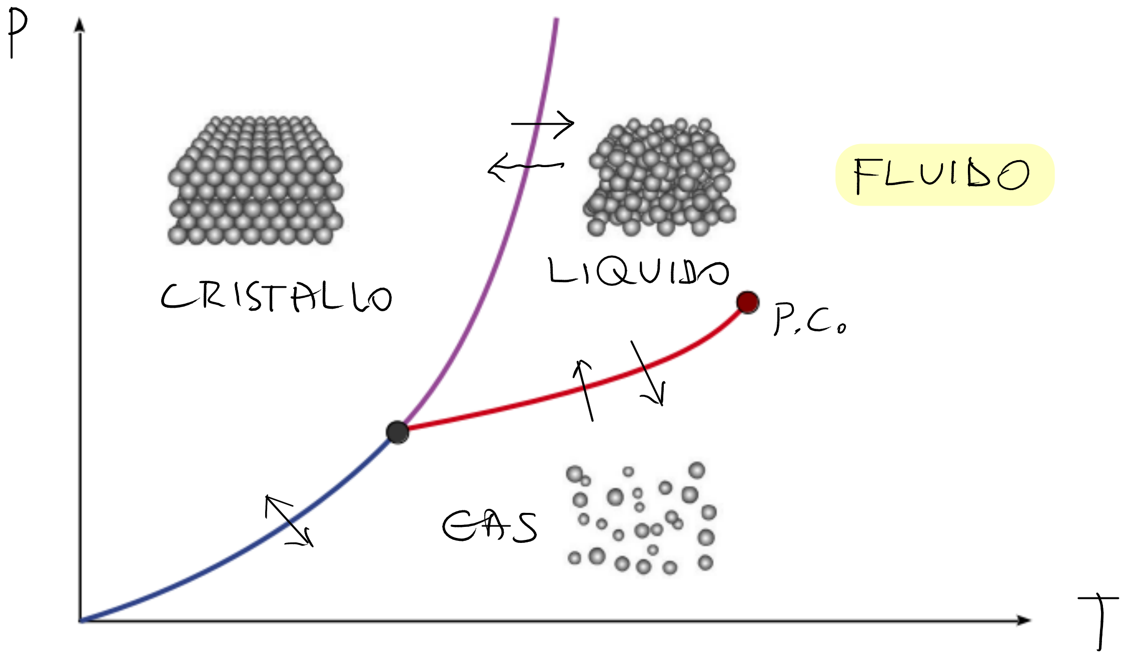
MECCANICA DEI FLUIDI

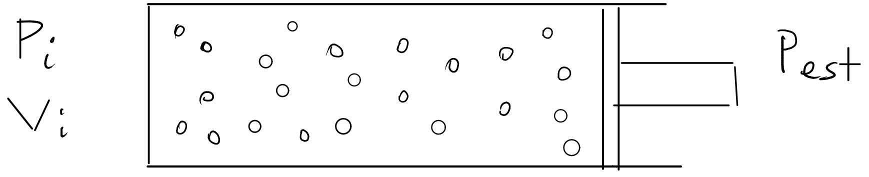


Fluido alta densità $\equiv$ liquido

Fluido bassa densità $\equiv$ gas

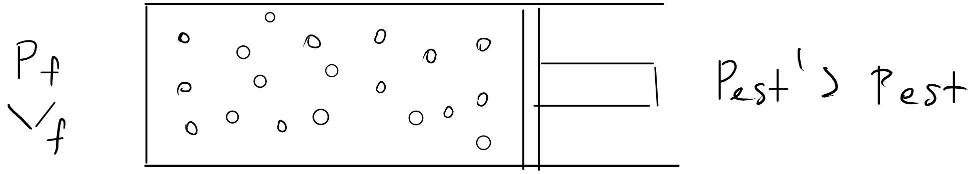
Gas : alta comprimibilità

Liquido : bassa comprimibilità



$$\Delta V = V_f - V_i$$

$$\Delta P = P_f - P_i$$



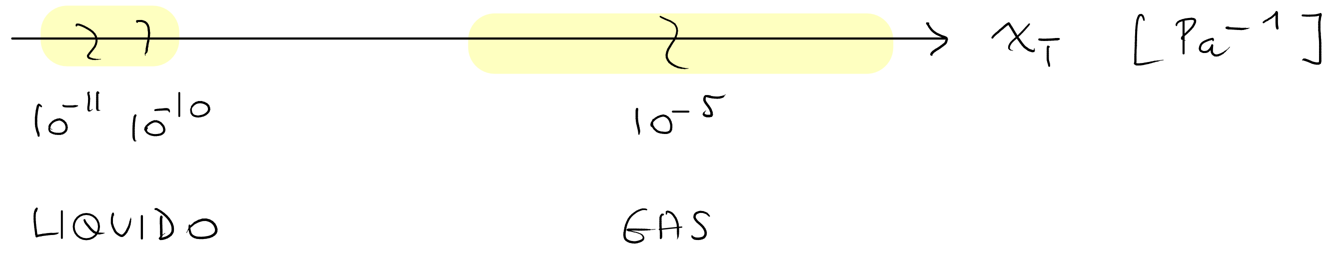
$\Delta V \sim \Delta P$ $\qquad$ $\Delta V = \underset{<0}{k}\, \Delta P$

Comprimibilità isoterma $\quad T = \text{cost} \quad T_i = T_f \quad (N = \text{cost})$

$$\frac{\Delta V}{V} = - \chi_T \, \Delta P \qquad \chi_T \equiv - \frac{1}{V} \frac{\Delta V}{\Delta P} \qquad SI: \; Pa^{-1}$$

$$\chi_T \equiv - \frac{1}{V} \frac{dV}{dP} \qquad T = \text{cost}$$

$$\chi_T \equiv - \frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T$$

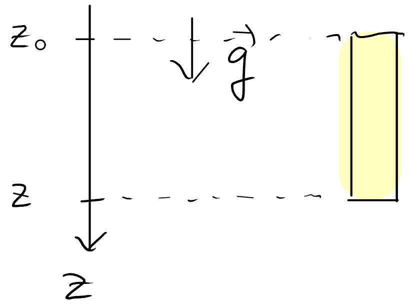


Es: χ_T per un gas perfetto $\quad PV = nRT$

$P = 10^5 \, Pa \sim P_{atm}$

FLUIDOSTATICA

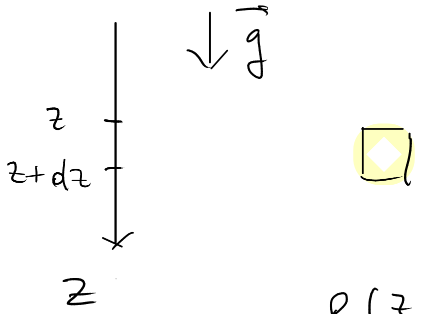
Legge di Stevino : fluido $\rho = \text{cost}$



$$P = P_0 + \rho g (z - z_0)$$

$$\Delta P = \rho g \Delta z$$

Legge fondamentale della fluidostatica



$\rho(z)$

$$P(z+dz) = P(z) + \rho g \, dz$$

$$dP = \rho g \, dz$$

$$\frac{dP}{dz} = \rho g$$

⚠

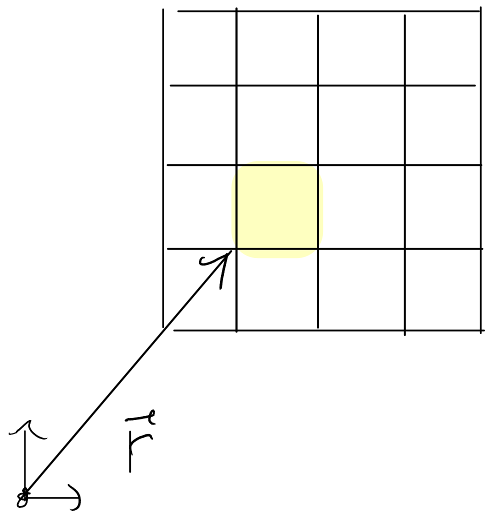
$\rho = \rho(z)$

$$T = T(\bar{r})$$

$$P = P(\bar{r})$$

$$\rho = \rho(\bar{r})$$

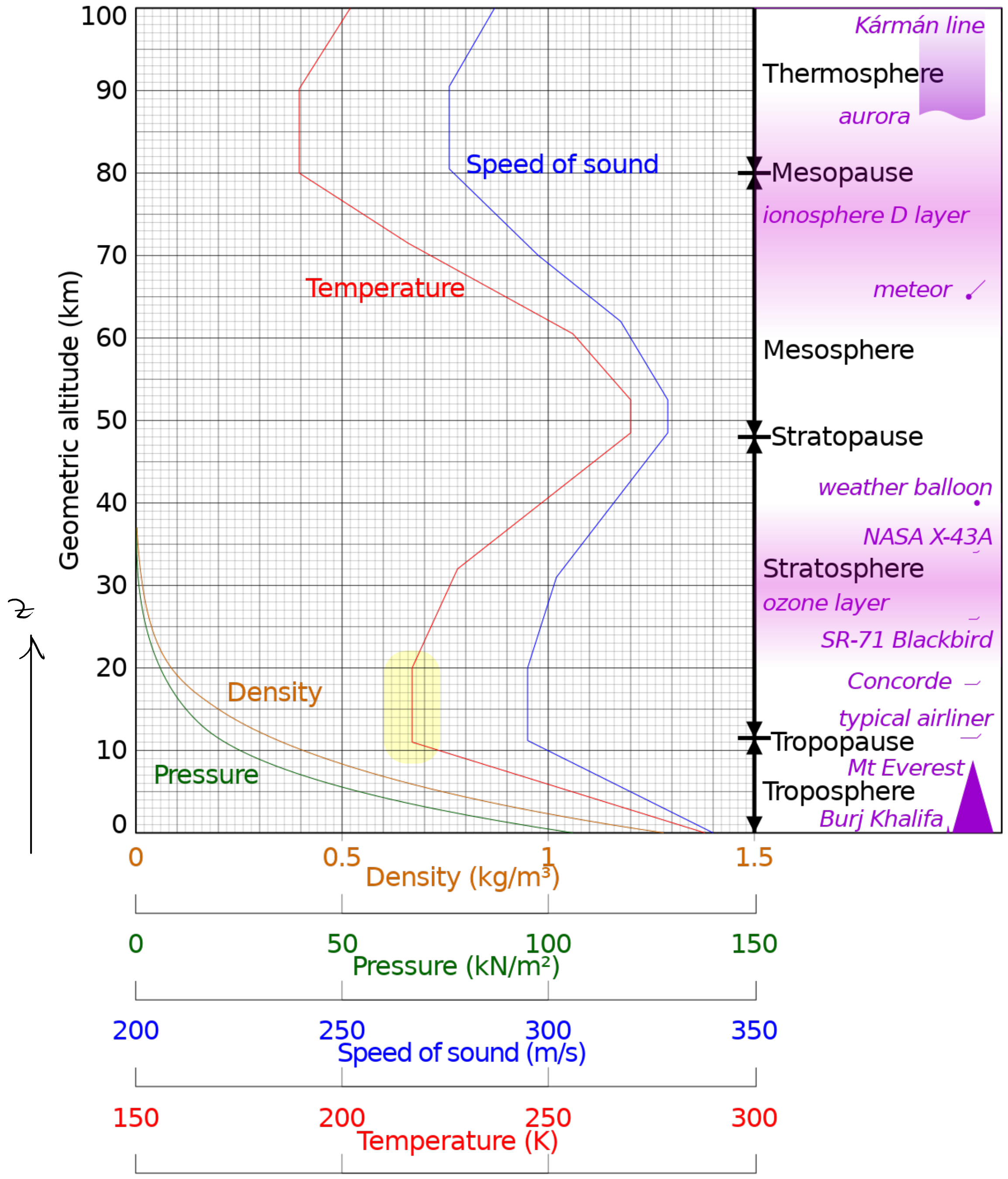
campi



z

$$dP = -\rho g \, dz$$

$$\frac{dP}{dz} = -\rho g$$



Modellizzazione dell' atmosfera terrestre

- atmosfera a riposo ①
- $T = \text{cost}$
- atmosfera = gas perfetto ②
- massa molare $M_A = 28\ g/mol$

$$\begin{cases} \dfrac{dP}{dz} = -\rho g & (1) \\ PV = nRT & (2) \end{cases}$$

$$\downarrow$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$\rho = \frac{M}{V} = \frac{n M_A}{V} = \frac{P M_A}{RT} \Rightarrow ①$$

$$\frac{dP}{dz} = -\frac{P M_A g}{RT} = -\underbrace{\frac{M_A g}{RT}}_{\equiv \frac{1}{\ell}} P = -\frac{1}{\ell} P$$

Eq. differenziale : $P = P(z)$

$$\frac{dP}{dz} = -\frac{1}{\ell} P \quad \rightarrow \quad \frac{dy}{dx} = -ky \quad \rightarrow \quad y \sim \exp(-kx)$$

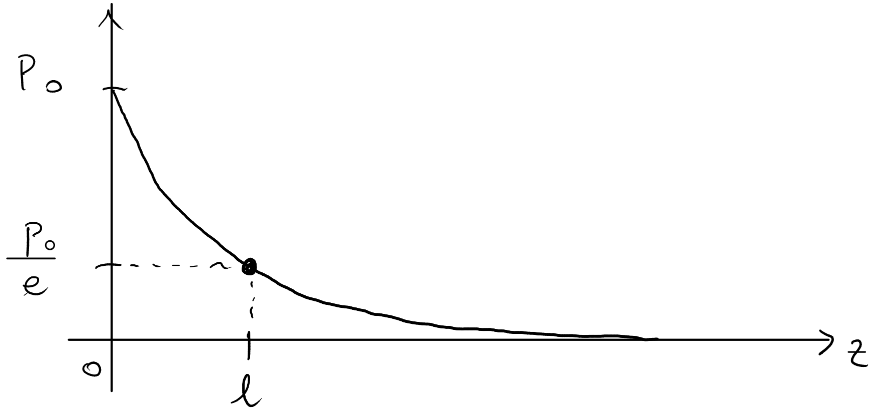
separazione delle variabili

$$\frac{dP}{P} = -\frac{1}{\ell} dz$$

$$\int_{P_0}^{P} \frac{dP'}{P'} = \int_{z_0}^{z} -\frac{1}{\ell} dz' \Rightarrow \ln P - \ln P_0 = -\frac{1}{\ell}(z - z_0)$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{1}{l}(z-z_0)$$

$$\boxed{P = P_0 \exp\left[-\frac{1}{l}(z-z_0)\right]} = P_0 \exp\left[-\frac{M_A g}{RT}(z-z_0)\right]$$



$$z_0 = 0$$

$$P(z=l) = P_0 \exp(-1) = \frac{P_0}{e}$$

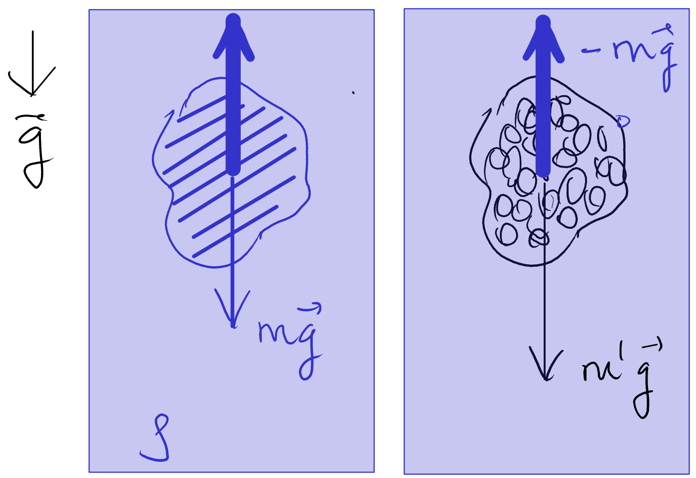
$$l = \frac{RT}{M_A g} \approx \frac{8 \times \cancel{3}00}{\cancel{28} \times 10^{-\cancel{3}\,2} \times \cancel{9{,}8}}\ m \approx 8 \times 10^3\ m \approx 10^4\ m$$

10 Km

Es:

$$\rho = \rho(z)\ ?$$

Principio di Archimede

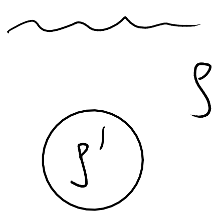


Un corpo immerso in un fluido subisce una forza uguale in modulo e direzione al peso del fluido spostato dal corpo e di verso opposto a tale peso.

Forza di Archimede : $\boxed{\vec{F}_A = -\rho V \vec{g}}$

Casi particolari:

1) Corpo completamente immerso in un fluido ($\rho = \text{cost}$)



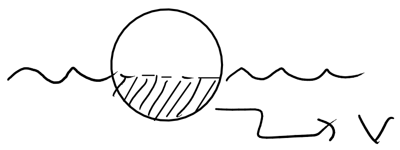
$$\Sigma \vec{F} = \vec{P} + \vec{F}_A = \rho' V \vec{g} - \rho V \vec{g} = (\rho' - \rho) V \vec{g}$$

$\rho' > \rho$: $\Sigma \vec{F} \sim \vec{g}$ $\Downarrow$

$\rho' < \rho$: $\Sigma \vec{F} \sim -\vec{g}$ $\Uparrow$

$\rho' = \rho$: $\Sigma \vec{F} = \vec{0}$ "galleggia" $\rightarrow$ equilibrio meccanico INDIFFERENTE

2) corpo parzialmente immerso $\rho' \leq \rho$



$$\sum \vec{F} = \rho' V' \vec{g} - \rho V \vec{g} = (\rho' V' - \rho V)\vec{g}$$

Eq. meccanico

V' = volume corpo

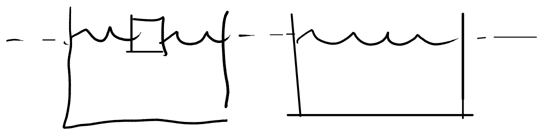
V = volume spostato

$$\sum \vec{F} = \vec{0}$$

$$\rho' V' = \rho V \Rightarrow V = \frac{\rho'}{\rho} V' \Rightarrow \frac{V}{V'} = \frac{\rho'}{\rho} \leq 1$$

$\uparrow$ frazione volume immerso

Es: mostrare che il livello dell' acqua non cambia!



Es.! mostra che l'equilibrio è stabile nel caso 2) ($\rho' < \rho$)