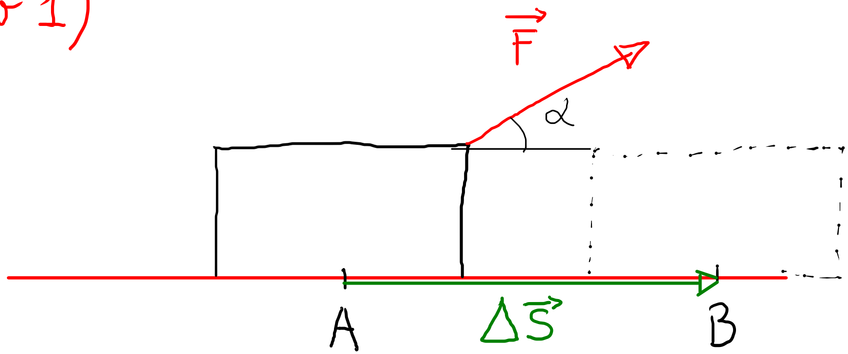
LAVORO (liv 1)



$$\mathcal{L} = \vec{F}\cdot\Delta\vec{S} \overset{!}{=} |\vec{F}|\cdot|\Delta\vec{S}|\cos\alpha$$

$\mathcal{L} = \vec{F}\cdot\Delta\vec{S}$ è valida se:

1) $\vec{F}$ è costante su tutto lo spost.
2) Lo spostamento è rettilineo

$$\mathcal{L} \gtreqless 0 \quad \begin{cases} \alpha \in [0, \frac{\pi}{2}[& \mathcal{L} > 0 \\ \alpha = \frac{\pi}{2} & \mathcal{L} = 0 \\ \alpha \in]\frac{\pi}{2}, \pi] & \mathcal{L} < 0 \end{cases}$$

$$\boxed{1\,J = 10^7\,erg}$$

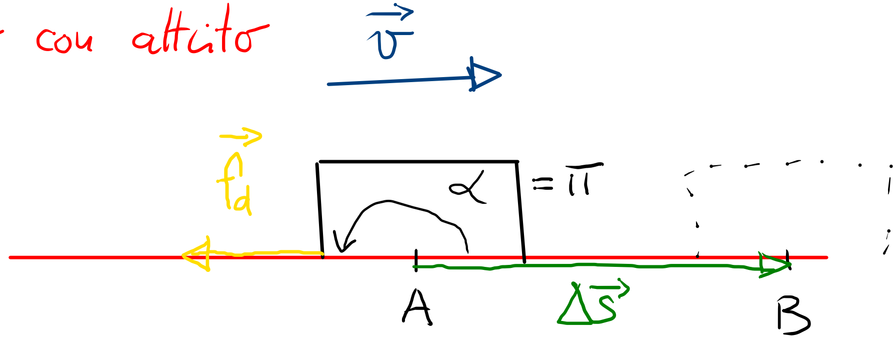
$[\mathcal{L}] = N \cdot m = J$ (Joul) in SI

erg in c.g.s.

$$10^5\,dine \cdot 10^2\,cm = 10^7\,\overbrace{dine\cdot cm}^{erg}$$

ESEMPI

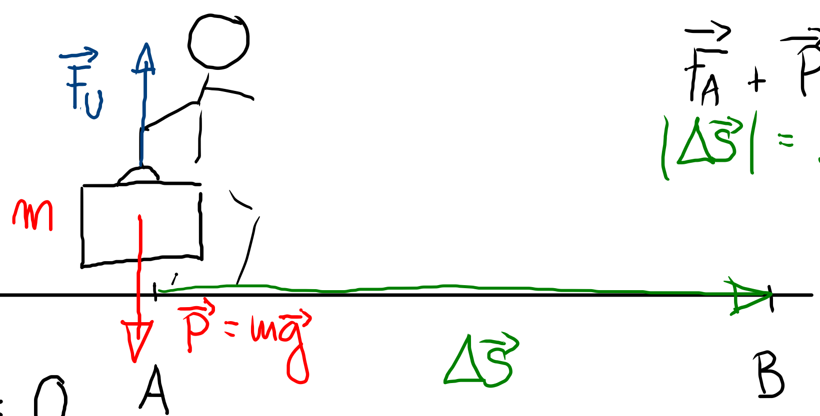
piano con attrito



$\alpha = \pi$

$\mathcal{L} < 0$

uomo con la valigia



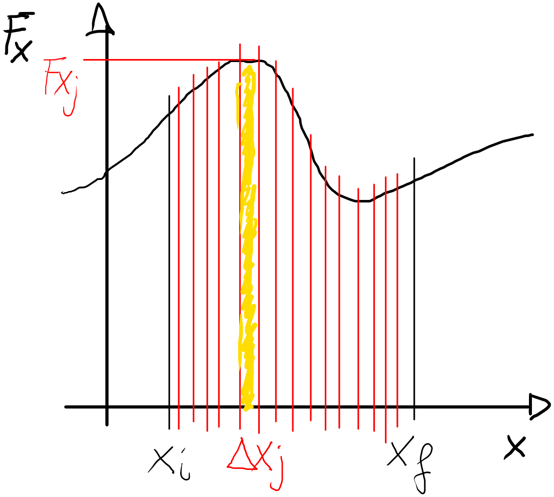
$\vec{F_A} + \vec{P} = 0$

$|\Delta\vec{s}| = 300\ m$

$\mathcal{L}_P = \vec{P} \cdot \Delta\vec{s} = 0$

$\mathcal{L}_U = \vec{F_U} \cdot \Delta\vec{s} = 0$

[liv 2] $\vec{F}$ non è costante
spostamento rettilineo (1D)



$\Delta x = x_f - x_i$

~~$\mathcal{L} = F_x \cdot \Delta x$~~
~~? che valore metto ?~~

$\mathcal{L}_j = F_{x_j} \cdot \Delta x_j$ area del rettangolino

suff. piccolo che F_{x_j} è costante

$$\mathcal{L} \simeq \sum_j \mathcal{L}_j \cong \sum_j F_{x_j} \cdot \Delta x_j$$

$\cong$ area sotto la curva

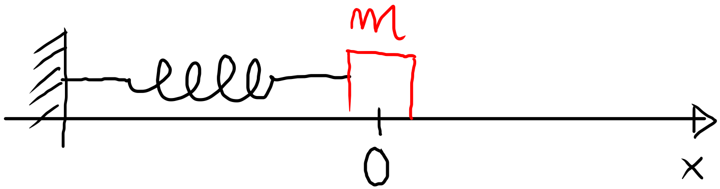
$\lim_{\Delta x_j \to 0}$

$$\mathcal{L} = \lim_{\Delta x_j \to 0} \sum_j F_{x_j} \Delta x_j = \int_{x_i}^{x_f} F_x \cdot dx$$

= area sotto la curva

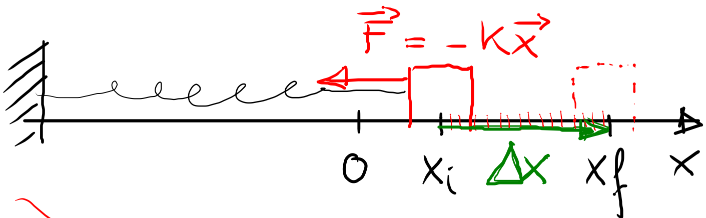
ESEMPIO

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$



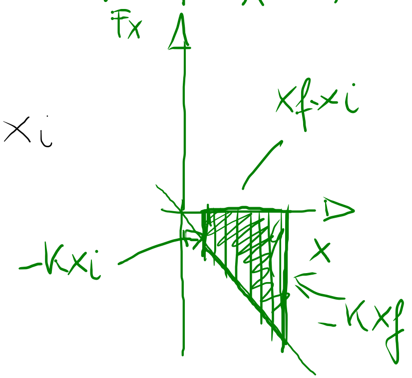
0 = equilibrio

$$A = \frac{1}{2}(x_f - x_i)(-kx_f - kx_i)$$



$\Delta x = x_f - x_i$

$x_f > x_i$



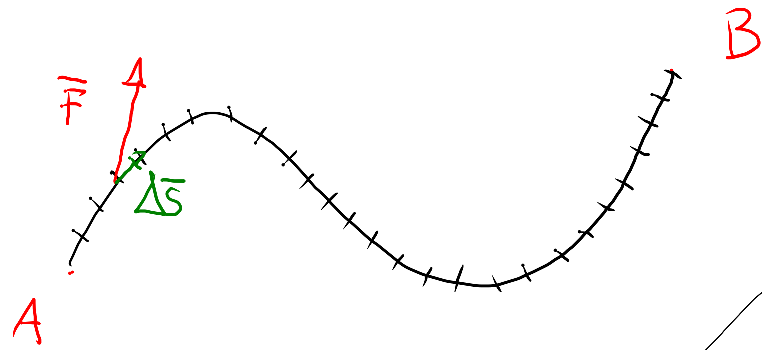
$\cancel{\mathcal{L} = F \cdot \Delta x}$ F cresce continuamente

$$\mathcal{L} = \int_{x_i}^{x_f} F \cdot dx = \int_{x_i}^{x_f} (-kx)\,dx = -k \int_{x_i}^{x_f} x\,dx =$$

$$= -k \left[\frac{x^2}{2}\right]_{x_i}^{x_f} = -\frac{1}{2} k \left[x^2\right]_{x_i}^{x_f} = -\frac{1}{2} k \left[x_f^2 - x_i^2\right] = \frac{1}{2} k \left(x_i^2 - x_f^2\right)$$

$$\mathcal{L} < 0$$

[liv. 3]



$$\mathcal{L} = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r}$$

integrale di linea
(lungo un particolare percorso)

(il risultato dipende dal percorso)

– GAME OVER –

ENERGIA (capacità di compiere lavoro)

ENERGIA CINETICA ($\vec{v} \neq 0$)

$$K = \frac{1}{2} m v^2$$

massa

velocità, modulo della

$$[K] = kg \frac{m^2}{s^2} = kg \frac{m}{s^2} \cdot m = N \cdot m = J$$

TEOREMA LAVORO ENERGIA

$$\boxed{\mathcal{L} = \Delta K}$$

Lavoro della risultante delle forze o*
Somma dei lavori di tutte le forze

che agiscono su un certo p.m.

$\Delta K = K_f - K_i$

variazione di en. cin. del punto materiale

$$* \quad \mathcal{L} = \Sigma \vec{F} \cdot \Delta\vec{s} = \left(\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_N\right) \cdot \Delta\vec{s}$$

$$= \vec{F}_1 \cdot \Delta\vec{s} + \vec{F}_2 \cdot \Delta\vec{s} + \dots + \vec{F}_N \cdot \Delta\vec{s} = \mathcal{L}_1 + \mathcal{L}_2 + \dots \mathcal{L}_N$$

DIMOSTRAZIONE

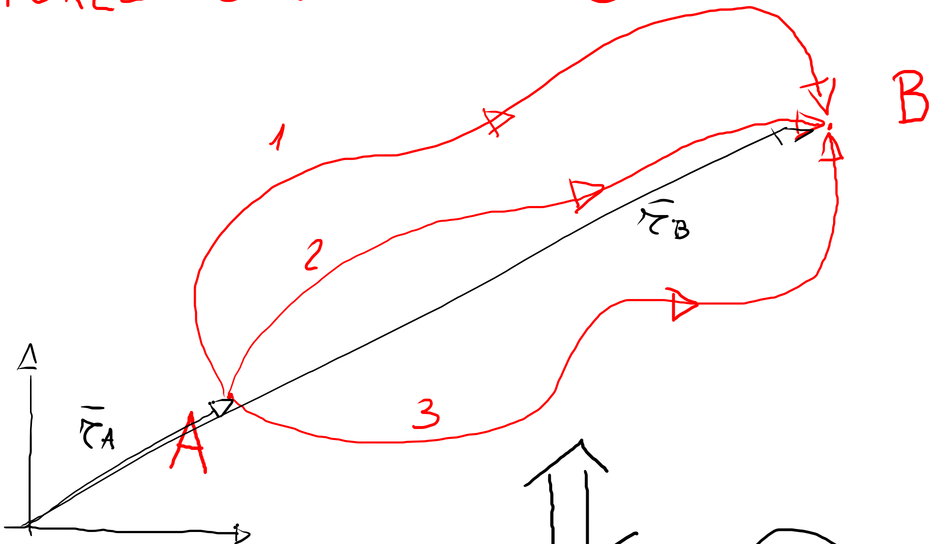
$$\mathcal{L} = \int_{x_i}^{x_f} \Sigma F \, dx \overset{\text{II PRINCIPIO}}{=} \int_{x_i}^{x_f} ma \cdot dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx =$$

$$= \int_i^f m \, dv \frac{dx}{dt} = \int_{v_i}^{v_f} m \, v \, dv = m \int_{v_i}^{v_f} v \, dv$$

$$= m \left[\frac{1}{2} v^2 \right]_{v_i}^{v_f} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = K_f - K_i = \Delta K$$

FORZE CONSERVATIVE

A)

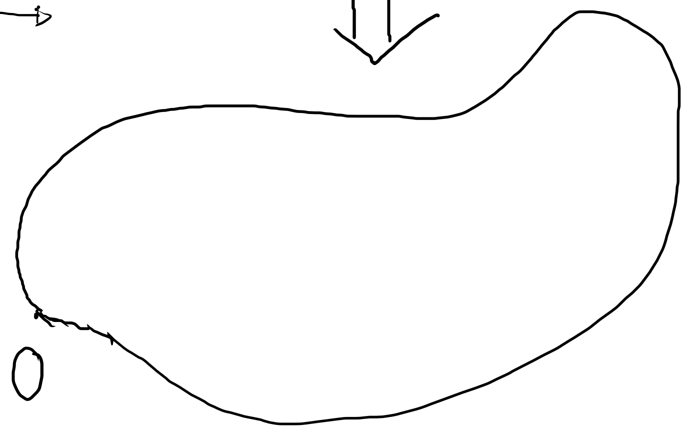


$$\mathcal{L}_{AB}^{1} = \mathcal{L}_{AB}^{2} = \mathcal{L}_{AB}^{3}$$

(in generale per ogni percorso)

$$\Updownarrow$$

B)



$$\mathcal{L} = 0$$

per ogni percorso chiuso

ENERGIA POTENZIALE
(solo per forze conservative)

$$U(\vec{r})$$

$$\mathcal{L}_{AB} = U(\bar{r}_A) - U(\bar{r}_B) = U_A - U_B$$

$$\boxed{\mathcal{L}_{AB} = -\Delta U}$$

Esempi di forze conservative / non conservative (dissipative)

- $\bar{P} = m\vec{g}$
- $\vec{F}_g = -G \dfrac{m_1 m_2}{r^2} \hat{r}_{12}$
- $\vec{F}_e = -K\bar{x}$

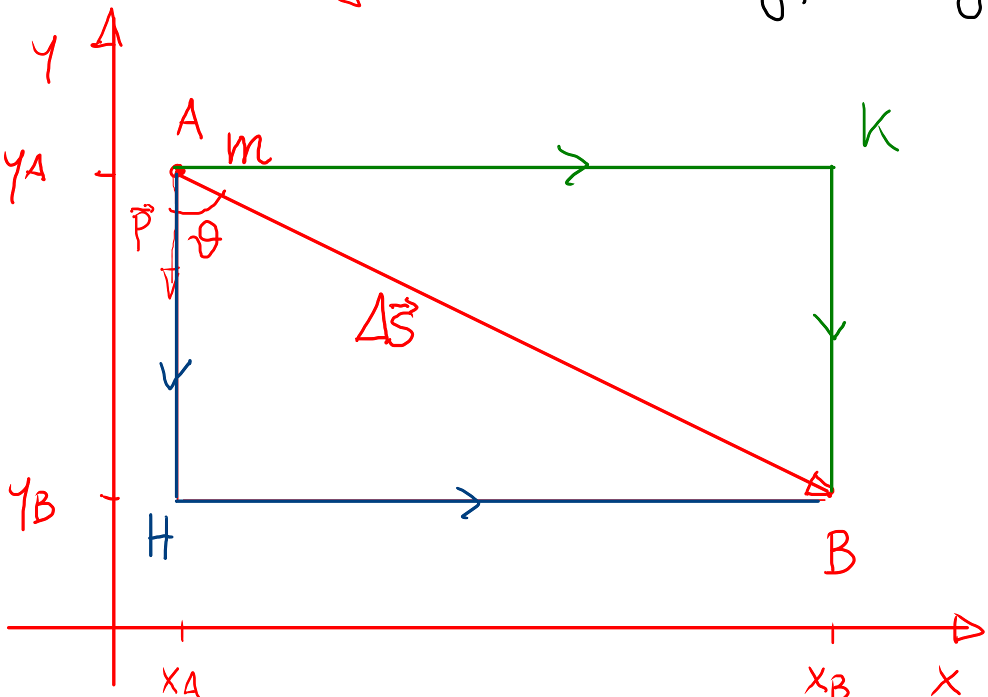
- attrito

ESEMPIO: $\vec{P} = m\vec{g}$

$$\mathcal{L} = mgy_A - mgy_B = U_A - U_B$$

m $\quad \boxed{U = mgy}$

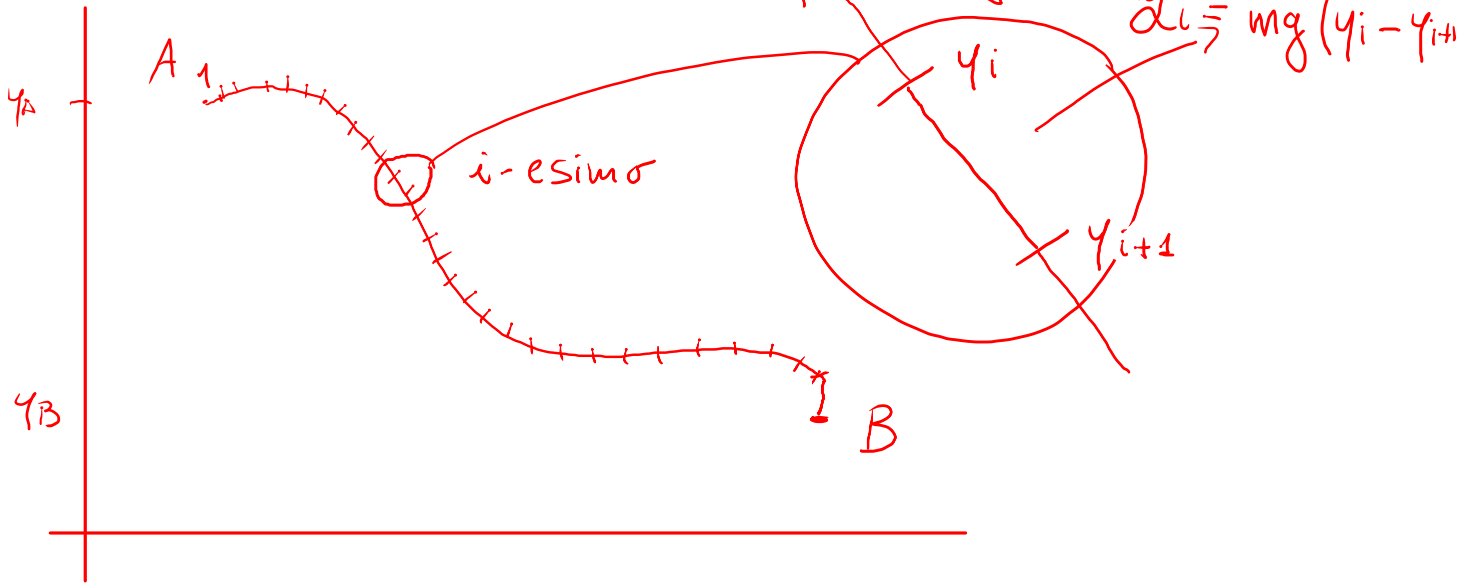
$\downarrow \vec{g}$



$$\mathcal{L}_{AB} = \vec{P} \cdot \Delta\vec{s} = mg \cdot \underbrace{|\Delta\vec{s}|\cos\vartheta}_{(y_A - y_B)} = mg\,(y_A - y_B)$$

$$\mathcal{L}_{AB} = \mathcal{L}_{AH} + \mathcal{L}_{HB} = mg\,(y_A - y_B) + 0 = mg\,(y_A - y_B) = \mathcal{L}_{AB}$$

CASO GENERALE (improvvisazione fuori programma)



$$\mathcal{L} = \Sigma_i \mathcal{L}_i = mg(y_A - y_1) + mg(y_1 - y_2) + \ldots$$

$$\ldots + mg(y_i - y_{i+1}) + mg(y_{i+1} - y_{i+2}) + \ldots$$

$$\ldots\ mg(y_N - y_B)$$

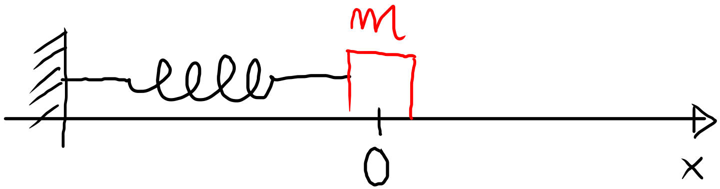
$$= mg\left(y_A - \cancel{y_1} + \cancel{y_1} - \cancel{y_2} + \cancel{y_2} + \ldots + \cancel{y_{N-1}} - \cancel{y_N} + \cancel{y_N} - y_B\right)$$

$$\mathcal{L} = mg(y_A - y_B)$$

ESEMPIO (RELOADED)

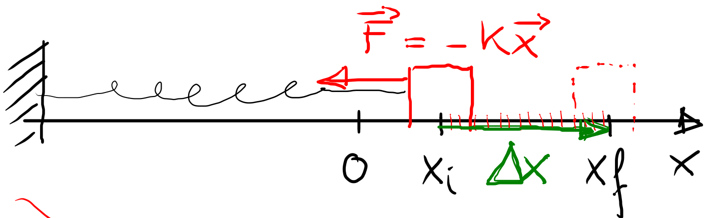
$$\mathcal{L} = \frac{1}{2}k\left(x_A^2 - x_B^2\right) = U_A - U_B$$

$$\boxed{U = \frac{1}{2}kx^2}$$



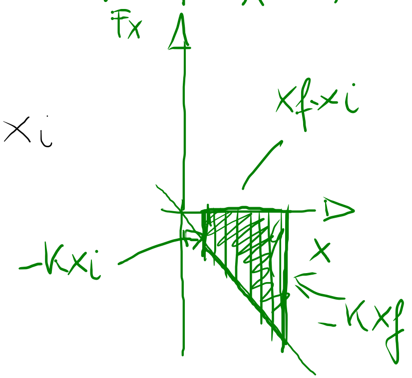
0 = equilibrio

$$A = \frac{1}{2}(x_f - x_i)(-kx_f - kx_i)$$



$\Delta x = x_f - x_i$

$x_f > x_i$



$\cancel{\mathcal{L} = F \cdot \Delta x}$

F cresce continuamente

$$\mathcal{L} = \int_{x_i}^{x_f} F \cdot dx = \int_{x_i}^{x_f} (-kx)\,dx = -k\int_{x_i}^{x_f} x\,dx =$$

$$= -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = -\frac{1}{2}k\left[x^2\right]_{x_i}^{x_f} = -\frac{1}{2}k\left[x_f^2 - x_i^2\right] = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$$

$$\mathcal{L} < 0$$

CONSERVAZIONE DELL'ENERGIA MECCANICA

$$\mathcal{L} = \Delta K \quad \text{(vale per tutte le forze)}$$
$$\mathcal{L} = -\Delta U \quad \text{(solo per forze conservative)}$$
$$\Delta K = -\Delta U$$

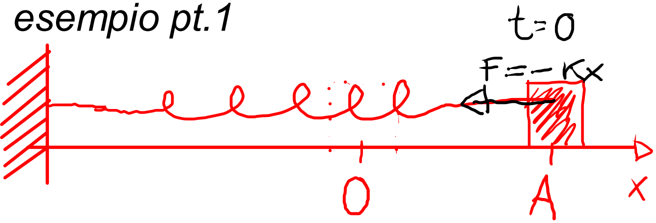
$$\Delta K + \Delta U = 0$$

definisco: $E_{mecc} = K + U$

$$\Delta E_{mecc} = 0$$

In un sistema meccanico conservativo, E_{mecc} si conserva!

esempio pt.1



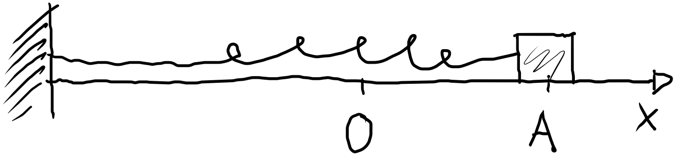
- $x(t) = A\cos(\omega t)$
  $v(t) = -A\omega\ \text{sen}(\omega t)$
  $a(t) = -A\omega^2 \cos(\omega t) = -\omega^2 x(t)$

- $\left.\begin{array}{l} F = -Kx \\ F = ma \end{array}\right\} \quad -Kx = ma \qquad a = -\frac{K}{m}x \longrightarrow \omega^2 = \frac{K}{m} \qquad \omega = \sqrt{\frac{K}{m}}$

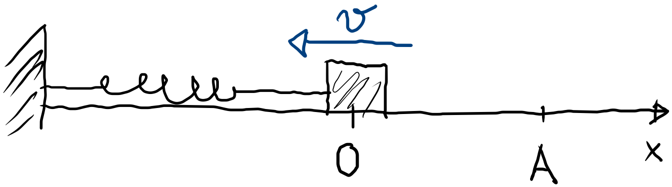
  $K = m\omega^2$

- $$E_{mecc} = K + U = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$
  $$= \frac{1}{2}m\left[-A\omega\ \text{sen}(\omega t)\right]^2 + \frac{1}{2}k\left[A\cos(\omega t)\right]^2$$
  $$= \frac{1}{2}mA^2\omega^2\ \text{sen}^2(\omega t) + \frac{1}{2}\underset{m\omega^2}{K}A^2\cos^2(\omega t)$$
  $$= \frac{1}{2}A^2 m\omega^2 \underbrace{\left[\text{sen}^2(\omega t) + \cos^2(\omega t)\right]}_{1} = \frac{1}{2}mA^2\omega^2 = \frac{1}{2}kA^2$$

esempio pt.2



$K=0 \quad U=\frac{1}{2}kA^2 \quad E_{mecc}=\frac{1}{2}kA^2$

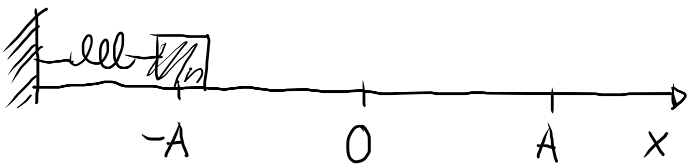


$K=\frac{1}{2}mv^2 \quad U=0$

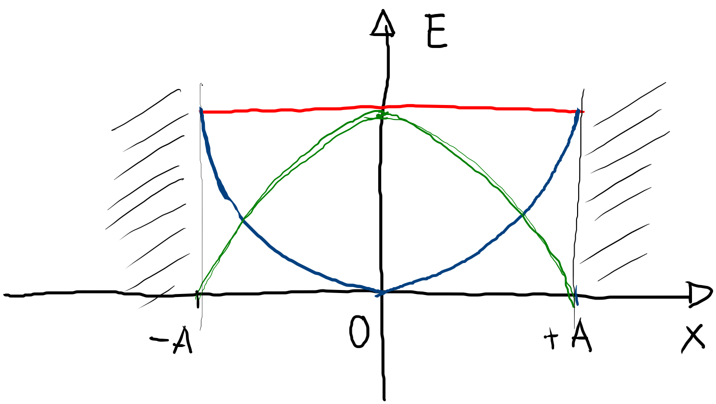
$\frac{1}{\cancel{2}}mv^2=\frac{1}{\cancel{2}}kA^2$

$v^2=\frac{k}{m}A^2=\omega^2A^2$

$v=\pm\omega A$



$U=\frac{1}{2}kA^2 \Rightarrow K=0$



$E_{mecc}=\frac{1}{2}kA^2$

$U=\frac{1}{2}kx^2$

$K=\frac{1}{2}mv^2$

$$\mathcal{L} = \Delta K \quad \text{(vero sempre)}$$

1) $\mathcal{L}_C + \mathcal{L}_D$

C → forze conservative

D → " non conservative o dissipative

2) $\mathcal{L}_C = -\Delta U$ (solo x forze conservative, U en. potenziale)

$$\mathcal{L}_C + \mathcal{L}_D = \Delta K$$

$$\mathcal{L}_D = \Delta K - \mathcal{L}_C = \boxed{\Delta K + \Delta U = \Delta(K+U) = \Delta E_{mecc}} \to *$$

$\mathcal{L}_D < 0$ $\qquad$ $\Delta K + \Delta U < 0$ $\qquad$ En. meccanica non si conserva

$$\Delta E_{mecc} < 0$$

SISTEMA ISOLATO (non scambia materia o energia con l'ambiente esterno)

" CHIUSO (non scambia materia ma può scambiare energia con amb. est.)

In un sistema isolato: $-\Delta E_{mecc} = \Delta E_{int}$ (**)

E_{int} = E interna del sistema

$$* \rightarrow \quad \Delta K + \Delta U = \Delta E_{mecc}$$

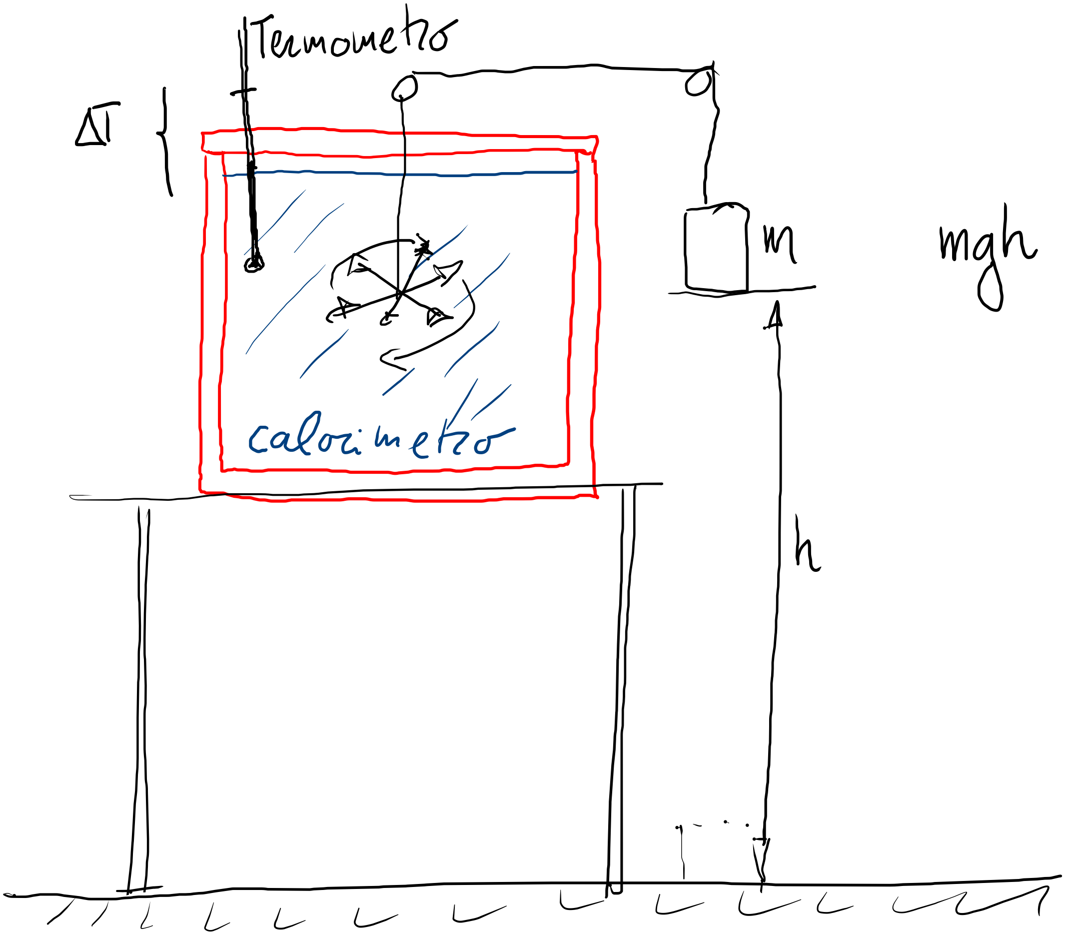
$$\Delta K + \Delta U - \Delta E_{mecc} = 0$$

$$** \rightarrow \quad \Delta K + \Delta U + \Delta E_{int} = 0$$

$$\Delta \underbrace{(K + U + E_{int})}_{E} = 0$$

E energia totale del sistema

DIGRESSIONE : ESP. DEL CALORIMETRO



mgh

ΔE_{int}

POTENZA

$$P_m = \frac{\mathcal{L}}{\Delta t}$$

lavoro compiuto

intervallo di tempo in cui si è compiuto

(potenza media)

$$P = \frac{d\mathcal{L}}{dt} = \frac{d}{dt}\left(\bar{F} \cdot \bar{s}\right) = \bar{F} \cdot \frac{d\bar{s}}{dt} = \vec{F} \cdot \vec{v}$$

(potenza istantanea)

$$\frac{1\,J}{1\,s} = 1\ \text{Watt} \quad (W)$$

RENDIMENTO

$$\eta = \frac{\mathcal{L}}{E} \cdot 100 \quad (\%)$$

lavoro compiuto

energia necessaria a far funzionare la macchina

CAMPI DI FORZE (*** APPROFONDIMENTO ***)
(caso particolare di campo vettoriale)

$\vec{E}(\vec{r};t)$

$\vec{B}(\vec{r};t)$

(equazioni di Maxwell)

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Forza di Lorentz

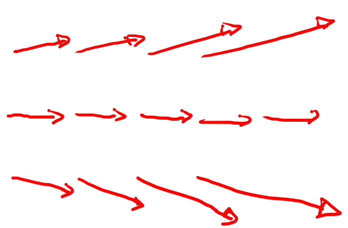
carica elettrica q

velocità $\vec{v}$

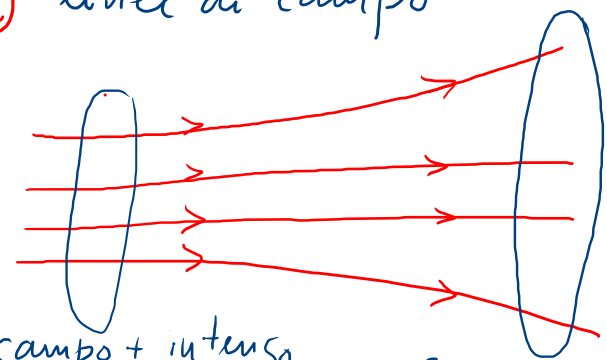
della particella

Come rappresentare un campo vettoriale?

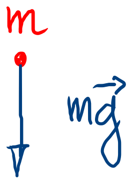
① su un insieme di punti

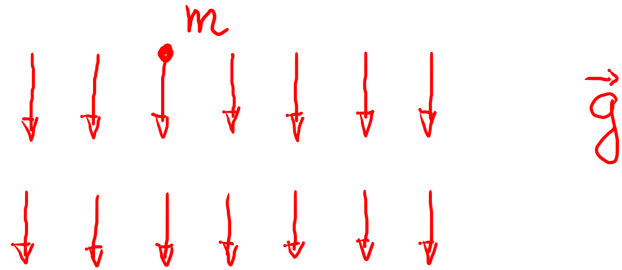


② linee di campo



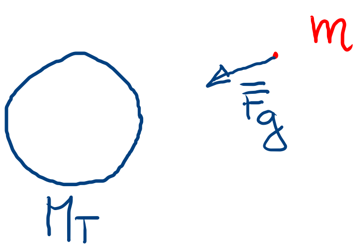
ESEMPIO : FORZA PESO $\Longleftrightarrow$ campo $\bar{g}$

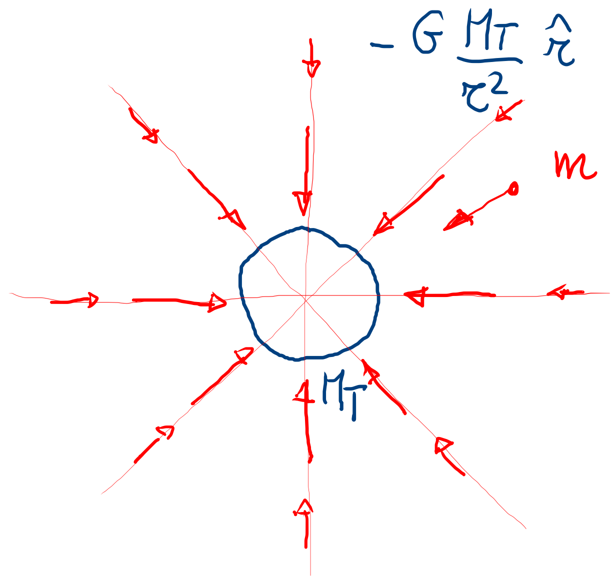




FORZA GRAVITAZIONALE DELLA TERRA

$$\vec{F}_g = -G \frac{M_T m}{r^2} \hat{r}$$





ENERGIA POTENZIALE E CAMPI DI FORZA

$$\mathcal{L} = -\Delta U$$

$\vec{F}$ costante

$\Delta\vec{x}$ rettilineo e parallelo ad $\vec{F}$

(problema è 1D $\xrightarrow{\quad\quad} x$)

$$\mathcal{L} = F \cdot \Delta x = -\Delta U$$

$$F = -\frac{\Delta U}{\Delta x}$$

$$\boxed{F = -\frac{dU}{dx}}$$

in 3D $\quad \vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k} \right)$

$$\boxed{\vec{F} = -\vec{\nabla} U}$$

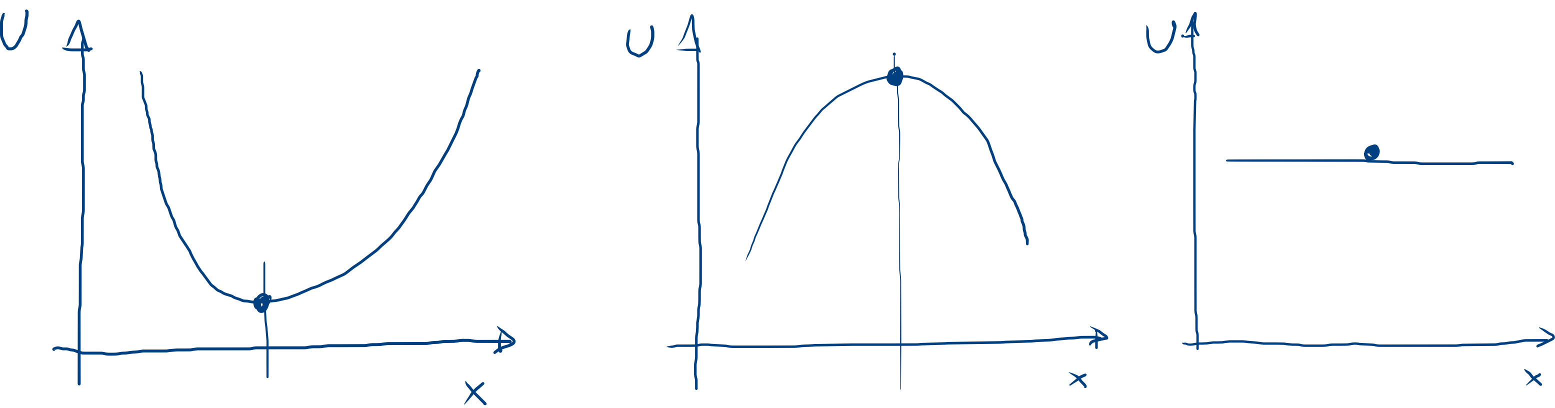
esempio $\quad U = mgz \quad$ (campo scalare)

$$\vec{F} = -\frac{\partial U}{\partial z}\hat{k} = -mg\hat{k} = +m\vec{g}$$

esempio: oggetto su tavolo orizzontale

$$U = mgz\Big|_{z=\text{cost}}$$

$$\vec{F} = -\vec{\nabla} U = 0 \qquad \text{(equilibrio)}$$



stabile (come nel caso della molla)

instabile

indifferente