$$\frac{dP}{P} = -\frac{1}{\ell}\, dz$$

$$\int_{P_i}^{P_f} \frac{dP}{P} = -\frac{1}{\ell} \int_{z_i}^{z_f} dz \qquad z_i\ ,\ P_i = P(z_i) \qquad z_f\ ,\ P_f = P(z_f)$$

$$\ln\left(\frac{P_f}{P_i}\right) = -\frac{1}{\ell}\left(z_f - z_i\right)$$

$$P_f = P_i \exp\left[-\frac{1}{\ell}\left(z_f - z_i\right)\right] \qquad z_i \to z_0\ ,\ z_f \to z$$

$$P = P_0 \exp\left[-\frac{1}{\ell}\left(z - z_0\right)\right]$$

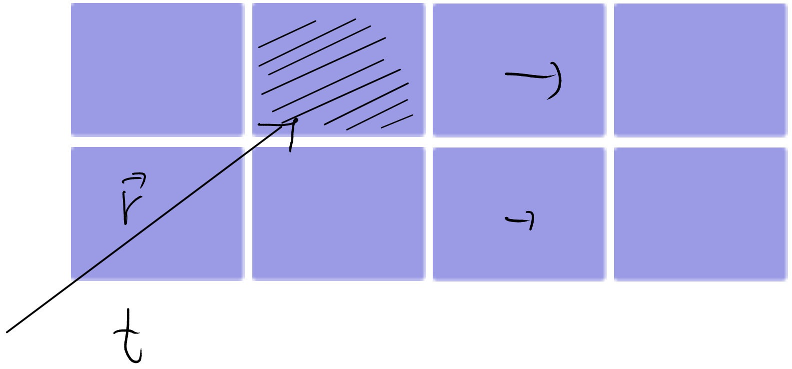
DINAMICA DEI FLUIDI

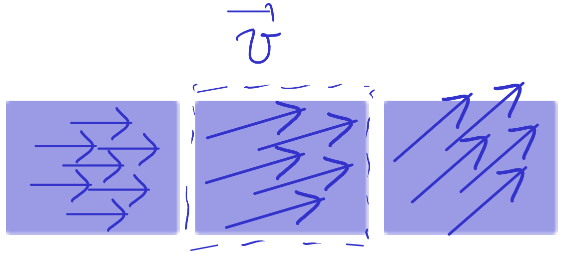
Fluidodinamica
→ fluidi comprimibili

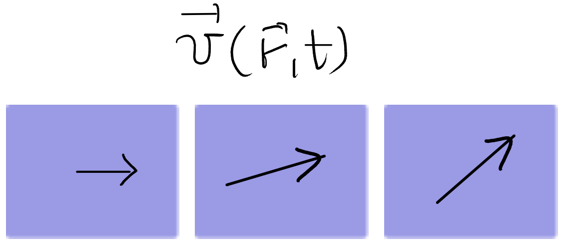
Idrodinamica
→ fluidi incomprimibili = liquidi
bassa / alta velocità

③ + ④ V

③, ④ X







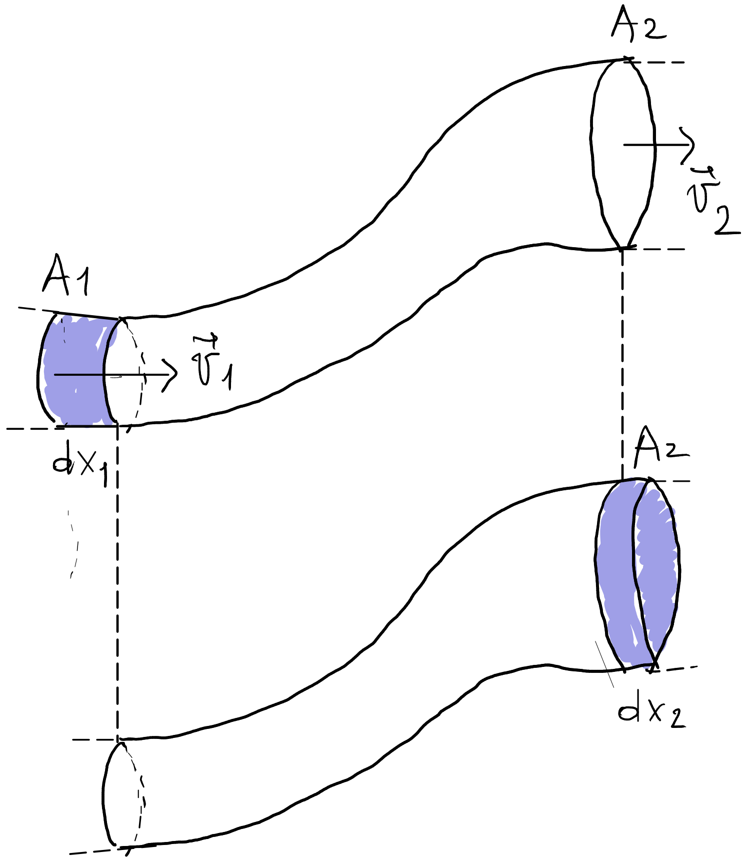
velocità media delle particelle

① fluido incomprimibile : ρ = cost
② fluido non viscoso : no attrito tra le diverse parti del fluido
⇒ fluido ideale

③ corrente stazionaria : $\vec{v} = \vec{v}(\vec{r})$
④ corrente irrotazionale

Equazione di continuità per fluidi incomprimibili

Fluido ideale : $\rho =$ cost in un tubo



istante iniziale

↓ intervallo dt

istante finale

Conservazione della massa

$$\cancel{\rho} A_1 v_1 \cancel{dt} = \cancel{\rho} A_2 v_2 \cancel{dt}$$

$$A_1 v_1 = A_2 v_2$$

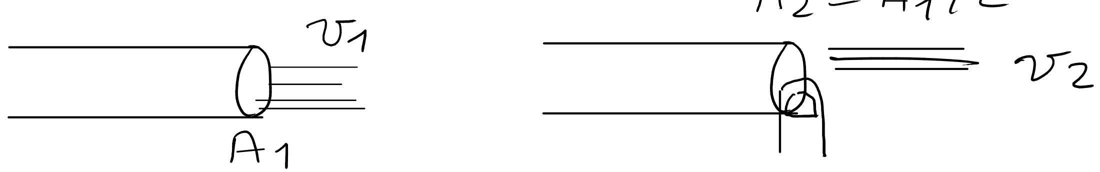
$$A v = \text{cost}$$

eq. continuità

$$[A v] = \frac{L^2 L}{T} = \frac{L^3}{T}$$

$A v$: corrente di volume

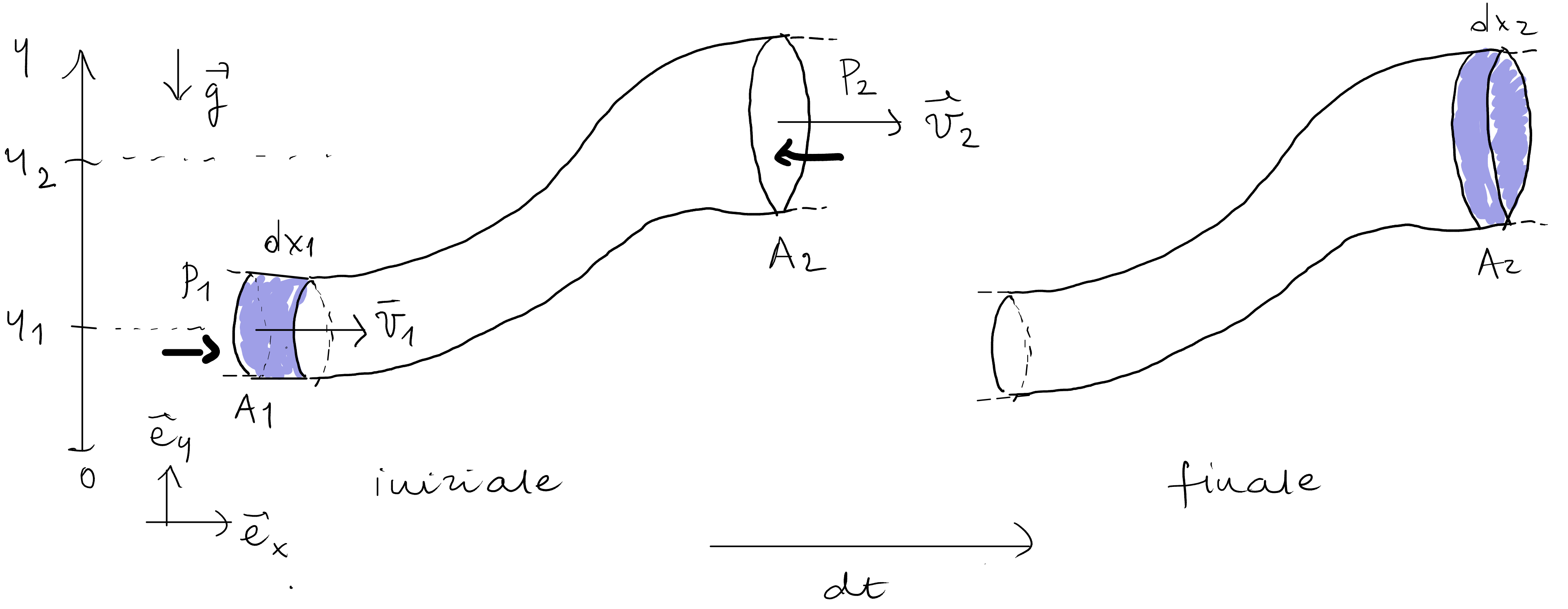
Es.:



$$v_1 A_1 = v_2 A_2 \Rightarrow v_2 = 2 v_1$$

Teorema di Bernoulli

Fluido ideale + forze : pressione del fluido circostante + gravità



Bernoulli 1738

- $F_1 = P_1 A_1$
  $\vec{F}_1 = P_1 A_1 \vec{e}_x$ $\qquad \vec{F}_2 = - P_2 A_2 \vec{e}_x$

corrente stazionaria 3

- $E_p = mgy$

$\underline{\text{Dim.}}$:
$$\underset{(3)}{\Delta E_c} = W(\Sigma \vec{F}) = W(\Sigma \vec{F}_{est})$$ teor. energia cinetica

$(1)\ P \qquad \vec{g}\ (2)$

(1)
$$W_1 = P_1 A_1 \vec{e}_x \cdot \vec{v}_1 dt = P_1 A_1 dx_1 = P_1 V_1$$
$$W_2 = -P_2 A_2 \vec{e}_x \cdot \vec{v}_2 dt = -P_2 A_2 dx_2 = -P_2 V_2$$
$$\Rightarrow W_1 + W_2 = (P_1 - P_2) V$$

(2)
$$W = -\Delta E_p$$
$$\Delta E_p = (mgy_2 + E_p^0) - (mgy_1 + E_p^0) = mg(y_2 - y_1)$$

(3)
$$\Delta E_c = \left(\frac{1}{2} m v_2^2 + E_c^0\right) - \left(\frac{1}{2} m v_1^2 + E_c^0\right) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\Delta E_c + \Delta E_p = W_1 + W_2$$
$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1 = P_1 V - P_2 V$$
$$\frac{1}{2} \rho v_2^2 + \rho g y_2 + P_2 = \frac{1}{2} \rho v_1^2 + \rho g y_1 + P_1 \Rightarrow \boxed{\frac{1}{2} \rho v^2 + \rho g y + P = cost} \qquad \square$$

Teorema di Bernoulli

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{cost} \qquad P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

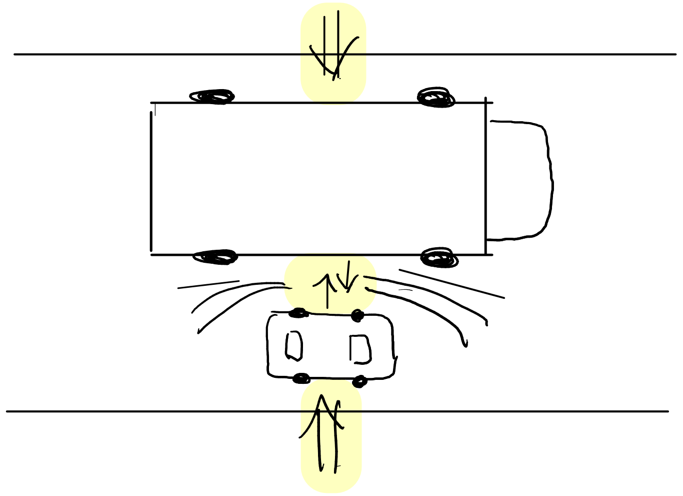
Casi particolari

1) $\vec{v} = \vec{0} \rightarrow P + \rho g y = \text{cost} \rightarrow P_1 + \rho g y_1 = P_2 + \rho g y_2 \rightarrow \Delta P = \rho g \Delta y$

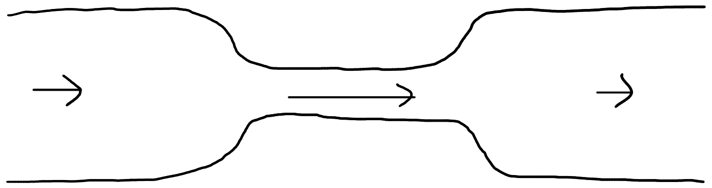
legge di Stevino

2) $\vec{v} = \text{cost} \rightarrow$ idem!

3) $y = \text{cost} \rightarrow P + \frac{1}{2}\rho v^2 = \text{cost} \qquad A v = \text{cost}$

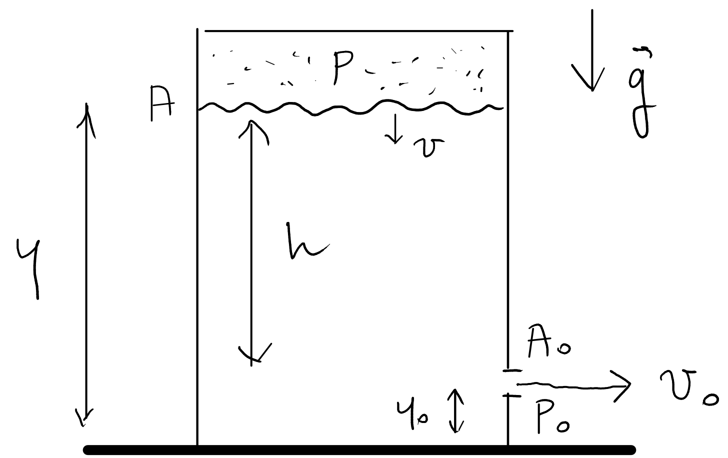


$A \downarrow \quad v \uparrow \quad P \downarrow$



effetto Venturi

Es: Velocità di uscita di un fluido da un contenitore



fluido ideale, $\rho = cost$

$A_0 << A \quad v << v_0 \Rightarrow v \approx 0$

Velocità di uscita = ?

Teor. Bernoulli:

$$P + \rho g y + \frac{1}{2}\rho v^2 = cost$$

$\downarrow v \approx 0$

$$P + \rho g y = P_0 + \rho g y_0 + \frac{1}{2}\rho v_0^2$$

$$\frac{1}{2}\rho v_0^2 = P - P_0 + \rho g (y - y_0) \qquad h \equiv y - y_0$$

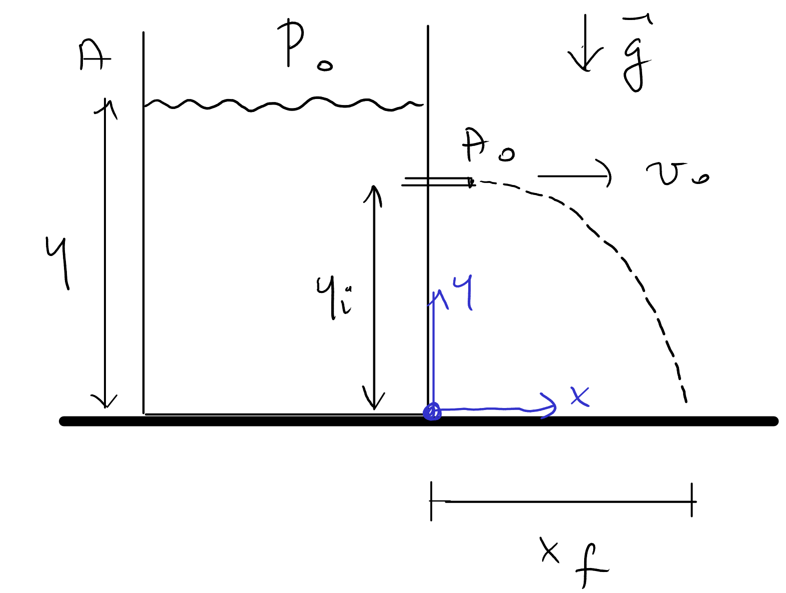
$$v_0 = \sqrt{\frac{2(P-P_0)}{\rho} + 2gh}$$

Casi limite:

- $P \gg P_0$: $\quad v_0 \approx \sqrt{\dfrac{2P}{\rho}}$

- $P = P_0$: $\quad v_0 = \sqrt{2gh} \rightarrow$ caduta libera (legge di Torricelli)

Variante : contenitore aperto, gittata massima ?



$A \gg A_0 \Rightarrow v \approx 0 \Rightarrow v_0 = \sqrt{2g(y - y_i)}$

cinematica : moto unif. accelerato

$$\begin{cases} x = x_i + v_{xi}\, t \\ y = y_i + v_{yi}\, t - \frac{1}{2} g t^2 \end{cases}$$

Condizioni iniziali : $x_i = 0$, $v_{xi} = v_0$, $v_{yi} = 0$

$$\begin{cases} x_f = \sqrt{2g(y-y_i)}\; t_f \\ 0 = y_i - \frac{1}{2} g t_f^2 \end{cases} \Rightarrow t_f = \sqrt{\frac{2y_i}{g}} \Rightarrow x_f = \sqrt{4(y-y_i)y_i} = 2\sqrt{yy_i - y_i^2}$$

gittata :

$x_f = x_f(y_i) \Rightarrow$ gittata massima = ? (es.)