UNIVERSITÀ DEGLI STUDI DI TRIESTE

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INDUSTRIAL PLANTS II

Chapter two ï part 1
Maintenance of Industrial Plants
Reliability Theory

DOUBLE DEGREE MASTER IN
ñPRODUCTION ENGINEERING AND MANAGEMENTò

CAMPUS OF PORDENONE
UNIVERSITY OF TRIESTE

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INDUSTRIAL PLANTS II

Chapter 2: Maintenance of Industrial Plants

SUMMARY:

INTRODUCTION TO MAINTENANCE

COMPANY/ORGANIZATION → PLANT/MACHINES INVESTMENT

TO ACCOMPLISH A MISSION / ACHIEVE ITS RESULTS → TO WORK FOR A SCHEDULED TIME

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INTRODUCTION TO MAINTENANCE

TO WORK FOR A SCHEDULED TIME

RELIABILITY

ĩthe capability of a device to fulfill a required function, under certain conditions, for a definite period of timeò. In other words it is the probability that a system (machine, subsystem, part) can work without stopping for a failure for a certain time t from its start-up and with certain ambient and company conditions

AVAILABILITY

It can be defined as the percentage of time of good performance in comparison to the total time where the performance is required

INTRODUCTION TO MAINTENANCE

RELIABILITY

Since the performance of any system inevitably tends to degrade along the time, the reliability of a system should be defined as the measure of its ability to provide a satisfying service over time.

The most complete definition is the one that indicates the reliability of a element or system as the probability that the element or system will perform a specific function:

- Å **Under specific operating conditions (C)**
- Å **Under specific ambient conditions (A)**
- Å **At a given instant and / or for a predetermined time interval. (t, ∆t)**

Reliability is a probability: it is not a deterministic quantity, which can be determined with analytical formulas, but a random variable, whose value can only be predicted through probabilistic considerations. This measure is nothing more than a numerical value, expressed on a scale of real numbers between 0 and 1.

RELIABILITY

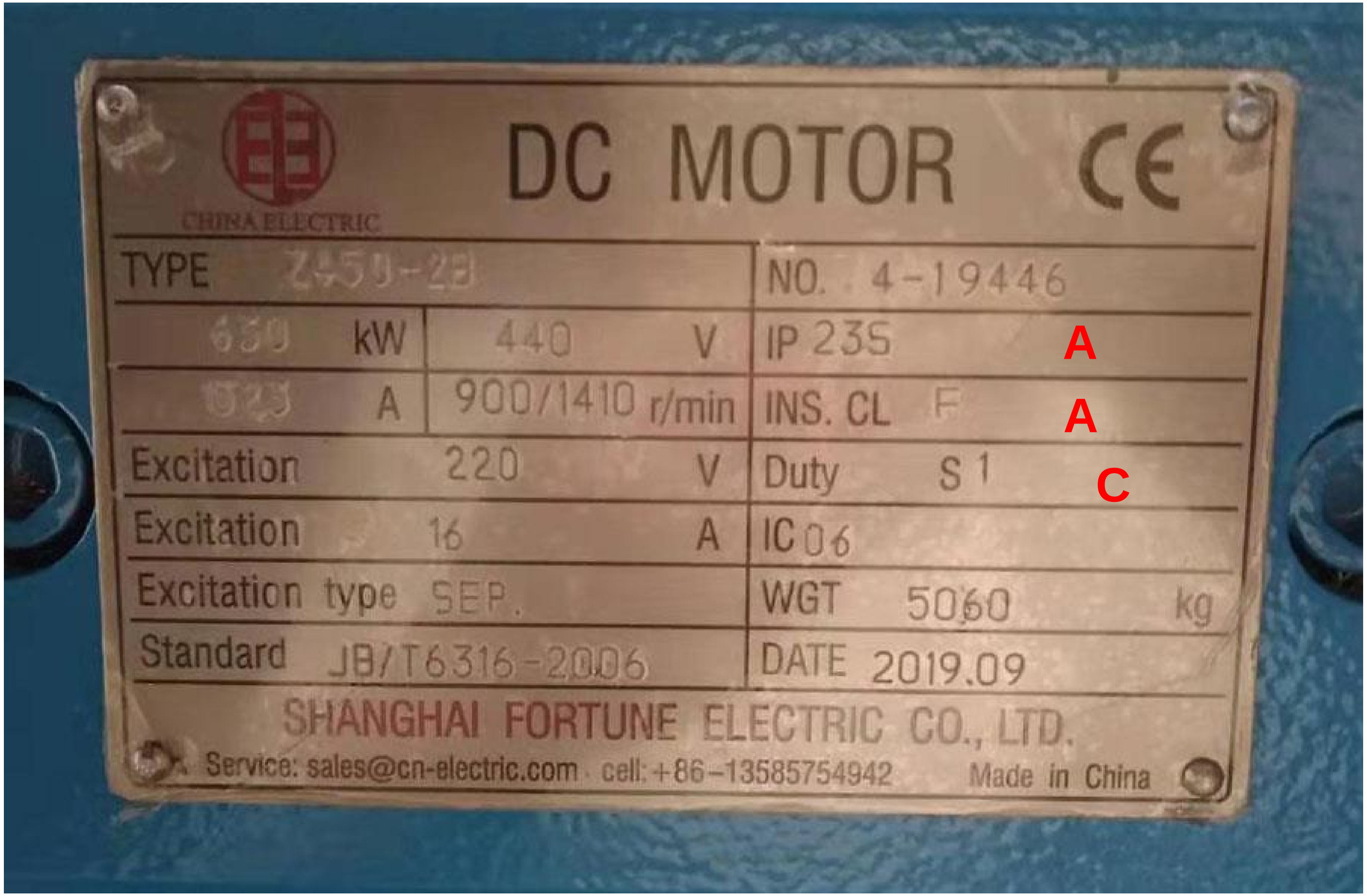
The definition of reliability is therefore linked to the specific function that the system has to perform and under the operating conditions in which it finds itself.
It is therefore necessary to define the design intent of the system

- **what is the function that the system must actually perform?**
- **what are the limit values of the operating and ambient conditions under which should the system work properly?**
- **at what instant or time interval should the system work?**
- **how diagnostic and maintenance techniques affect the operation of the system?**

(eg for a motor the type of service for which it is requested is important, see S1 = continuous, S2 = of limited duration, S3 = intermittent period, etc.)

INTRODUCTION TO MAINTENANCE

RELIABILITY



RELIABILITY

The definition of reliability assumes that:

1) the performance conditions are clearly defined (C). It means that the criteria for judging whether the element is working or not working are univocally defined.
For bistable systems (only 2 states of possible functioning) this criterion is obvious. For other systems it is possible also identify partial operating states that represent various levels of performance: in these cases the failure status can be defined once it is defined an admissible limit below which we consider the event as a fault (eg. efficiency of a motor, intensity of a light source);

2) the environmental conditions (A) of use are established and kept constant in the time period in question;

3) the time interval from instant 0 to instant t is defined mission time during which we expect the component to work.

RELIABILITY

The mathematical definition of reliability is therefore:

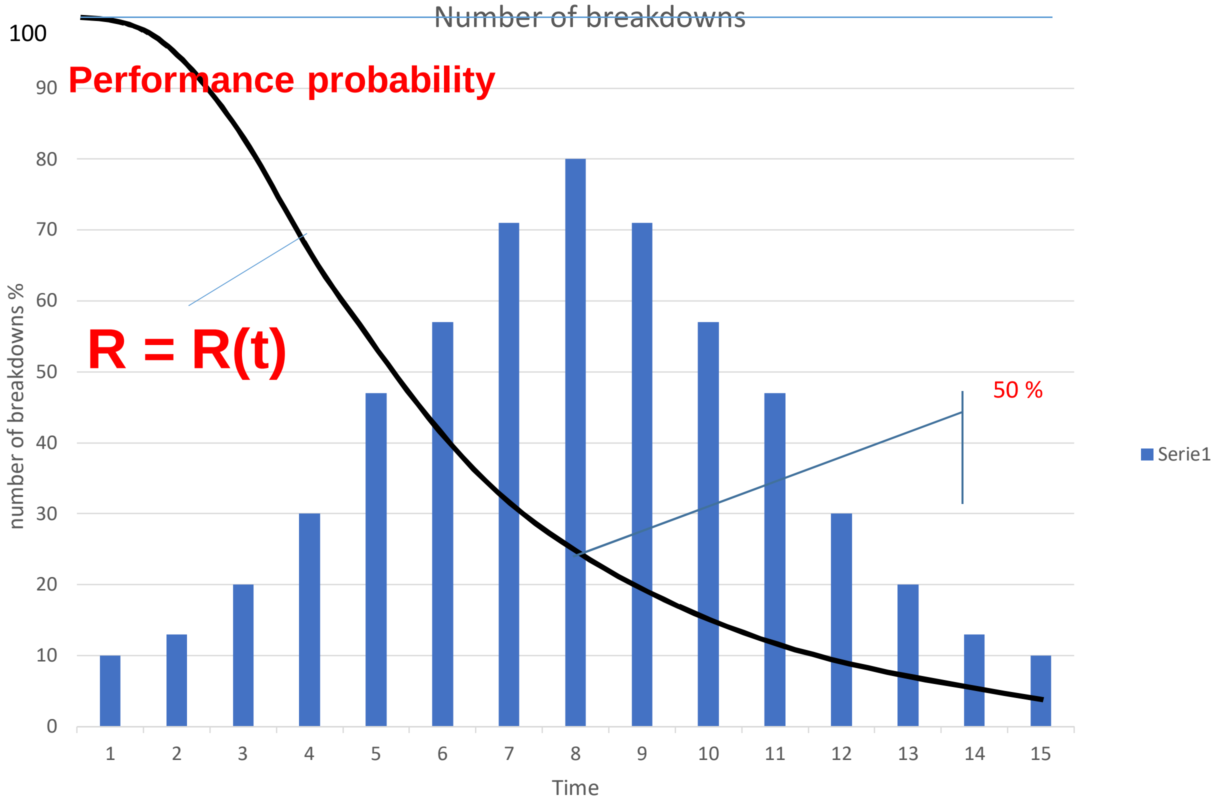
$$R = R(C, A, t)$$

And, if A and C are definite:

$$R = R(t)$$

INTRODUCTION TO MAINTENANCE

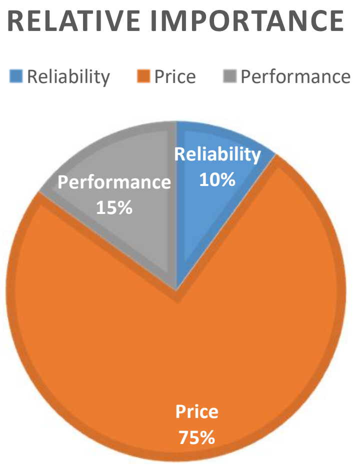
RELIABILITY

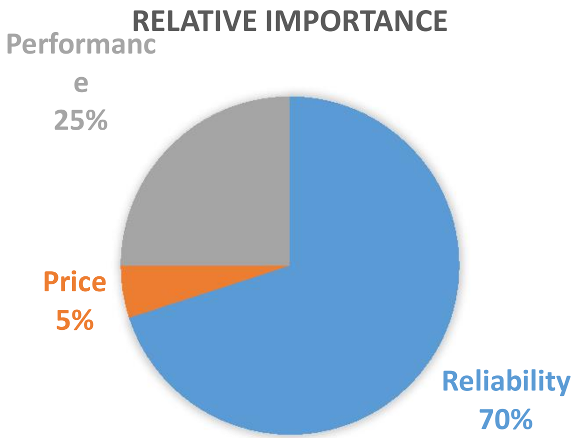


EXAMPLE (qualitative graph):
100 LAMPS
T= 1 H, BRD = 2, P%= 98%
T= 1000 H, BRD = 30, P%=70%

INTRODUCTION TO MAINTENANCE

RELIABILITY





INTRODUCTION TO MAINTENANCE

RELIABILITY

Let us consider a sample of components consisting of a large number N_0 of equal elements, all operating at instant t = 0 under certain operational and environmental conditions.

By measuring the functional parameters of the elements, we can establish, at each instant t, whether they are still functional or not.

If we indicate with:
N_v (t) the number of components operating at instant t;
N_g (t) the number of components fault at instant t.
Obviously, we must have for every instant t:

$$N_v(t) + N_g(t) = N_0$$

RELIABILITY

We can define:

1) **Reliability R (t)** as the probability for the single component to be still functioning at time t (i.e. after a time interval from 0 to t)

2) **Unreliability F (t)**, as the probability for the single component to be failure at time t (i.e. after a time interval from 0 to t):

$$R(t) = \frac{Nv(t}{N_0}$$

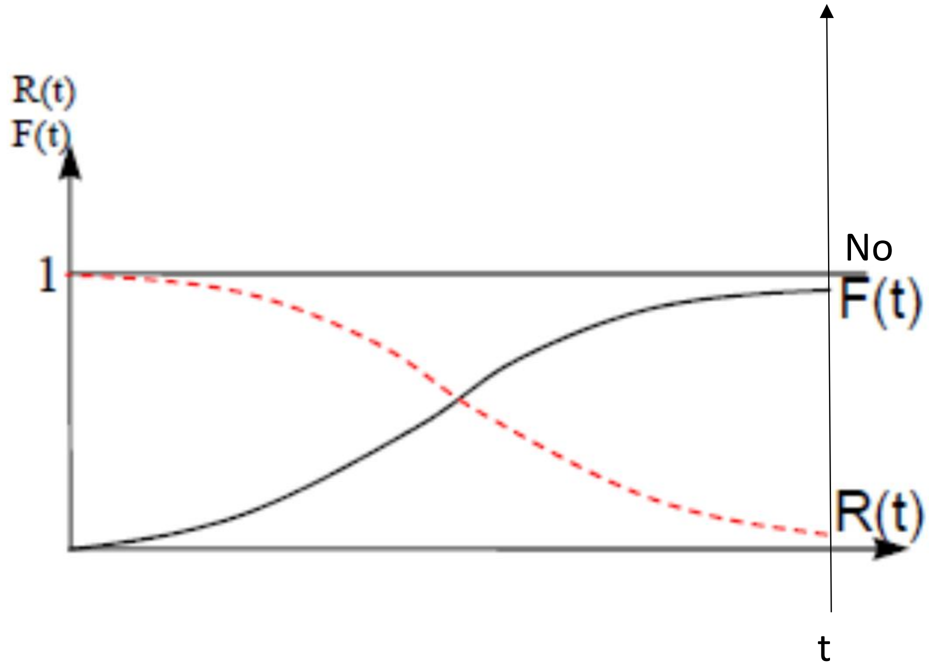
$$F(t) = \frac{Ng(t}{N_0}$$

$$\Rightarrow \quad R(t) + F(t) = \quad \Rightarrow \quad R(t) = 1 - F(t$$

RELIABILITY AND
CUMULATIVE FUNCTION (PROBABILITY) OF FAILURE

Graphically:

$$R(t) + F(t) =$$



Note: t represents the working time and F(t) is a cumulative function of failure, expressed as a percentage of faulty components in comparison with the total ones N_0.

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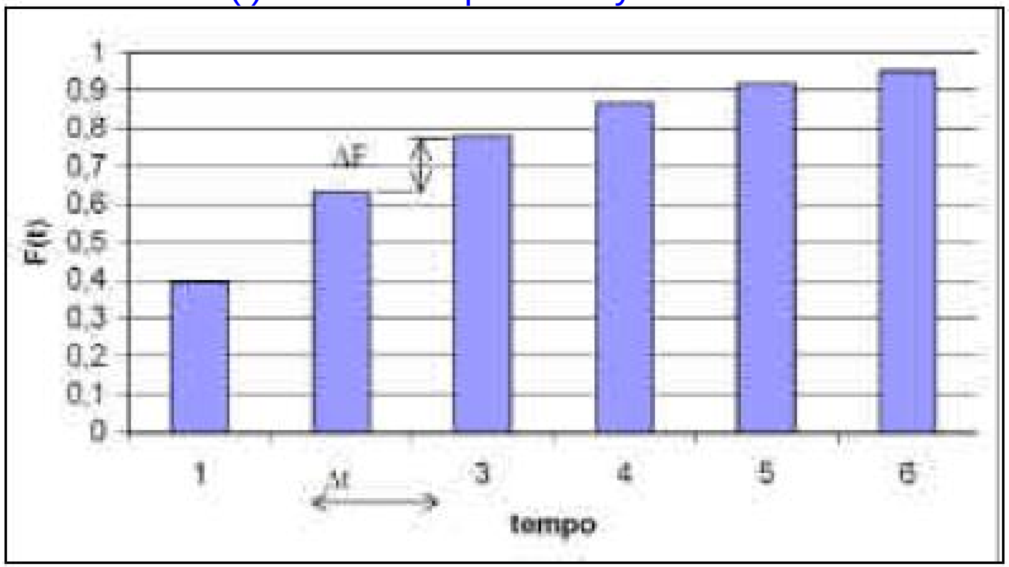
DENSITY OF PROBABILITY OF FAILURE

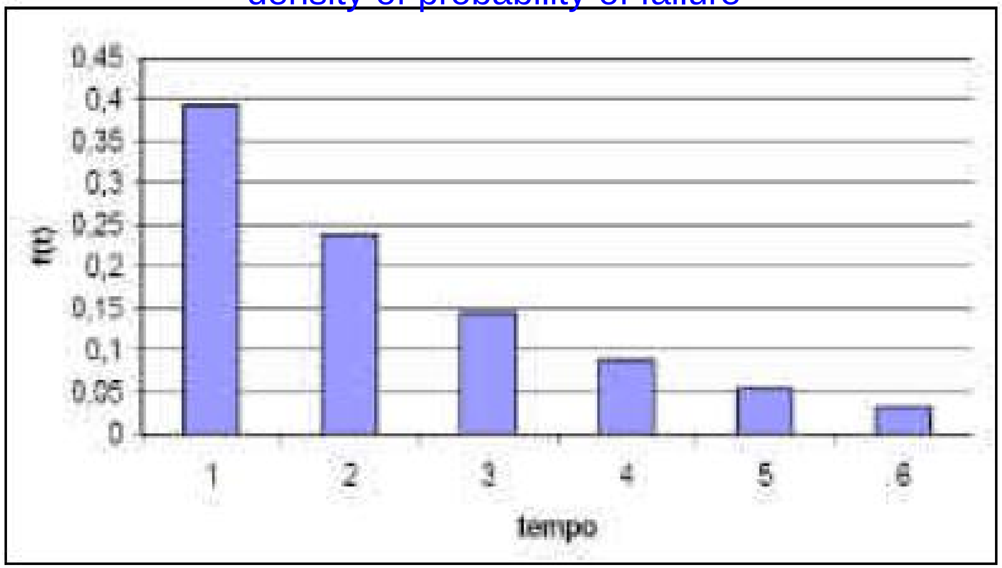
If we take into account the variations of the function F (t) at discrete intervals of amplitude qt, we can also define the function f (t) called density of probability of failure that is the probability f(t)dt that a component activated at t = 0,fails exactly between t and t+dt

$$f(t) = \frac{dF(t)}{dt} = \lim_{\Delta t \to 0} \frac{Ng(t+\Delta t) - Ng(t)}{\Delta t} \frac{1}{N_0} = \frac{dNg(t)}{dt} \frac{1}{N_0}$$

F(t) cumulative probability of failure



density of probability of failure



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DENSITY OF PROBABILITY OF FAILURE

The function f (t) represents a partial probability of failure relating to the interval [t, t + φt], in other terms, indicates with what frequency a component fails in the interval [t, t + φt].
For φt which tends to zero, considering the continuous function F (t), the f (t) is given by its derivative.
The f (t) can be expressed in p.u. ("Per unit"): per unit of time (second, hour, year, etc.).

$$f(t) = \frac{dF(t)}{dt} = \lim_{\Delta t \to 0} \frac{Ng(t+\Delta t) - Ng(t)}{\Delta t} \; \frac{1}{N_0} = \frac{dNg(t)}{dt} \; \frac{1}{N_0}$$

$$f(t) = \frac{dF(t)}{dt} = \frac{1}{N}\frac{dNg(t)}{dt} = -\frac{dR(t)}{dt}$$

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt = \int_t^\infty f(t)dt$$

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FAILURE RATE

Given a time unit, small as dt, failure rate is defined as the probability of a system that at time t is working properly, and fails in a time between t and dt.

$$\lambda(t) = \frac{f(t)}{N_v(t)/N_0} = f(t) \cdot \frac{N_0}{N_v(t)} = \left(\frac{dF(t)}{dt}\right) \cdot \frac{N_0}{N_v(t)}$$

Considering the discrete functions, we have:

$$f(t) = \frac{dNg(t)}{dt} \frac{1}{N_0}$$

$$\lambda(t) = f(t) \cdot \frac{N_0}{N_v(t)} = \left(\frac{\Delta N_g(t)}{\Delta t} \cdot \frac{1}{N_0}\right) \cdot \frac{N_0}{N_v(t)} = \frac{\Delta N_g(t)}{\Delta t} \cdot \frac{1}{N_v(t)}$$

The function $\lambda(t)$ represents the fraction of the population that fails in a interval Δt related to the number of components still functioning at the instant t.

FAILURE RATE

The failure rate can also be expressed as:

$$\lambda(t) = f(t)\cdot\frac{N_0}{N_v(t)} = \left(-\frac{dN_v(t)}{dt}\cdot\frac{1}{N_0}\right)\cdot\frac{N_0}{N_v(t)} = -\frac{dN_v(t)}{dt}\cdot\frac{1}{N_v(t)} = \frac{f(t)}{R(t)}$$

The failure rate is therefore the ratio, sign changed, between the derivative respect the time of the number of surviving objects at time t and the number of survivors themselves.
At the end:

$$\Lambda(t) = \frac{f(t)}{R(t)}$$

FAILURE RATE

In the particular case in which the failure rate ə-(t) remains constant over time (random failures) will simply be denoted by ə.
In this case failure rate can be derived by the following consideration:
The failure frequency is therefore given by:

$$N_g(t)+N_v(t)=N_g(t+\Delta t)+N_v(t+\Delta t)$$

$$N_g(t+\Delta t)-N_g(t)=N_v(t)-N_v(t+\Delta t)$$

$$\boxed{f(t)}=\lim_{\Delta t\to 0}\frac{N_g(t+\Delta t)-N_g(t)}{\Delta t}\cdot\frac{1}{N_0}=\lim_{\Delta t\to 0}\frac{N_v(t)-N_v(t+\Delta t)}{\Delta t}\cdot\frac{1}{N_0}=-\frac{dN_v(t)}{dt}\cdot\frac{1}{N_0}=\boxed{-\frac{dR(t)}{dt}}$$

FAILURE RATE

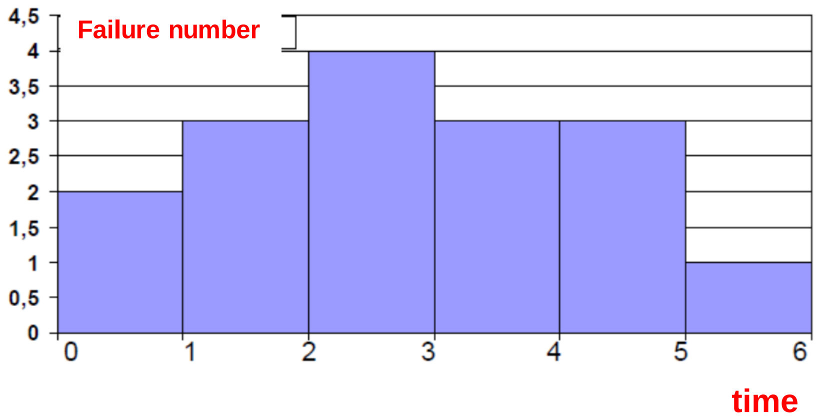
The failure rate can be interpreted as the ñnumber of failures in the unit of time ", or as a measure of speed of occurrence of the fault.

Difference between f (t) and ʼẠ(t)

$$\Lambda(t) = \frac{f(t)}{R(t)}$$

- f (t) refers to a healthy population at time t = 0
- ʼẠ(t) refers to a healthy population at time t, therefore less numerous than the original population at time t = 0

A reliability test on 16 identical bulbs gave the following results. Calculate ʼẠ(t) and f (t) in the time interval from 2 <t <3



f(2-3) = 4/16 = 0,25
ʼẠ̧(2-3) = 4/ (16-5) = 4/11= 0,36
R(2) = 11/16 = 0,68

FAILURE RATE

The unit of measurement of ə-(t) is the percentage of failures per unit of time and can assume values between:

$$\Lambda(t) = \frac{f(t)}{R(t)}$$

0 : when there are no faults around the instant considered;

: when all Nv (t) components still functioning fail at the same instant.

For a generic system, the failure rate of each can be determined of the components that constitute it, referring to both considerations of statistical type, and with parameters provided by the manufacturer, paying close attention to the life span of the considered component.

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FAILURE RATE

In the simplest case in which ə can be considered constant, generally assumes:

Åreliability: $$R(t) = e^{-\lambda t}$$

Consequently we obtain:

$$\Lambda(t) = \frac{f(t)}{R(t)}$$

Åprobability of failure: $$F(t) = 1 - R(t) = 1 - e^{-\lambda t}$$

Ådensity of probability of failure: $$f(t) = \frac{dF(t)}{dt} = \lambda e^{-\lambda t}$$

Åfailure rate: $$\frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

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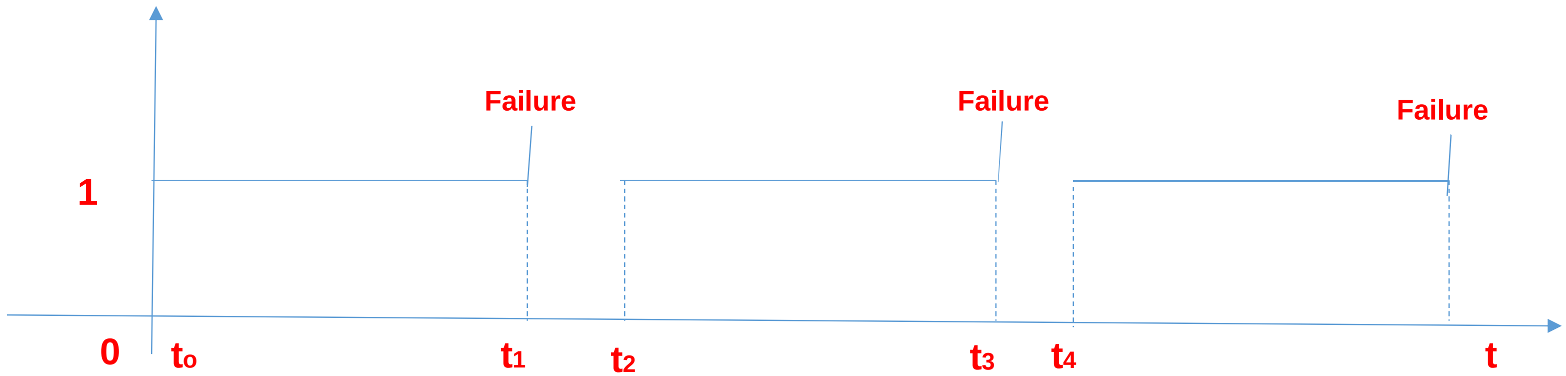
AVAILABILITY

If the systems or the components are repairable, it is also defined a function called
ñAvailabilityò A (t)

Availability can be defined as the percentage of time of good performance in comparison to the total time where the performance is required.

From the definition of reliability it is evident that, in the event scheduled maintenance, this must be performed when the system is idle, therefore when the system is not available.
Then we can say that maintenance makes the system unavailable too for the time necessary to repair it.
Hence Availability of a system is influenced by reliability of that system and by the maintenance criteria



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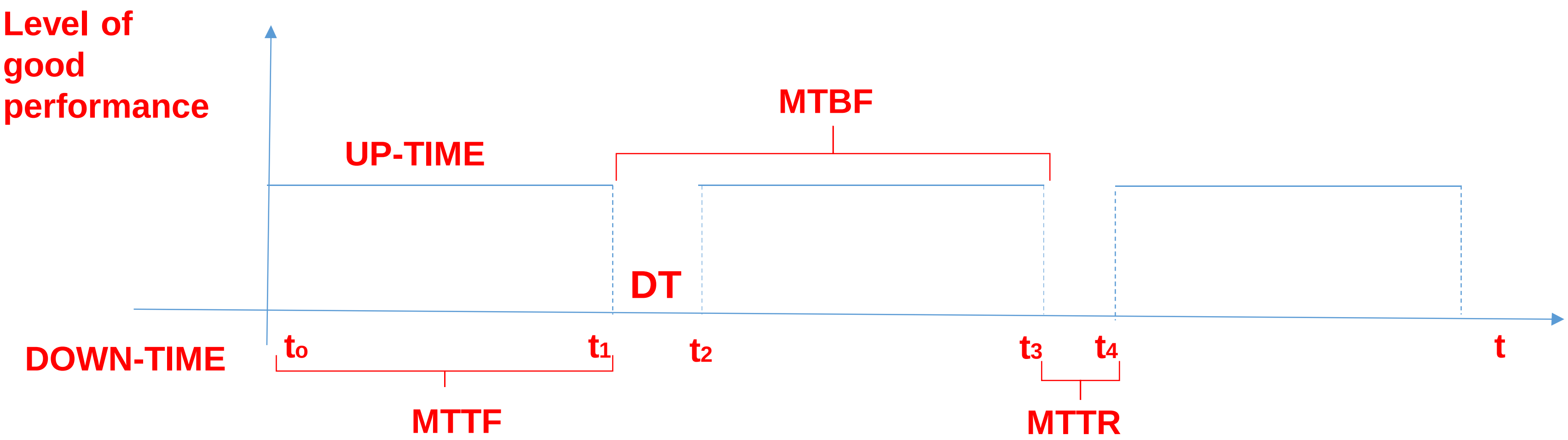
AVAILABILITY



UP-TIME = Time when the system is really operative and available

DOWN-TIME = Time when the system is not available for failure or maintenance

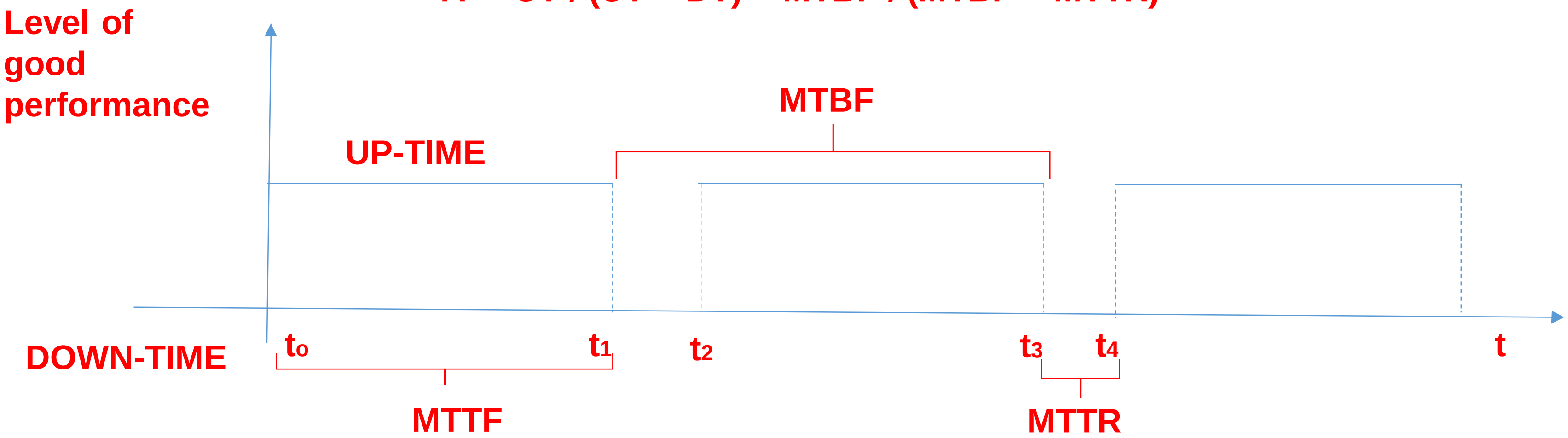
MTBF = Mean Time Before Failure

MTTR = Mean Time to Repair

A = UT / (UT + DT) = MTBF / (MTBF + MTTR)

AVAILABILITY

A = UT / (UT + DT) = MTBF / (MTBF + MTTR)



UP-TIME = Time when the system is really operative and available

DOWN-TIME = Time when the system is not available for failure or maintenance

MTBF = Mean Time Before Failure = MTTR + MTTF

MTTR = Mean Time to Repair

$$MTTF = \int_0^{\infty} t \cdot f(t)dt$$

AVAILABILITY

A = UT / (UT + DT) = MTBF / (MTBF + MTTR)

MTTF = Mean Time to Failure It represent the medium time between the instant 0 in which the component works and the failure instant. This is the medium functioning time of a component, that is the expected value of failure rate.

$$MTTF = \int_0^\infty t \cdot f(t)dt$$

In case ᾿Ạ is constant:

$$MTTF = \int_0^\infty t \cdot \lambda e^{-\lambda t} dt = \lambda \int_0^\infty t \cdot e^{-\lambda t} dt$$

As we know:

$$\int x \cdot e^{ax} dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

We get: MTTF $= \lambda \left[\frac{e^{-\lambda t}}{-\lambda}\left(t - \frac{1}{-\lambda}\right)\right]_0^\infty = \lambda \left[0 - \left(-\frac{1}{\lambda}\left(0 + \frac{1}{\lambda}\right)\right)\right] = \lambda \frac{1}{\lambda^2} = \frac{1}{\lambda}$

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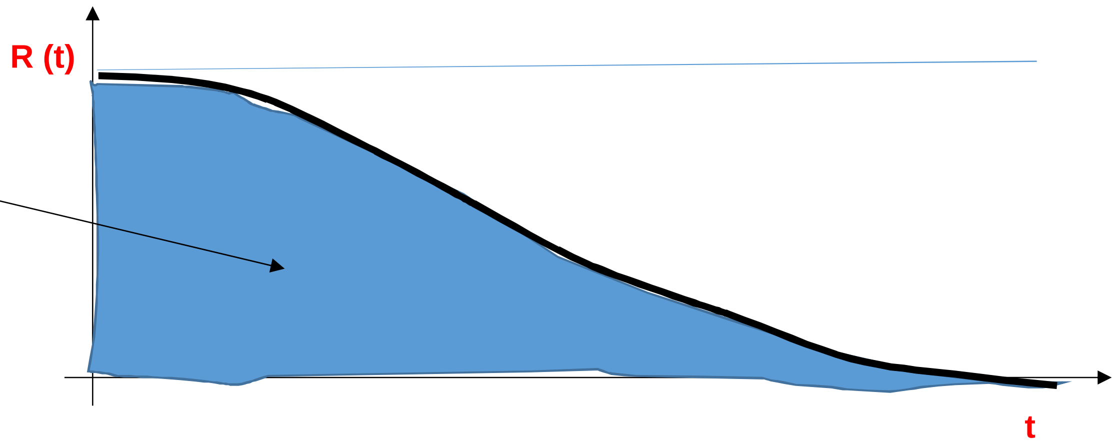
AVAILABILITY

MTTF = Mean Time to Failure It represent the medium time between the instant 0 in which the component works and the failure instant . This is the main parameter to define the reliability

$$MTTF = \int_0^{\infty} t \cdot f(t)dt$$

MTTF = $$\lambda\left[\frac{e^{-\lambda t}}{-\lambda}\left(t-\frac{1}{-\lambda}\right)\right]_0^{\infty} = \lambda\left[0-\left(-\frac{1}{\lambda}\left(0+\frac{1}{\lambda}\right)\right)\right] = \lambda\frac{1}{\lambda^2} = \frac{1}{\lambda}$$

$$MTTF = \int_0^{\infty} R(t)dt$$



AVAILABILITY/RELIABILITY

If we need to have an high reliability, what the MTTF value should be?
Letôs suppose that ʺẠ= constant, therefore we know that:

$$R(t) = e^{-\lambda t} \qquad MTTF = \frac{1}{\lambda}$$

Therefore, if t is the mission time:

$$t = MTTF = \frac{1}{\lambda} \Rightarrow R(t) = e^{-1} \cong 0{,}368$$

$$t = \frac{MTTF}{10} = \frac{1}{10\lambda} \Rightarrow R(t) = e^{-1/10} \cong 0{,}905$$

$$t = \frac{MTTF}{100} = \frac{1}{100\lambda} \Rightarrow R(t) = e^{-1/100} \cong 0{,}990$$

The conclusion is that, in order to have an high value of reliability, MTTF has to be at least 100 times longer that the mission time

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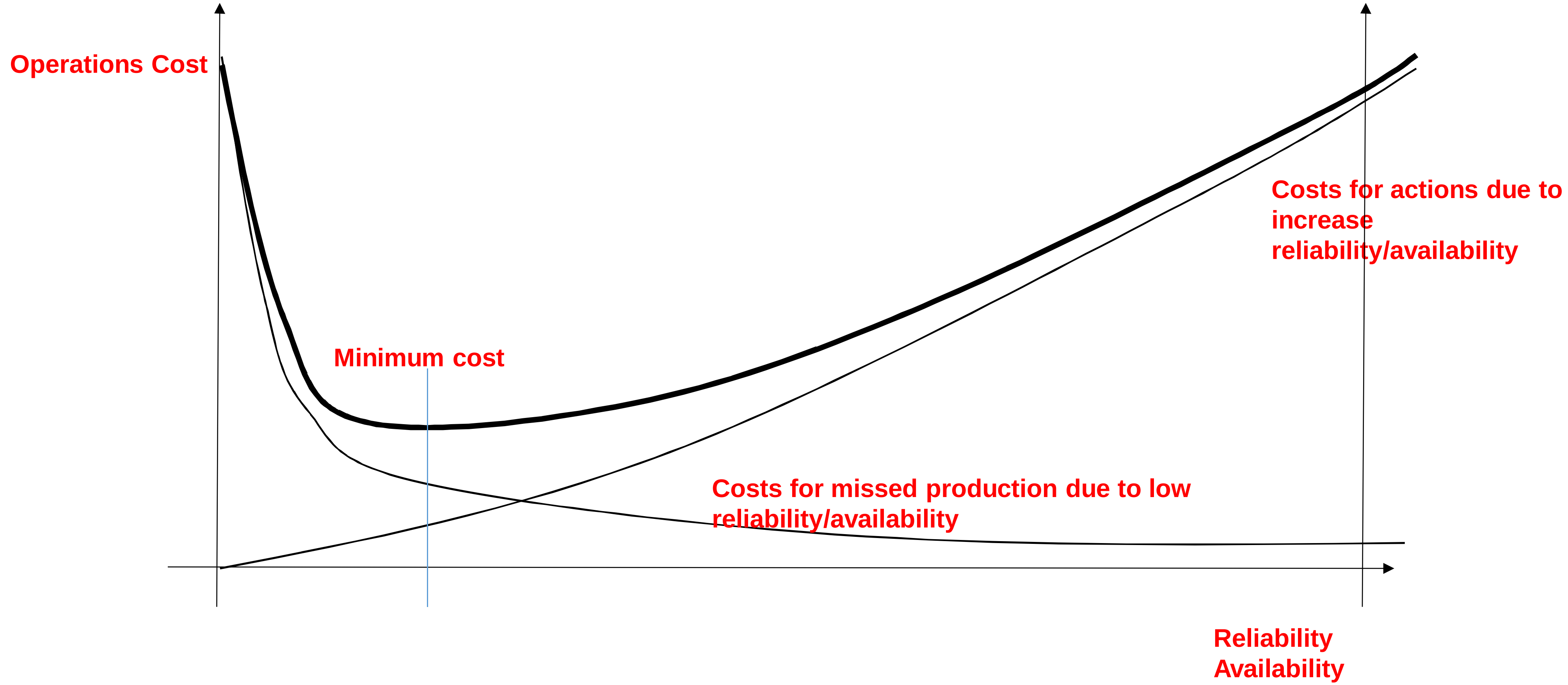
INTRODUCTION TO MAINTENANCE

TO WORK FOR A FORECASTED TIME CORRECTLY

Total cost vs. Reliability/ Availability



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THE CONCEPT OF ñFAILUREò

As already mentioned, the Italian Norm CEI 56-60 standard defines the term ñfailureò as the end of an object's ability to perform the required function, that is, a change in the performance of a device that makes it unusable for the use for which it is intended.

Here are some fault classification criteria:

	failure criteria		
	Entity	Impact	part's life
	total	critical	casual
failure type	partial	primary	infantile
	discontinous	secondary	wear

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THE CONCEPT OF ñFAILUREò ENTITY

A device fails even when it does not perform correctly the function for which it was designed.
According to this criterion we can divide the faults into 3 categories:

partial failures: they determine a change in the device performance such as not to completely compromise the functioning (e.g. performance degradation of an engine);

total failures: they cause a change in the device performance such as to completely prevent its operation;

intermittent failures: due to a random succession of failure periods and periods of operation, without any intervention of maintenance (typical example is a computer stop that resumes operation after resetting)

THE CONCEPT OF ñFAILUREò IMPACT

The fault condition generally refers only to the considered device: if this component is a part of a complex system, its failure may not even cause the entire system to fail, although it has negative effects on its reliability.
Eg. a mechanical failure of the engine makes a car useless, while a failure of the speedmeter the car continues to work, even if we don't know how fast we are driving.

We can then distinguish:

failures of secondary importance: those that do not reduce functionality of the entire system of which they are part;
failures of primary importance: those that reduce severly the functionality of the whole system of which they are part;
critical failures: even more serious than primary failures, represent a risk to the safety of people.

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THE CONCEPT OF "FAILURE" COMPONENT'S LIFE

There are hree types of failures based on their distribution during the life of a group of identical components (in the same operating and environmental conditions):

infantile failures: they occur in the first period of the components' life (First operations period) The nature of these failures is linked to intrinsic defects of the components that did not emerge during testing; if the design is correct, they are essentially due to construction errors and, mainly, assembly; the period during which faults occur of this type can vary from a few tens to several hundreds of hours operation. They can be minimized (but not zero) with an adequate quality control system.

Random failures: those that occur during the entire life of the components and have a probability of occurrence that is independent of the time; are due to rundom factors that not even a good project and

good execution can eliminate.

Failures due to wear: they happen only in the last period of life components and are due to aging and deterioration phenomena; therefore their probability of occurrence increases with the time of use. They can be reduced with an appropriate maintenance strategy.

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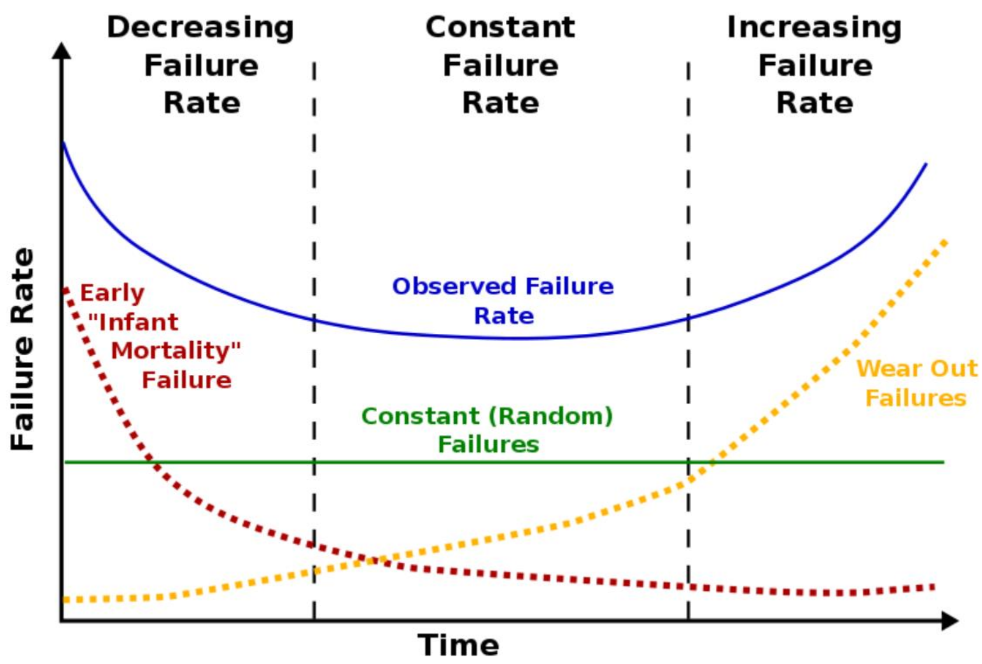
THE CONCEPT OF ñFAILUREò COMPONENTôS LIFE

If we consider a population of new components, all the same, and we do them operate under the same operating and environmental conditions starting from same instant t = 0, it is possible to draw the diagram shown in the figure, which reports the trend of the failure rate according to the age of the components instantaneous of the same.



Bathtub curve

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THE CONCEPT OF ñFAILUREò
COMPONENTÕS LIFE
WEIBULLÕS FUNCTION

The Weibull function is a two parameters function which, thanks to its flexibility, can be used to express the reliability function both during the stage of infantile breakdowns and during its useful life.

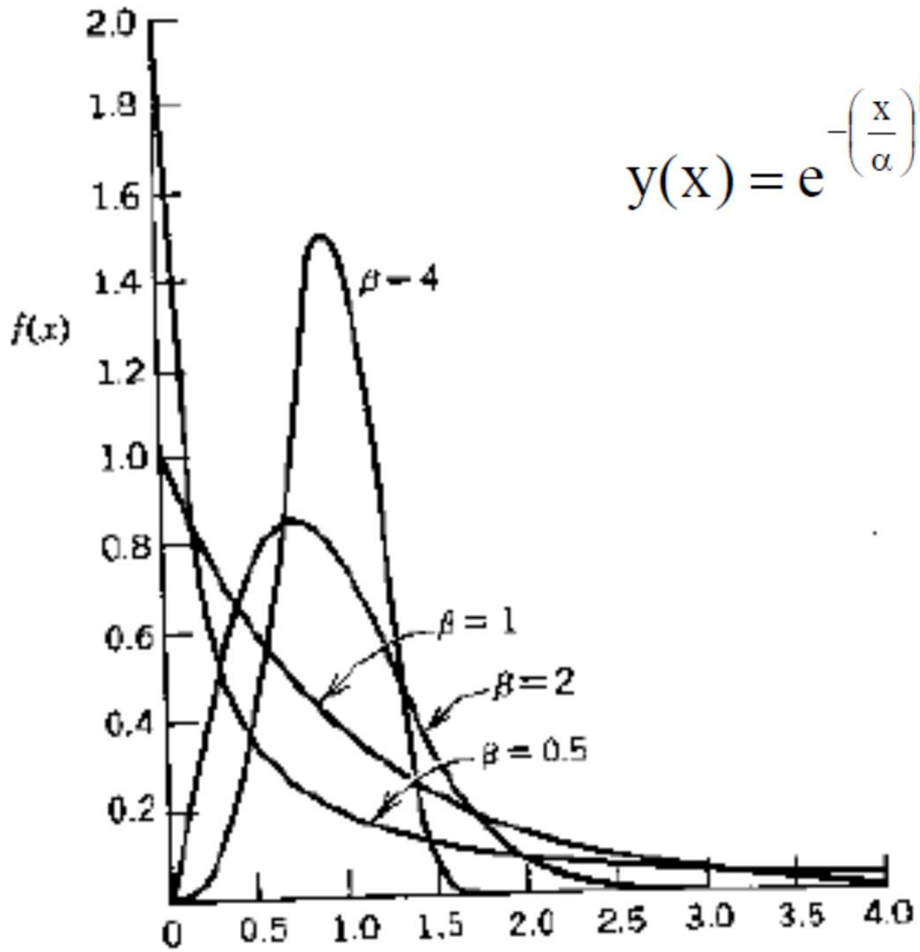
It is characterized by two parameters Ŭ and ß with positive value:

Ŭ shows the characteristic life (time)

ß is pure number that is a shape parameter, generally between 0.5 and 5.

If <1, the function is monotone decreasing,

if> 1,first it grows and then it decreases



$$y(x) = e^{-\left(\frac{x}{\alpha}\right)^{\beta}}$$

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THE CONCEPT OF ñFAILUREò
COMPONENTôS LIFE
WEIBULLôS FUNCTION

The machineôs initial life is described by a Weibull distribution of the function R(t):

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

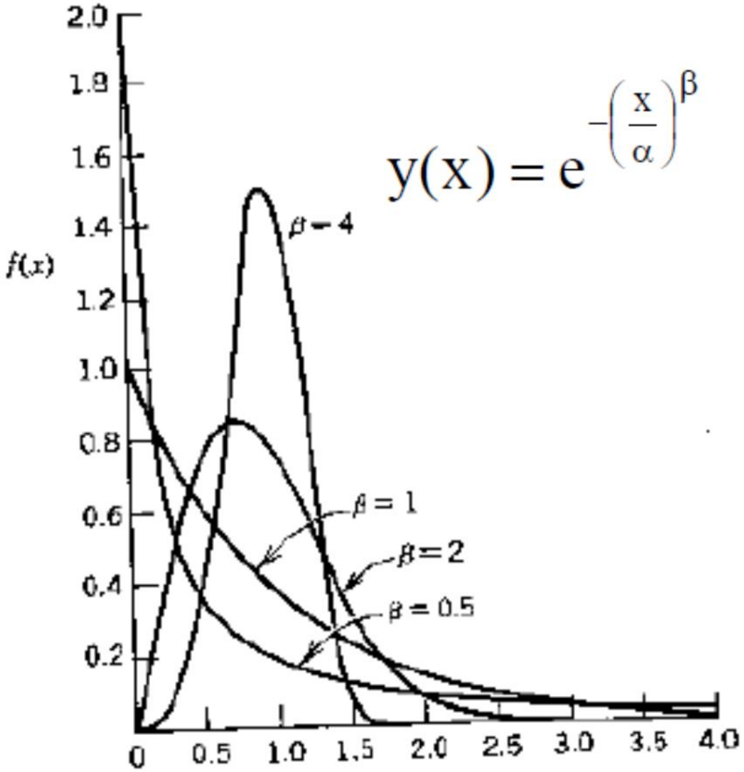
The percentage of parts population that fail at time t is:

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

The probability density of failures is:

$$f(t) = \underbrace{\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}}_{h(t)} * \underbrace{e^{-\left(\frac{t}{\alpha}\right)^{\beta}}}_{R(t)}$$

”Ạ=h(t)



THE CONCEPT OF ñFAILUREò COMPONENTôS LIFE WEIBULLôS FUNCTION

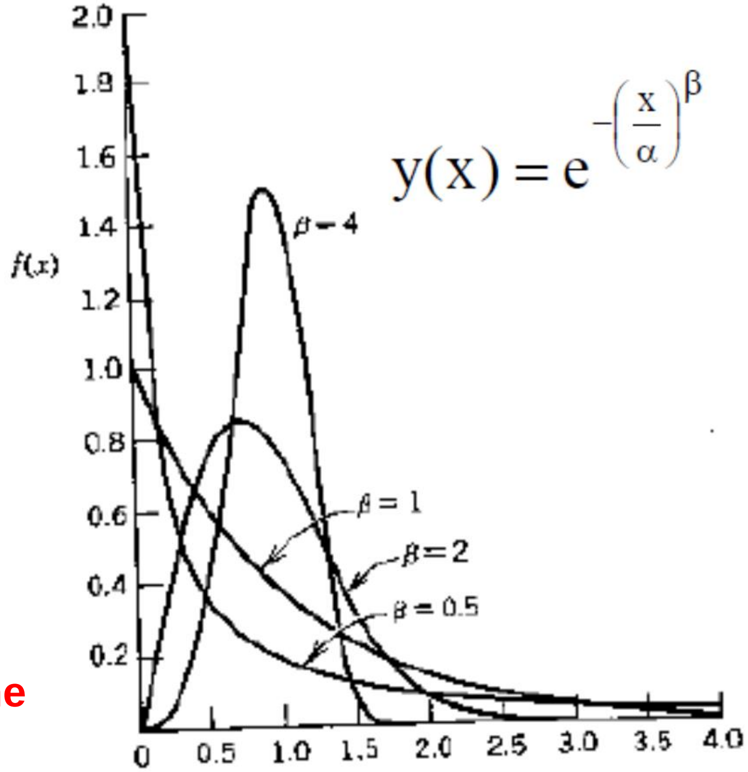
If the failure rate λ(t) is constant (the component fails casually)

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

With $\beta = 1 \quad \lambda = \frac{1}{\alpha}$

$$R(t) = e^{-\lambda t}$$

$$h(t) = \lambda$$



Therefore the infantile failures can be approximate with the exponential curve

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THE CONCEPT OF ñFAILUREò COMPONENTôS LIFE WEIBULLôS FUNCTION

If we consider a condensation unit, its life can be represented by a Weibull distribution with Ŭ = 100.000 and ɓ = 0,5.

After a year operation (8760 h):

The probability of good performance is :

$$R(t) = e^{-\left(\frac{8760}{100000}\right)^{0,5}} = 74\%$$

The probability of failures is:

$$F(t) = 1 - e^{-\left(\frac{8760}{100000}\right)^{0,5}} = 26\%$$

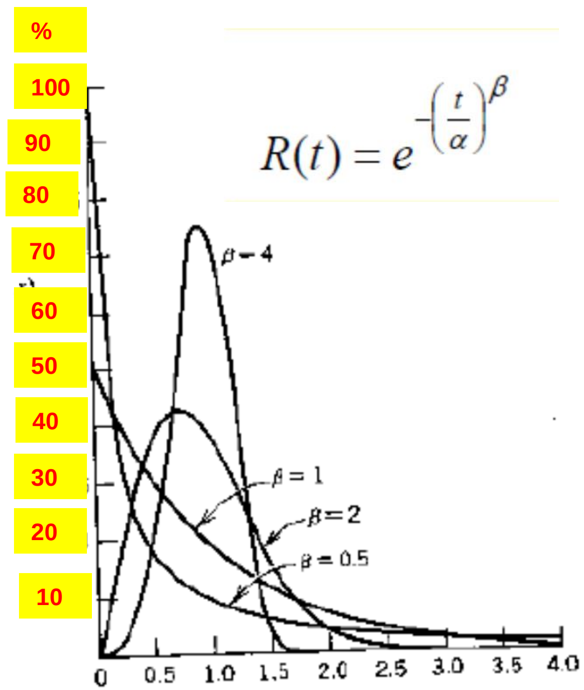
After two years:

$$R(t) = e^{-\left(\frac{17520}{100000}\right)^{0,5}} = 66\%$$

$$F(t) = 1 - e^{-\left(\frac{17520}{100000}\right)^{0,5}} = 34\%$$

8760/100000 =0,09

17520/100.000 = 0,175



Materiale riservato
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THE CONCEPT OF ñFAILUREò
COMPONENTôS LIFE
WEIBULLôS FUNCTION

The last part of the curve can be approximate with a normal distribution of the function f(t)

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

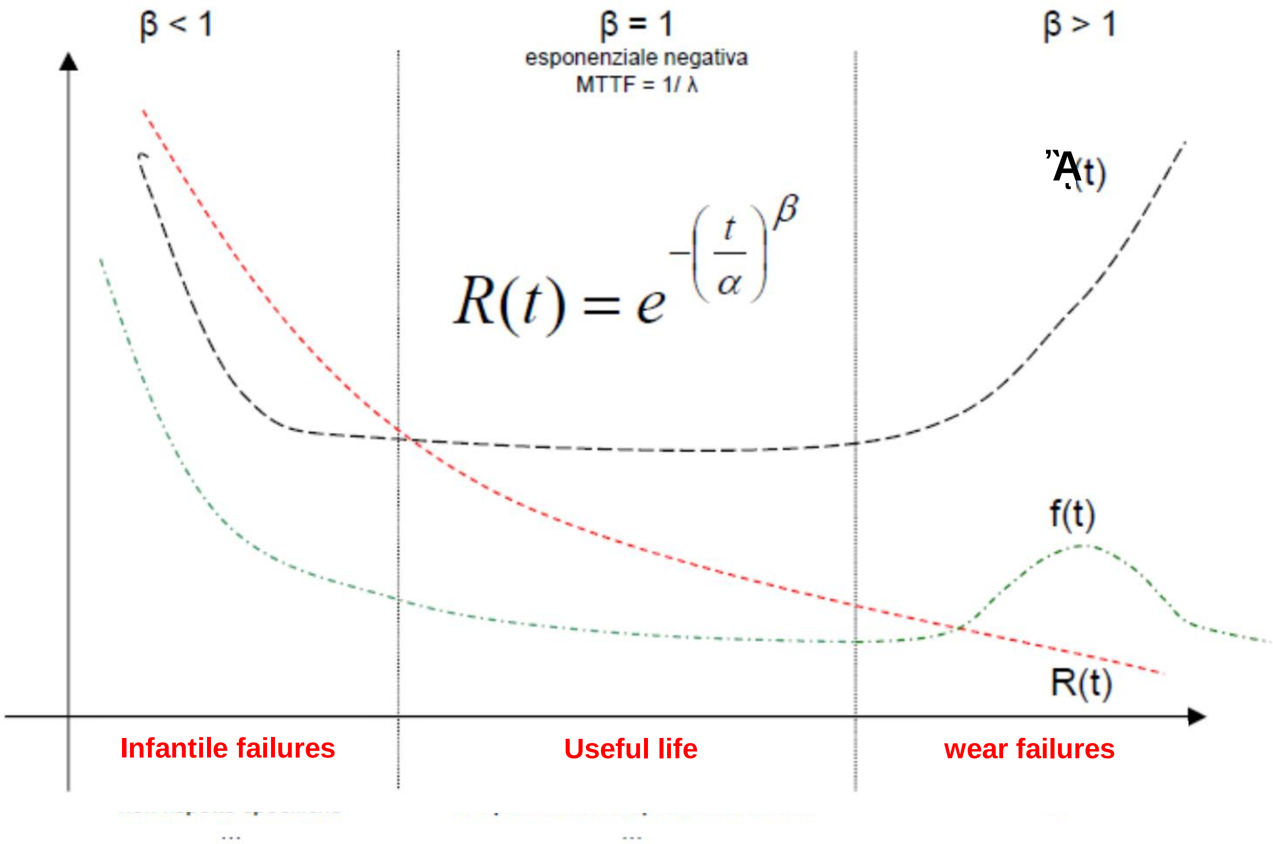
With:

$$\lambda(t) = \frac{e^{-1/2\left[\frac{t-\mu}{\sigma}\right]^2}}{\int_t^{\infty} e^{-1/2\left[\frac{t-\mu}{\sigma}\right]^2} dt}$$

And: MTBF = μ

THE CONCEPT OF ñFAILUREò
COMPONENTôS LIFE
WEIBULLôS FUNCTION

Finally, summarizing:



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THE CONCEPT OF ñFAILUREò
OTHER CLASSIFICATION CRITERIA

Progressive failures: They could be predicted (and therefore avoided) with a suitable equipment monitoring. Furthermore, the failures can be distinguished in slowly or rapidly progressive, according to the speed of failure.

Sudden failures: they cannot be foreseen and avoided.

Intrinsic failures: probably due to inherent weaknesses in the device;

Extrinsic failures: probably due to overstresses beyond the possibilities of the device (e.g. ambient temperatures higher than the ones reported in the design specifications like overloads, overvoltages,

RELIABILITY OF THE COMPLEX SYSTEMS

- **Non Redundant Systems (series systems): Failures result as soon as a component fails**

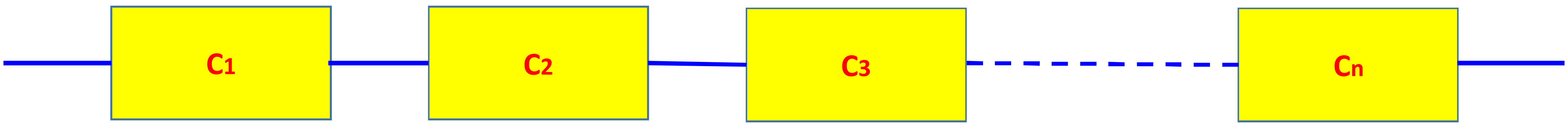
- **Redundant systems (parallel systems): They do not fail if one of their components fails**

- **Non-Repairable Systems : They are no longer repairable when one of their components fails**

- **Repairable Systems (Maintainability): They can be repaired when one of their components fails**

RELIABILITY OF THE COMPLEX SYSTEMS
SERIES CONNECTION

The reliability of a system in series is the probability that at instant t all the connected components are functioning.



If we assume that the possible faults are each other statistically independent, and if we define Ri (t) the reliability of the i-th element, the overall reliability of the entire Rs (t) system will be:

$$Rs(t) = R_1(t) \,.\, R_2(t) \,.\, \ldots\ldots R_n(t)$$

In the case of a constant failure rate, there will be:

$$R_s(t) = e^{-(\lambda_1+\lambda_2+\ldots+\lambda_n)t}$$

HYPOTHESIS - Non-repairable systems
- They work if and only if all components work (Non Redundant)
- There are only two states: good or bad
- The components are statistically independent

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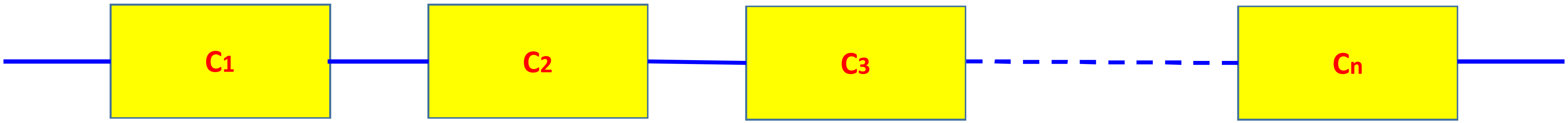
RELIABILITY OF THE COMPLEX SYSTEMS
SERIES CONNECTION

- **From the previous expression, it can be deduced that the reliability of a complex system with series connection decreases as the number of its components increases.**
- **The overall failure rate is equal to the sum of the individual failure rates elements.**
- **The overall reliability is numerically less than the smallest value of reliability present among the various components.**
- **There is a greater percentage increase of total reliability if we do actions to increase the reliability of the less reliable component.**



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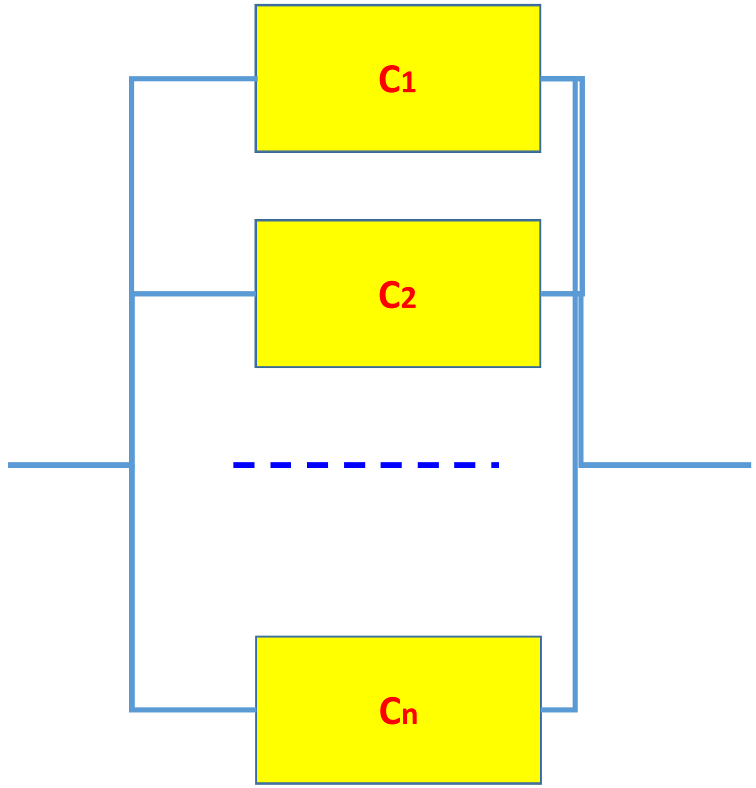
RELIABILITY OF THE COMPLEX SYSTEMS
PARALLEL CONNECTION

A complex system has a parallel (redundant) structure when the fault of a single component does not compromise the function of the whole system is the probability that at instant t all the connected components are functioning.

It means that, in case of an n component system, all the components should fail in the same time, to provoke the stop of the whole system



HYPOTHESIS - Non-repairable systems

- They work if only one component works (Redundant)
- There are only two states: good or bad
- The components are statistically independent

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RELIABILITY OF THE COMPLEX SYSTEMS
PARALLEL CONNECTION

- **In case of a complex system with parallel connection, the failure probability (unreliability) is:**

$$F_p(t) = F_1(t) \cdot F_2(t) \cdot \ldots \cdot F_n(t)$$

$$\Downarrow$$

$$1 - R_p(t) = (1 - R_1(t)) \cdot (1 - R_2(t)) \cdot \ldots \cdot (1 - R_n(t))$$

$$\Downarrow$$

$$R_p(t) = 1 - (1 - R_1(t)) \cdot (1 - R_2(t)) \cdot \ldots \cdot (1 - R_n(t))$$

$$\Downarrow$$

$$R_p(t) = 1 - \left(1 - e^{-\lambda_1 t}\right) \cdot \left(1 - e^{-\lambda_2 t}\right) \cdot \ldots \cdot \left(1 - e^{-\lambda_n t}\right)$$

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RELIABILITY OF THE COMPLEX SYSTEMS
PARALLEL CONNECTION

Å In case of a complex system with parallel structure with only two components, the total reliability is:

$$R_{p12}(t)=1-\left(1-e^{-\lambda_1 t}\right)\cdot\left(1-e^{-\lambda_2 t}\right)=1-\left(1-e^{-\lambda_1 t}-e^{-\lambda_2 t}+e^{-\lambda_1 \lambda_2 t}\right)$$

$$R_{p12}(t)=e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-\lambda_1 \lambda_2 t}$$

Å In a complex system with parallel structure reliability increases with the number of components and, from a numerical point of view, total reliability is higher than the most reliable component.

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KEY MESSAGES

- There are many math functions related to the performances of a system:
  - Reliability
  - Cumulative probability of failure
  - Function of distribution of probability
  - Failure rate
- The curve of Weibull can represent convenientely the reliability curves
- Availability, MTTR, MTBF
- Series and parallel systems