Cyber-Physical Systems

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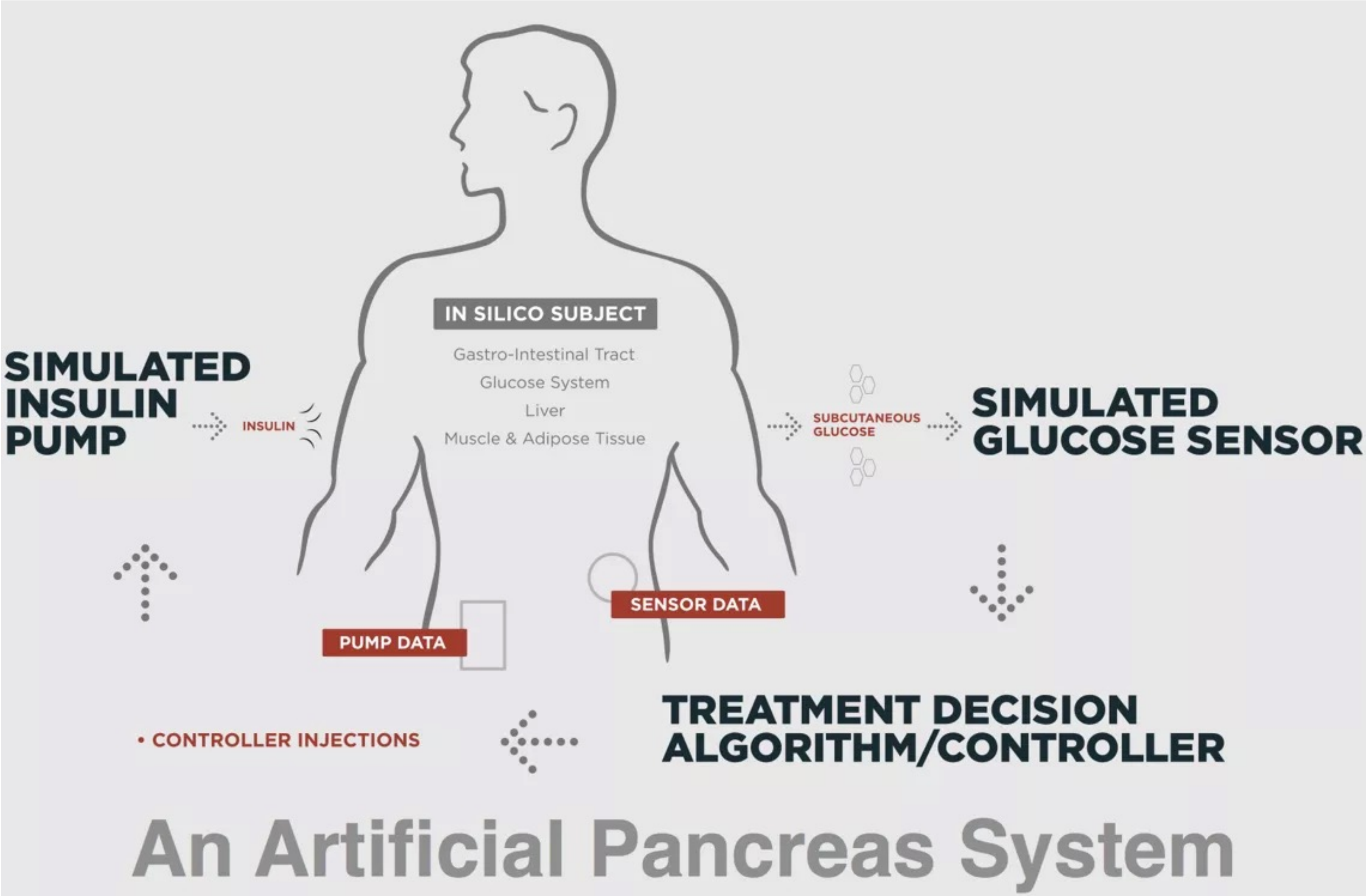
Lecture 10: Examples

Artificial Pancreas

Type 1 diabetes occurs when the pancreas produces little or none of the insulin needed to regulate blood glucose

They rely on external ad- ministration of insulin to manage their blood glucose levels.

Artificial Pancreas



Stochastic Hybrid Systems Of Glucose

$$\frac{d}{dt}\boldsymbol{x}(t) = F(\boldsymbol{x}(t);\ u(t);\Theta); \qquad y(t) = x_1(t)$$

Infusion rate of bolus insulin → $u(t)$

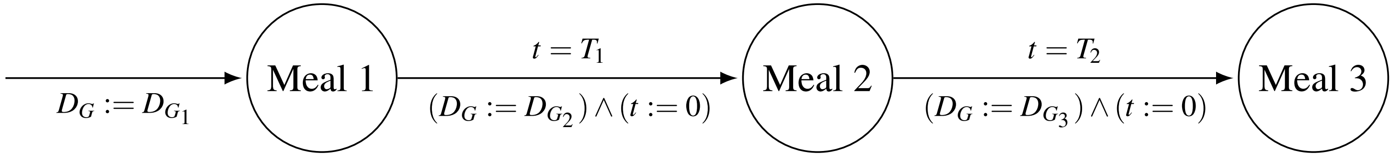
the control parameters → Θ

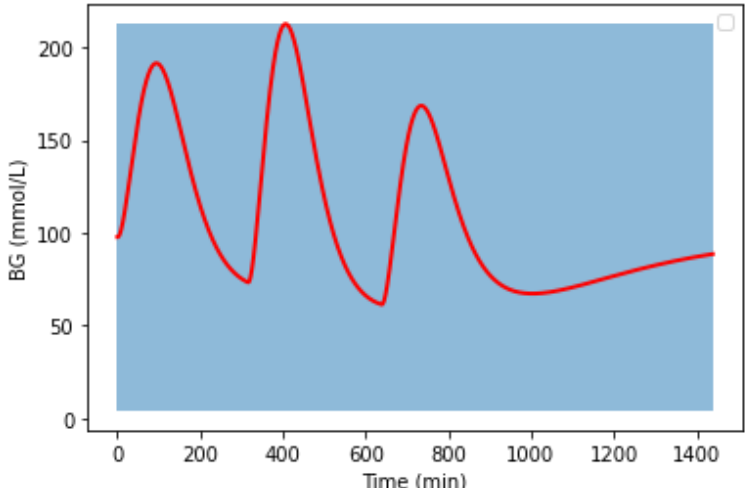
glucose concentration → $y(t)$

$\Theta = (D_{G_1}; D_{G_2}; D_{G_3};\ T_1;\ T_2)$ are the control parameter

$(D_{G_1}; D_{G_2}; D_{G_3}) \in (N(40;\ 10); N(90;\ 10); N(60;\ 10))$ are the three daily meals

$(T_1;\ T_2) \in \sim N$ (300, 10) and $T2 \sim N$ (300, 10) are the inter-times between each of them





Stochastic Hybrid Systems Of Glucose

$$\frac{d}{dt}Q_1(t) = -F_{01} - x_1Q_1 + k_{12}Q_2 - F_R + EGP_0(1-x_3) + \frac{D_G A_G}{t_{maxG}^2} t e^{-\frac{t}{t_{maxG}}}$$

$$\frac{d}{dt}Q_2(t) = x_1Q_1 - (k_{12} + x_2)Q_2;$$

$$\frac{d}{dt}S_1(t) = u(t) + u_b - \frac{S_1}{t_{maxI}};$$

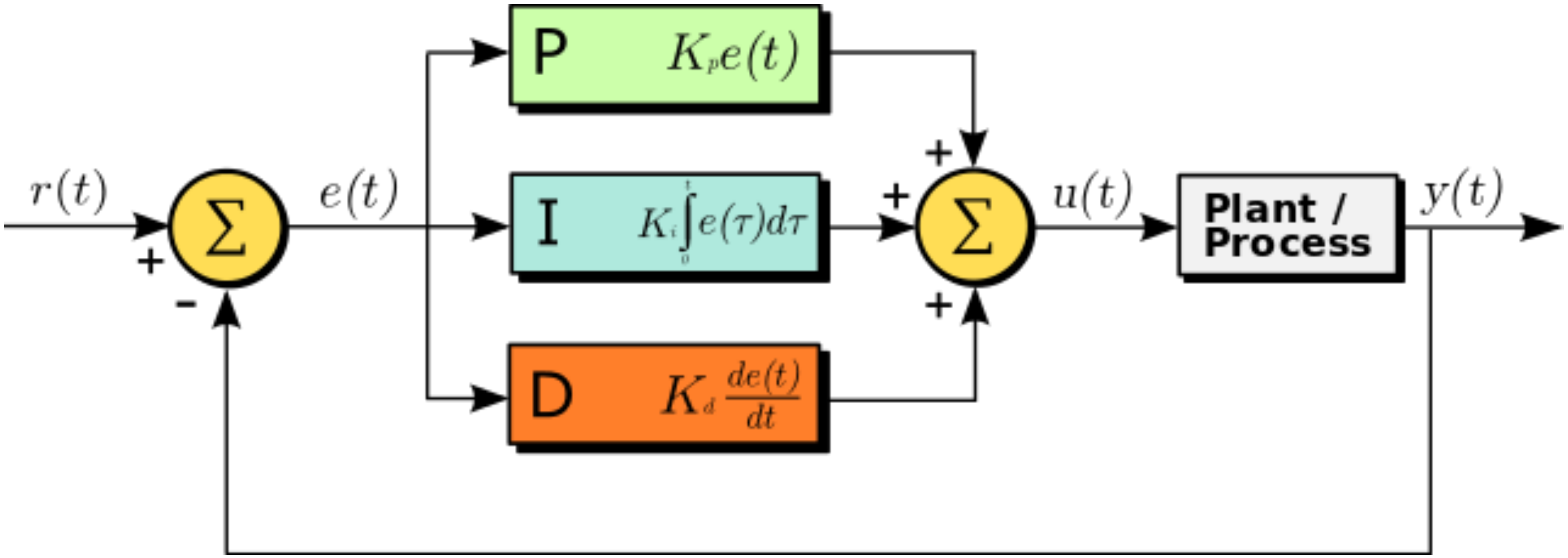
$$\frac{d}{dt}S_2(t) = S_1 - \frac{S_2}{t_{maxI}};$$

$$\frac{d}{dt}I(t) = \frac{S_2}{t_{maxI}V_I} - keI;$$

$$\frac{d}{dt}x_i(t) = -k_{a_i}x_i + k_{b_i}I; \qquad (i = 1,2,3)$$

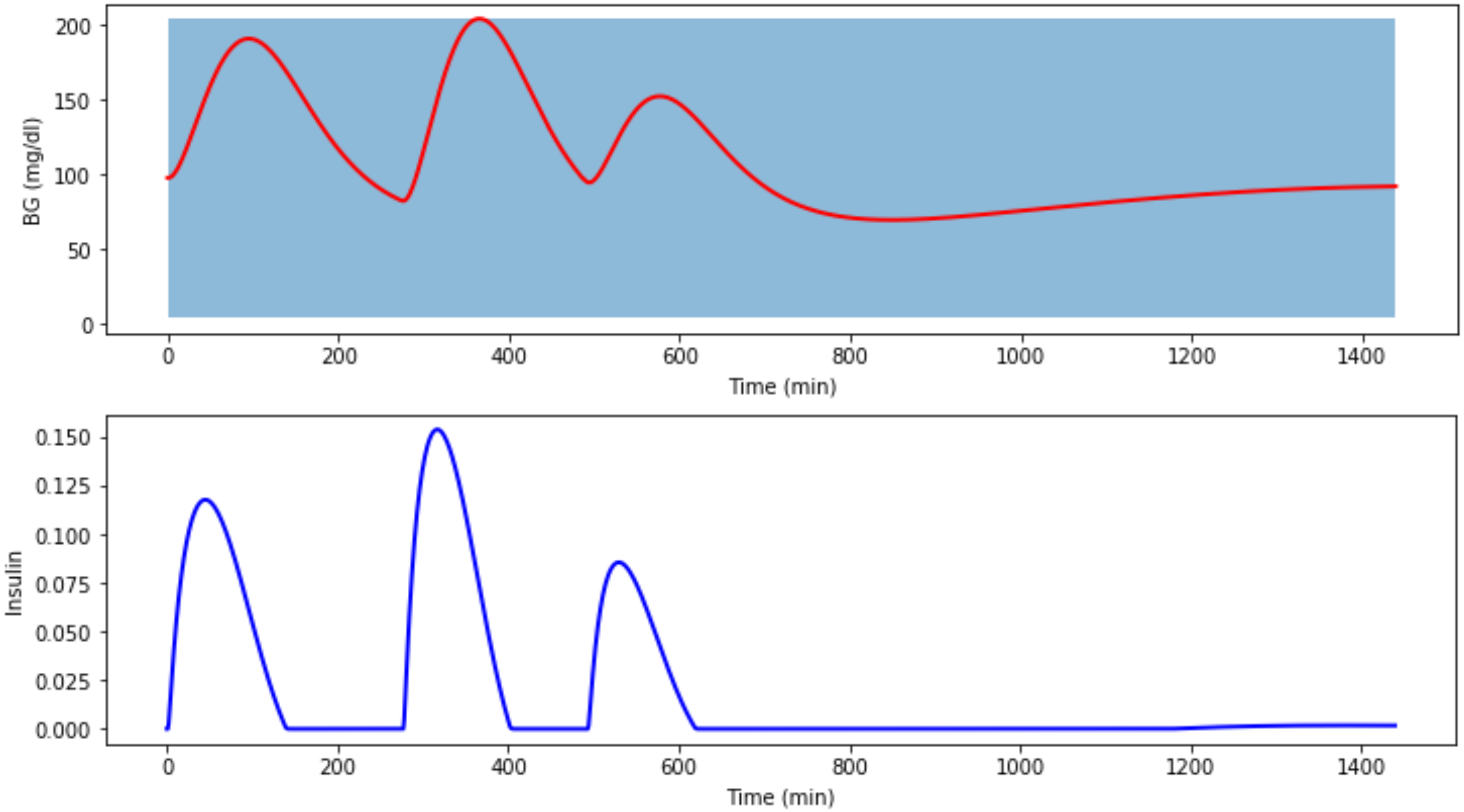
$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt}e(t), \qquad e(t) = r(t) - y(t)$$

PID Control



$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t), \qquad e(t) = r(t) - y(t)$$

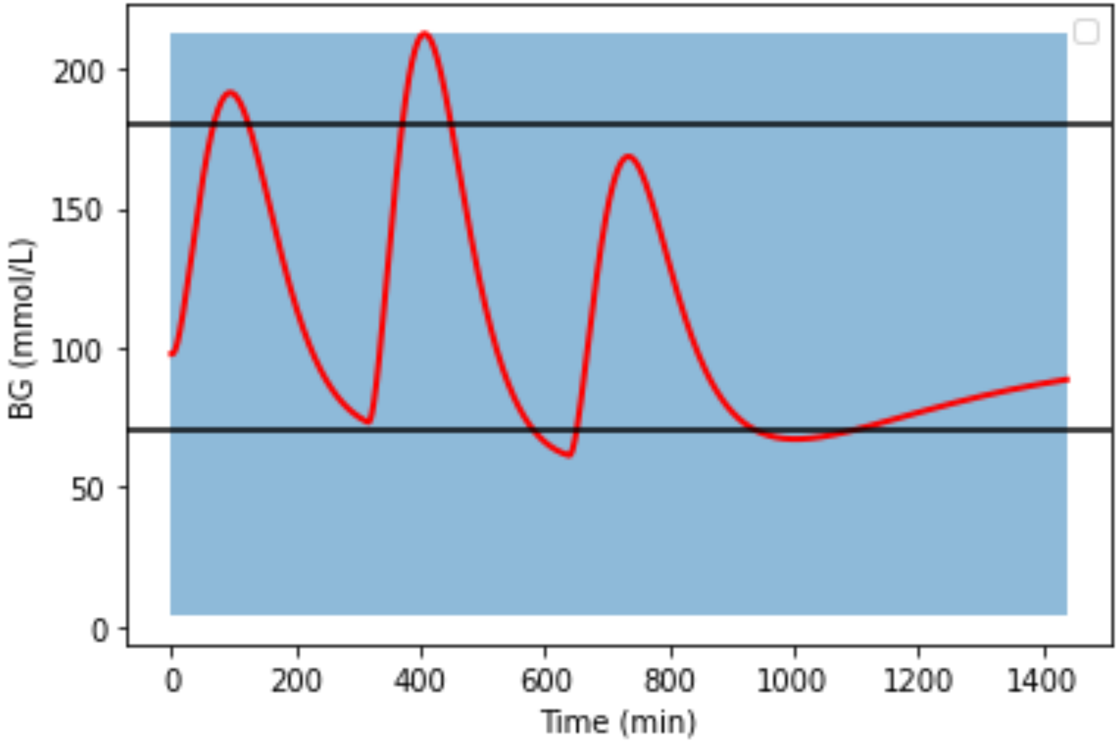
Artificial Pancreas Simulation



STL Properties for the Artificial Pancreas

- Hyperglycemia
  - "during the day the level of glucose goes above 180mg/dl"

$$G_{[0,24h]}(BG(t) < 180)$$



- Hypoglycemia
  - "during the day the level of glucose goes below 70mg/dl"

$$G_{[0,24h]}(BG(t) > 70)$$

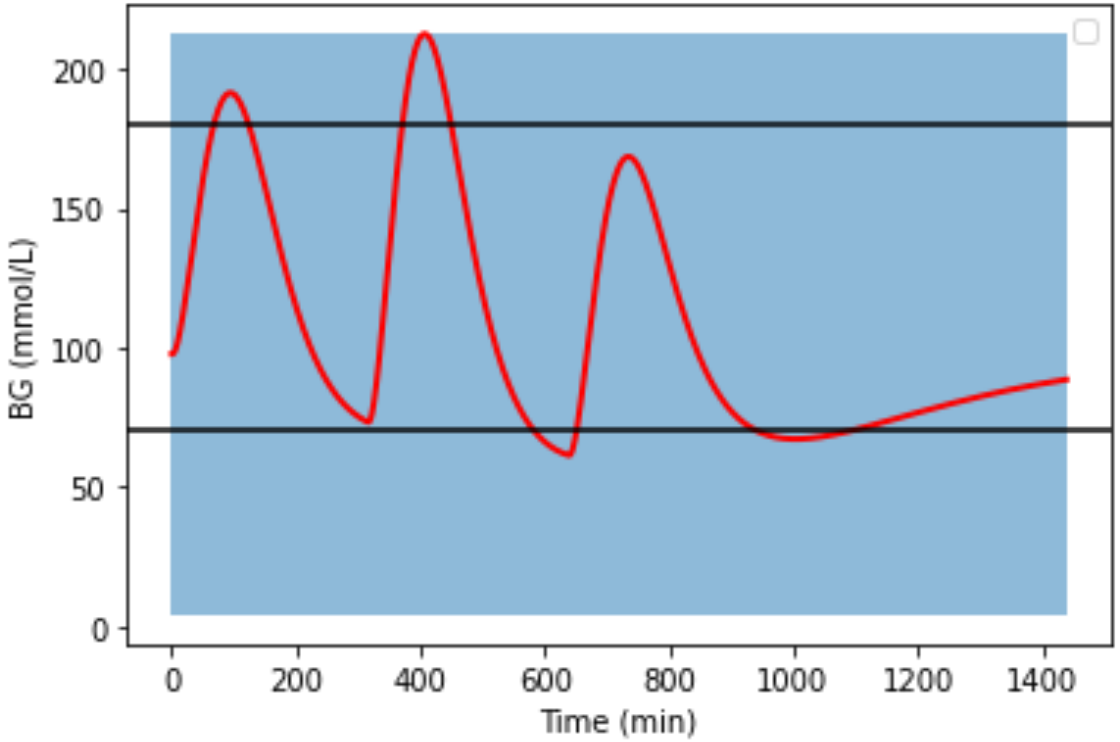
STL Properties for the Artificial Pancreas

- Prolonged Hyperglycemia
  - “during the day the level of glucose goes above 180mg/dl”

$$\neg F_{[0,21h]}(G_{[0,3]}(BG(t) \geq 180))$$

- Prolonged Hypoglycemia
  - “during the day the level of glucose goes below 70mg/dl”

$$\neg F_{[0,21h]}(G_{[0,0.5]}(BG(t) < 70)$$



Falsification

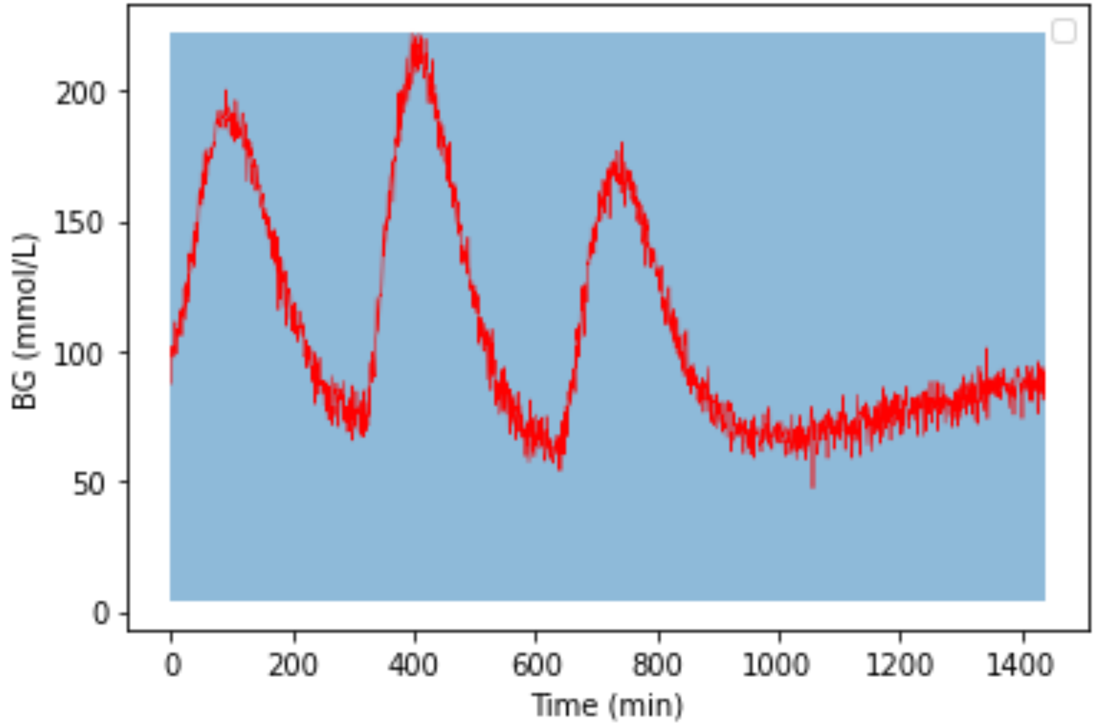
The most simple way to do falsification with respect a property ϕ is minimizing the robustness over N iterations considering random samples on control parameters, i.e:

```
minSTL = 'inf'
For i = 1,..,N:
    Θ  = sampling (D_G1, D_G2, D_G3, T_1, T_2)
    t ,y =simulation(Θ)
    stl= computeRobustness(y, ϕ)
    if (stl < minSTL):
        minSTL = stl
        vSTL = [D_G1, D_G2, D_G3, T_1, T_2]
```

For fixed control parameter spaces you can consider to sample with respect on grids over it.

Noise Robustness

- To consider noisy sensor we can add a Gaussian noise to the generated glucose trajectory, i.e. $GB(t) + \ \gamma$ with $\gamma \in N(0;\ 5)$



Bibliography

Nice survey on Specification-Based Monitoring of CPSs: http://www-verimag.imag.fr/PEOPLE/maler/Papers/monitor-RV-chapter.pdf

Artificial Pancreas:

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