{dr^2}(r_{eq})(r-r_{eq})^2 \rightsquigarrow m\frac{d^2r}{dt^2} = -\frac{d^2E_p}{dr^2}(r_{eq})(r-r_{eq})$$

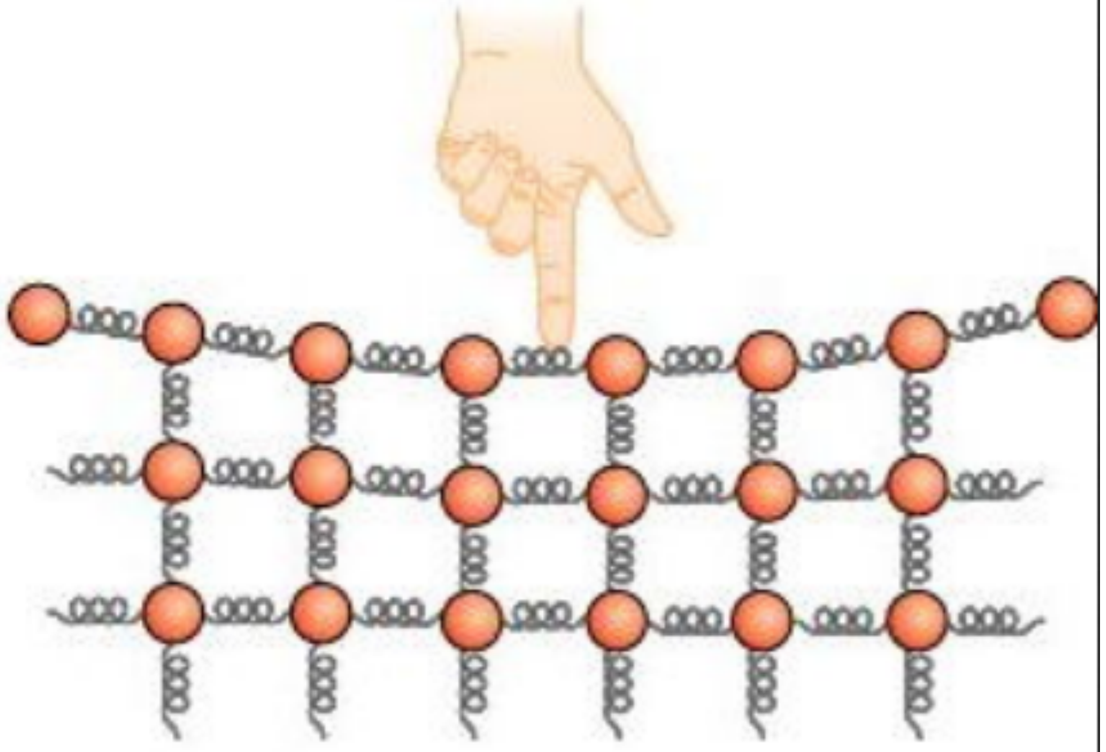
$$R = r - r_{eq} \qquad m\frac{d^2R}{dt^2} = -\frac{d^2E_p}{dr^2}(r_{eq})\,R$$

Vibrazioni e deformazioni nei solidi

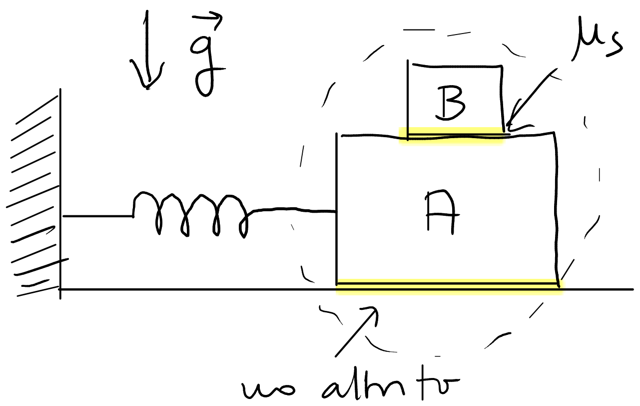
- oscillatori armonici accoppiati → modi normali
- oscillatori armonici indipendenti



Source: compadre.org



Problema di ricapitolazione (SJ 12.47)

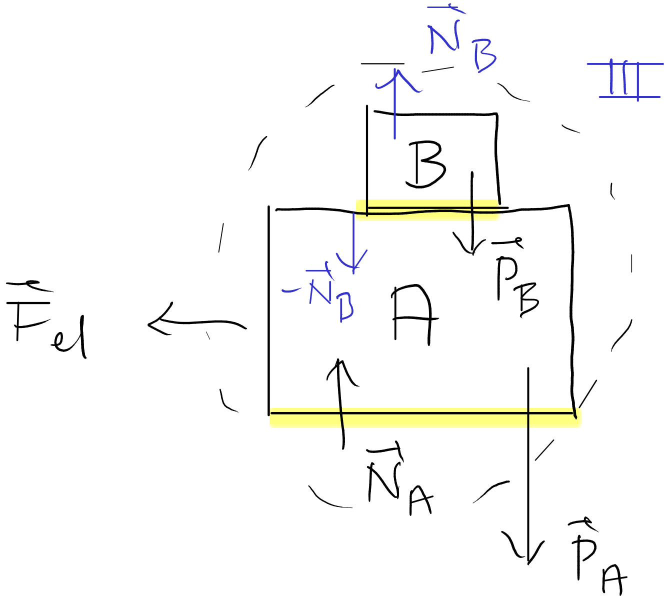


Attrito tra A e B ($\mu_s = 0.6$), non tra A e il suolo

Oscillazioni armoniche di A+B : $f = 1.5\ \text{Hz}$

Massima ampiezza Δx delle oscillazioni senza che B scivoli su A?

Sistema : $\{A, B\}$



III Newton

II Newton :

$$\sum_A \vec F = m_A \vec a_A$$

$$\sum_B \vec F = m_B \vec a_B$$

① solidali : $\vec a_A = \vec a_B$

② $|\vec F_a| \le \mu_s |\vec N_B|$

③ $f \to \boxed{K}$