Cyber-Physical Systems

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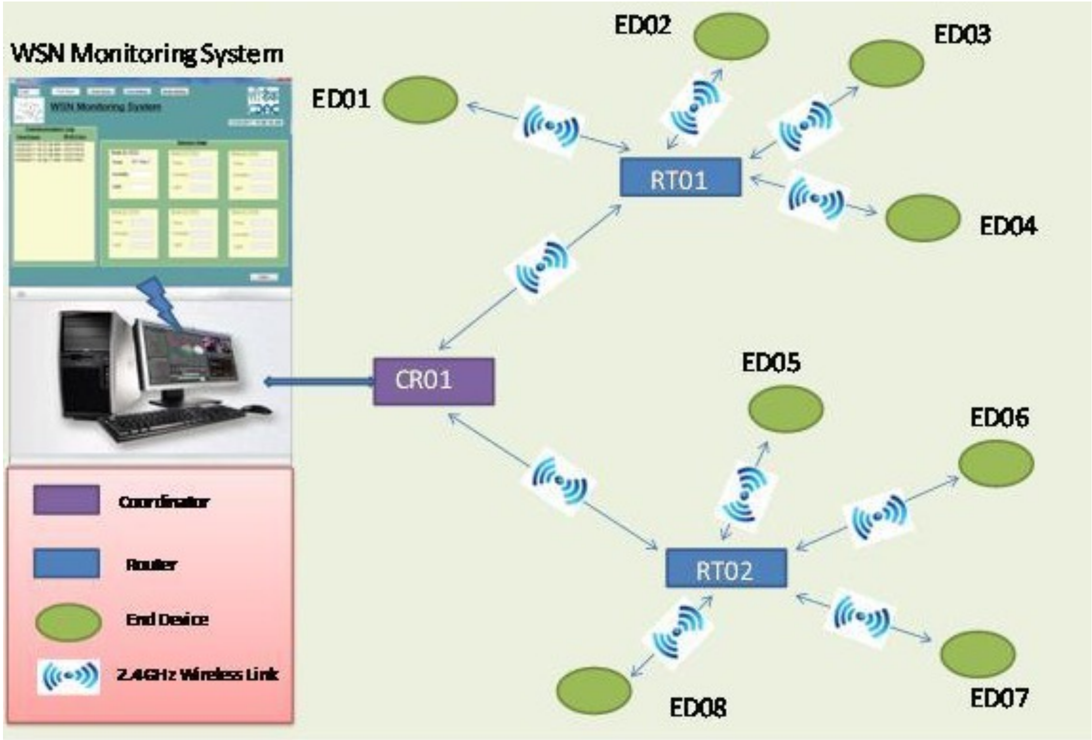
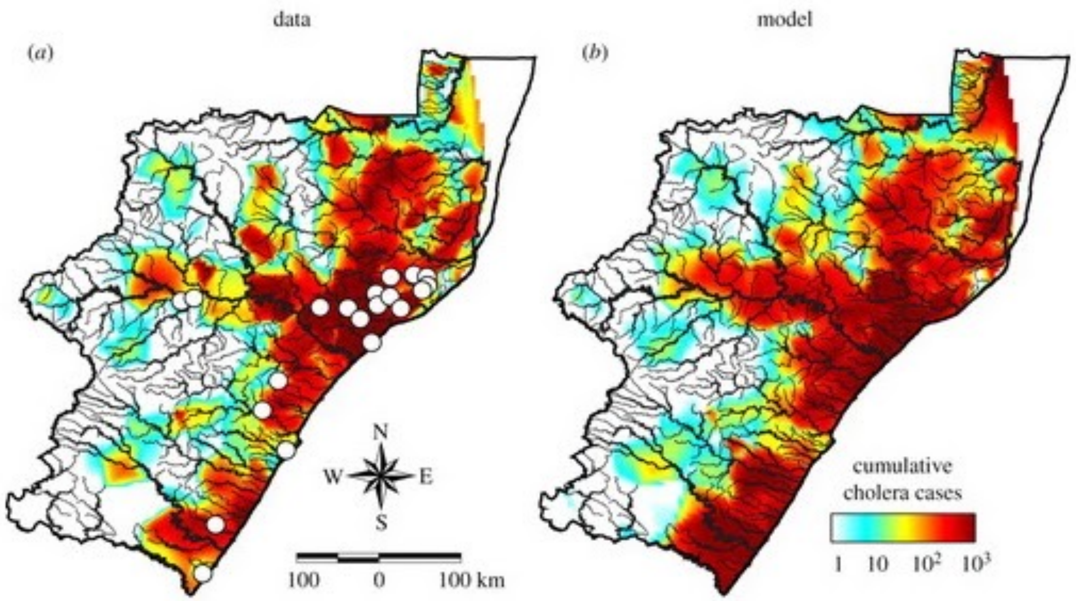
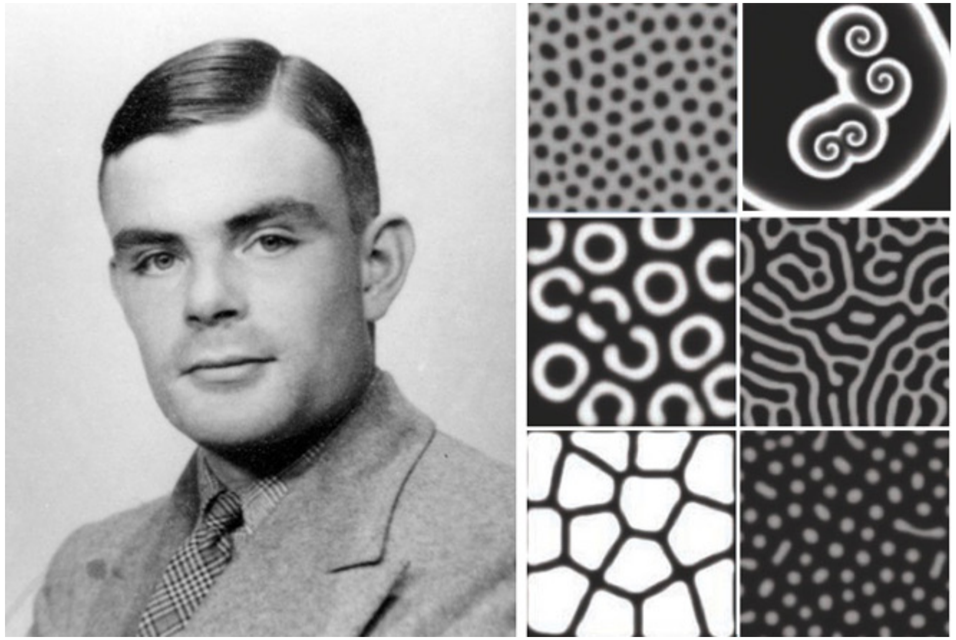
Lecture: Spatio-Temporal Reach and Escape Logic


WSN Monitoring System
ED01
ED02
ED03
ED04
ED05
ED06
ED07
ED08
RT01
RT02
CR01
Coordinator
Router
End Device
2.4GHz Wireless Link

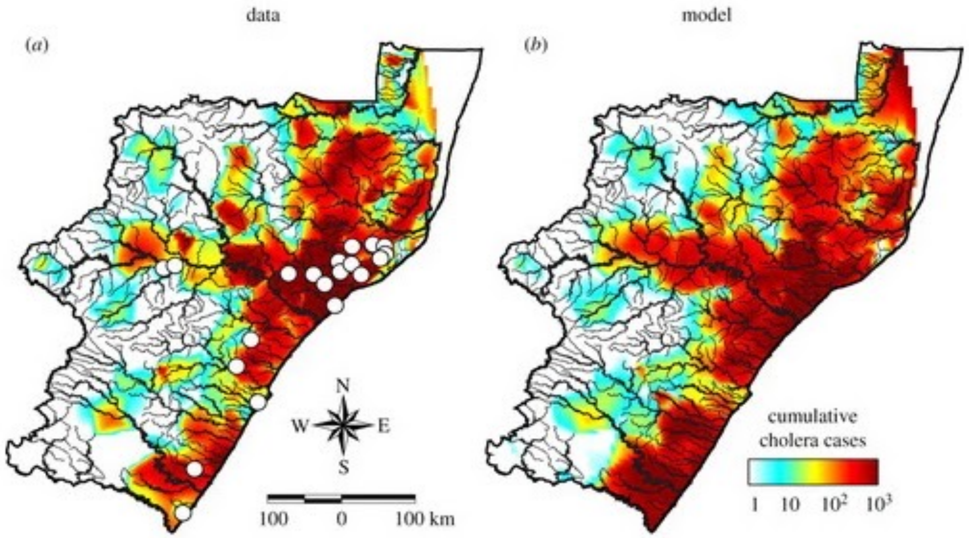

MELBOURNE
$5 helmets are closer than you think
RACV

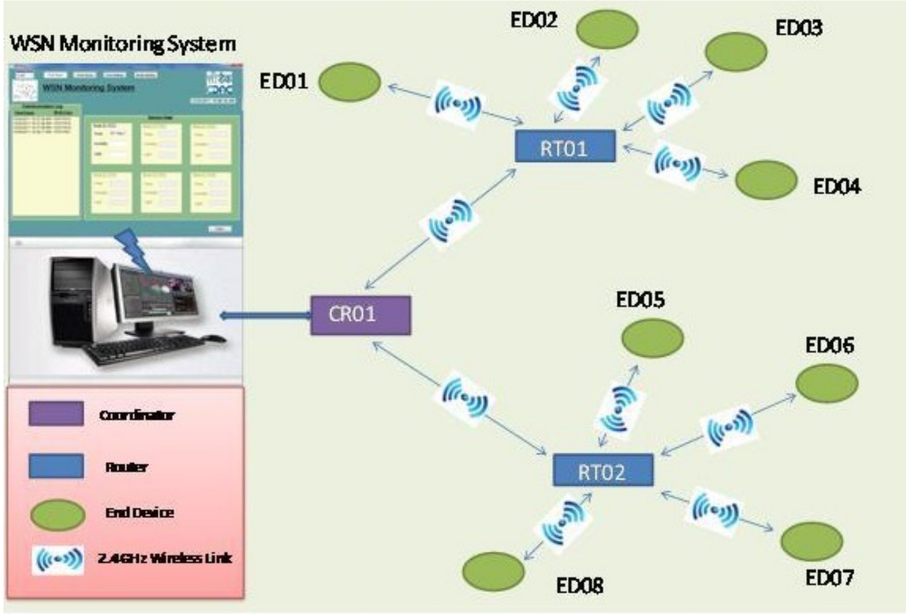

data
model
(a)
(b)
N
W
E
S
100
0
100 km
cumulative cholera cases
1
10
10^2
10^3



Availability: I can always find a station with at least one bike in a radius of 500 meters

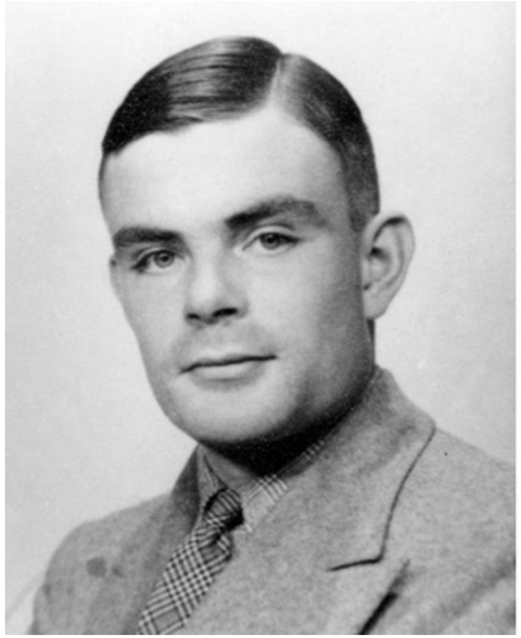
Spread: after 10 time units, there exists a location l′ at a certain distance from location l where the number of infected individuals is more than 50

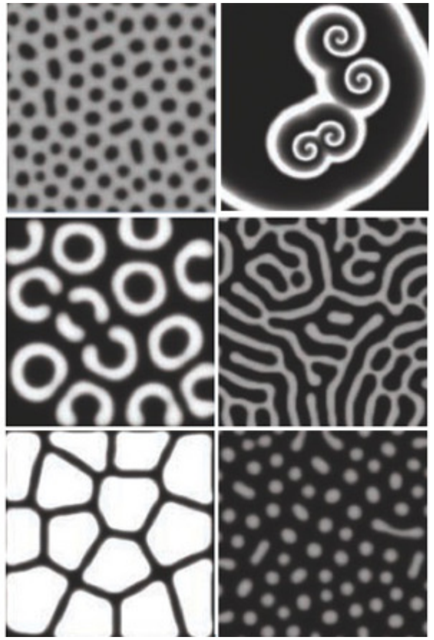




Reliability: we can always find a path of sensors such that all sensors have a battery level greater than 0.5

Spots: regions with low density of protein A are always surrounded by regions with high level of protein B





How to specify such spatio-temporal behaviours in a formal and human-understandable language ?

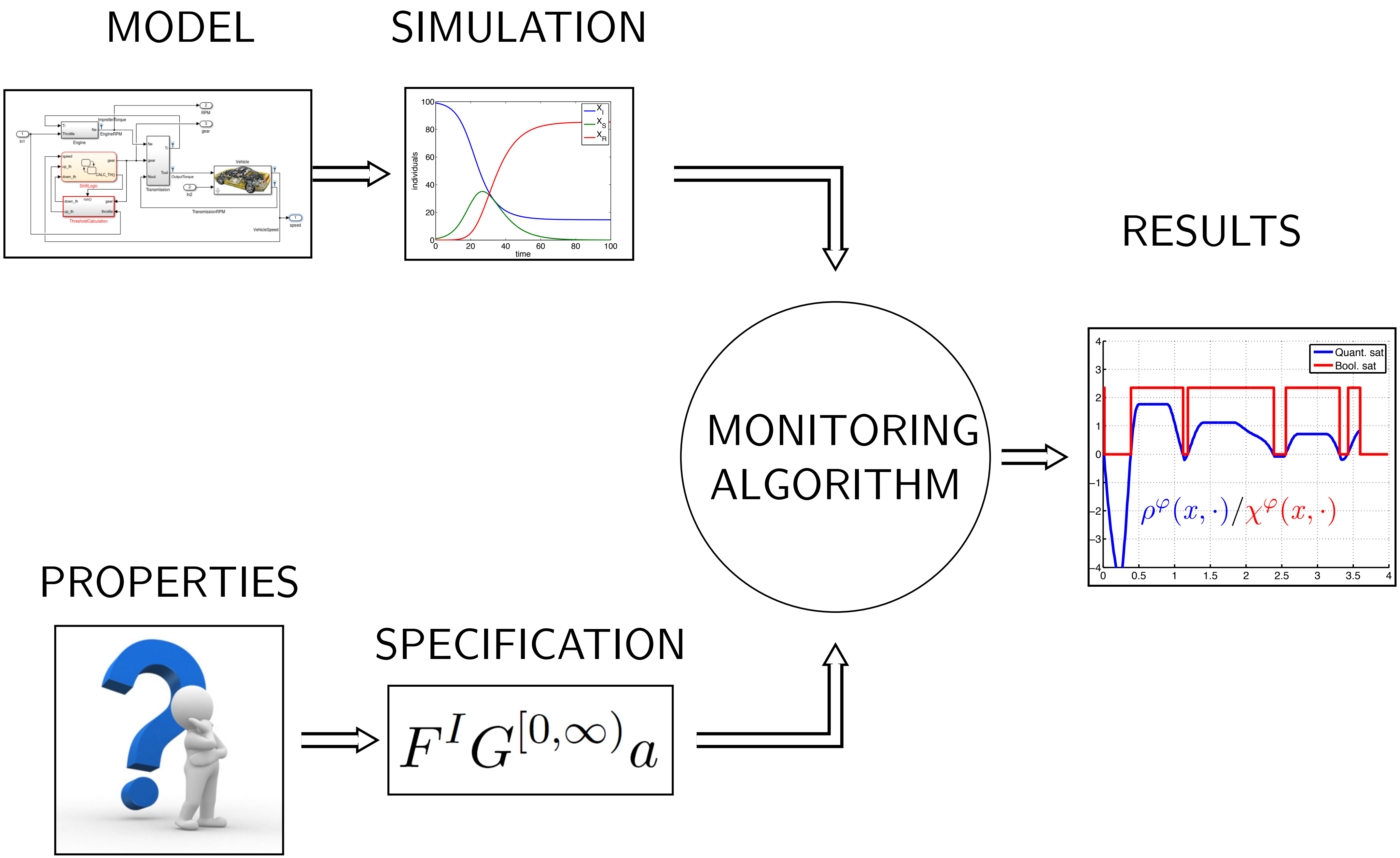
How to monitor their onset efficiently?

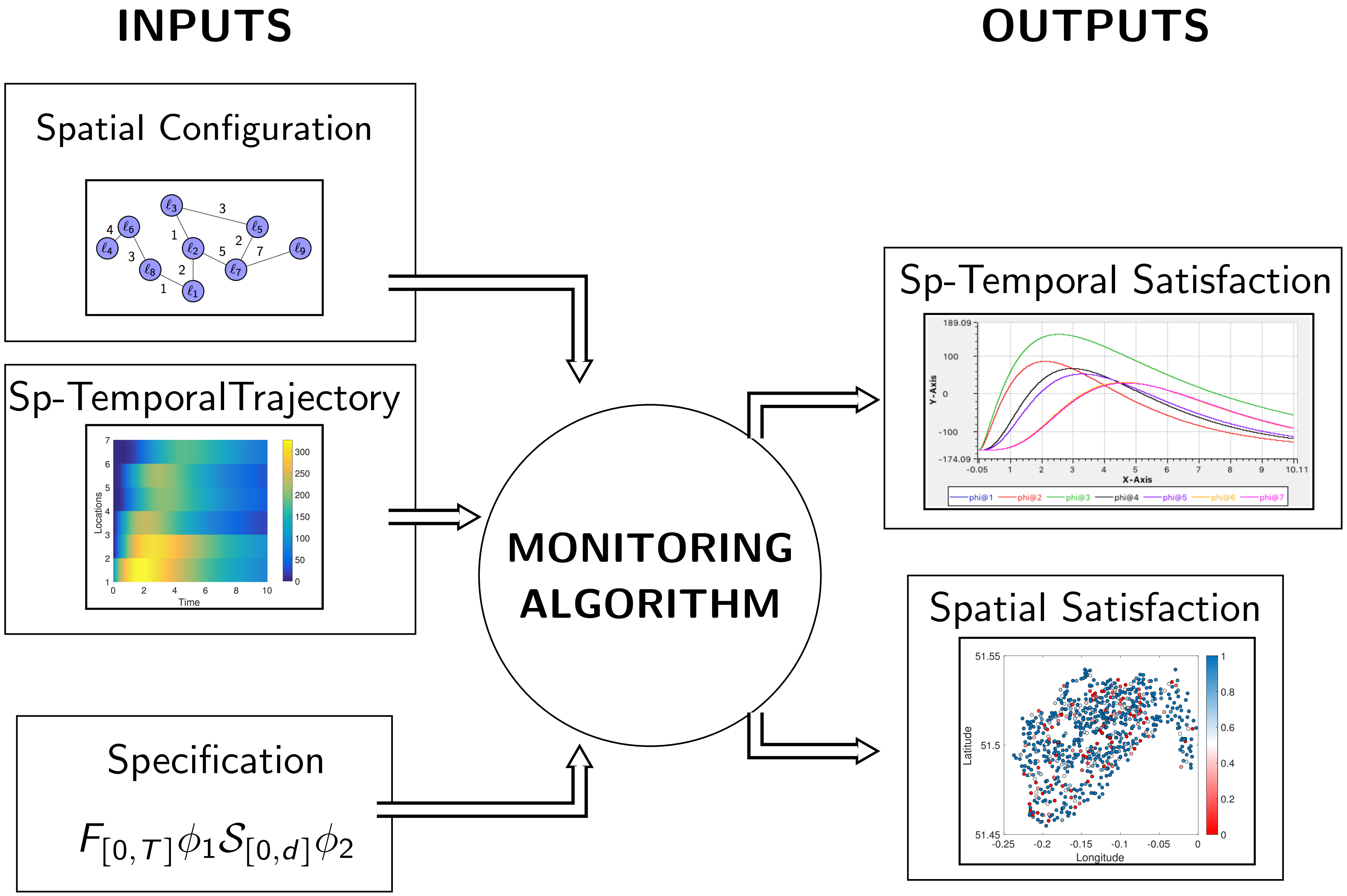
Part 1 :

- Space Model and traces
- Spatio- Temporal Reach and Escape Logic (STREL)

Part 2:

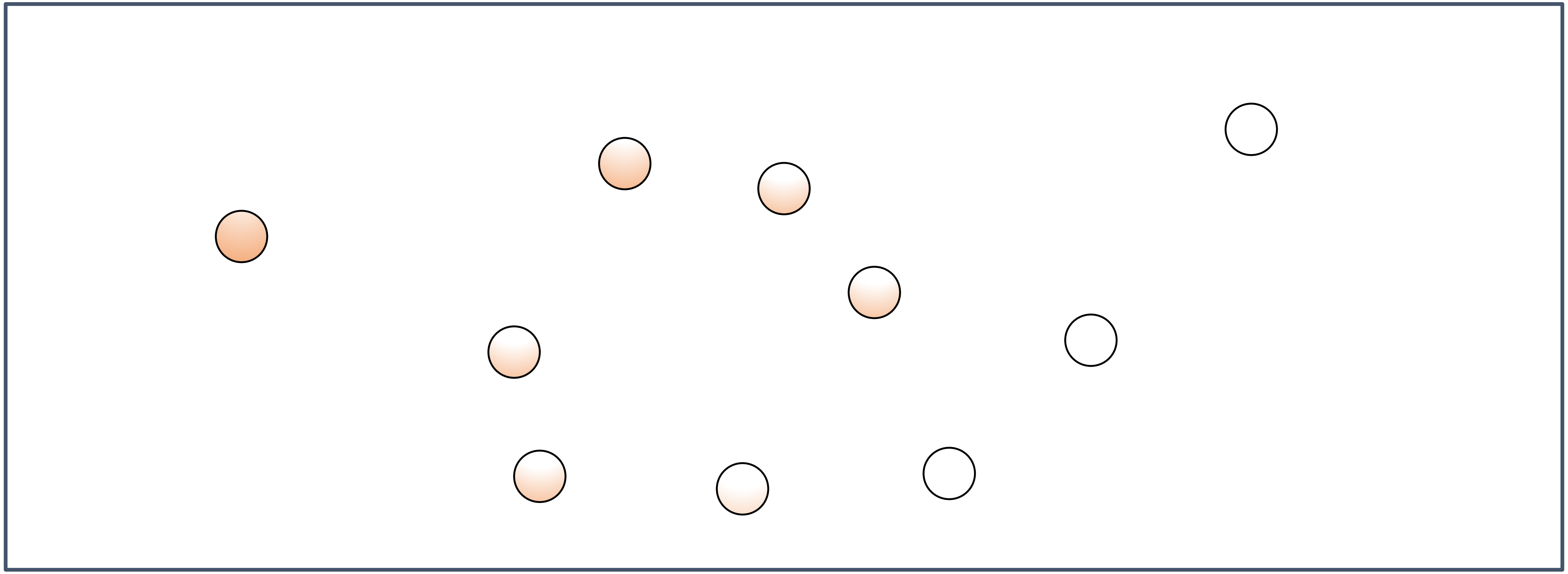
- Monitoring
- Applicability to different scenarios


MODEL
SIMULATION
individuals
time
RESULTS
MONITORING ALGORITHM
Quant. sat
Bool. sat
$\rho^{\varphi}(x,\cdot)/\chi^{\varphi}(x,\cdot)$
PROPERTIES
SPECIFICATION
$F^{I}G^{[0,\infty)}a$



INPUTS
OUTPUTS
Spatial Configuration
Sp-TemporalTrajectory
Locations
Time
Specification
$F_{[0,T]}\phi_1\mathcal{S}_{[0,d]}\phi_2$
MONITORING ALGORITHM
Sp-Temporal Satisfaction
X-Axis
Y-Axis
phi@1 phi@2 phi@3 phi@4 phi@5 phi@6 phi@7
Spatial Satisfaction
Latitude
Longitude

Running Example: Wireless Sensor Network



Space Model, Signal and Traces

Spatial Configuration

We consider a discrete space described as a weighted (direct) graph

Reasons:

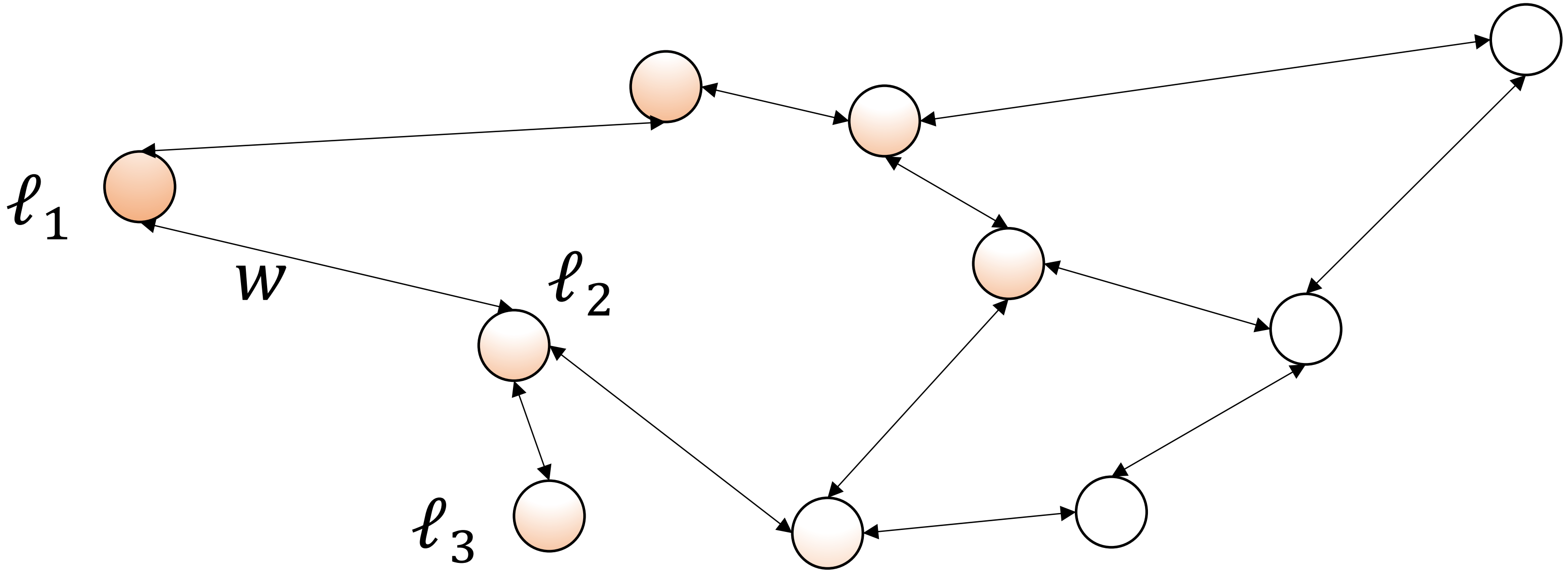
- many applications, like bike sharing systems, smart grid and sensor networks are naturally framed in a discrete spatial structure

- in many circumstances continuous space is abstracted as a grid or as a mesh, e.g. numerical integration of PDEs

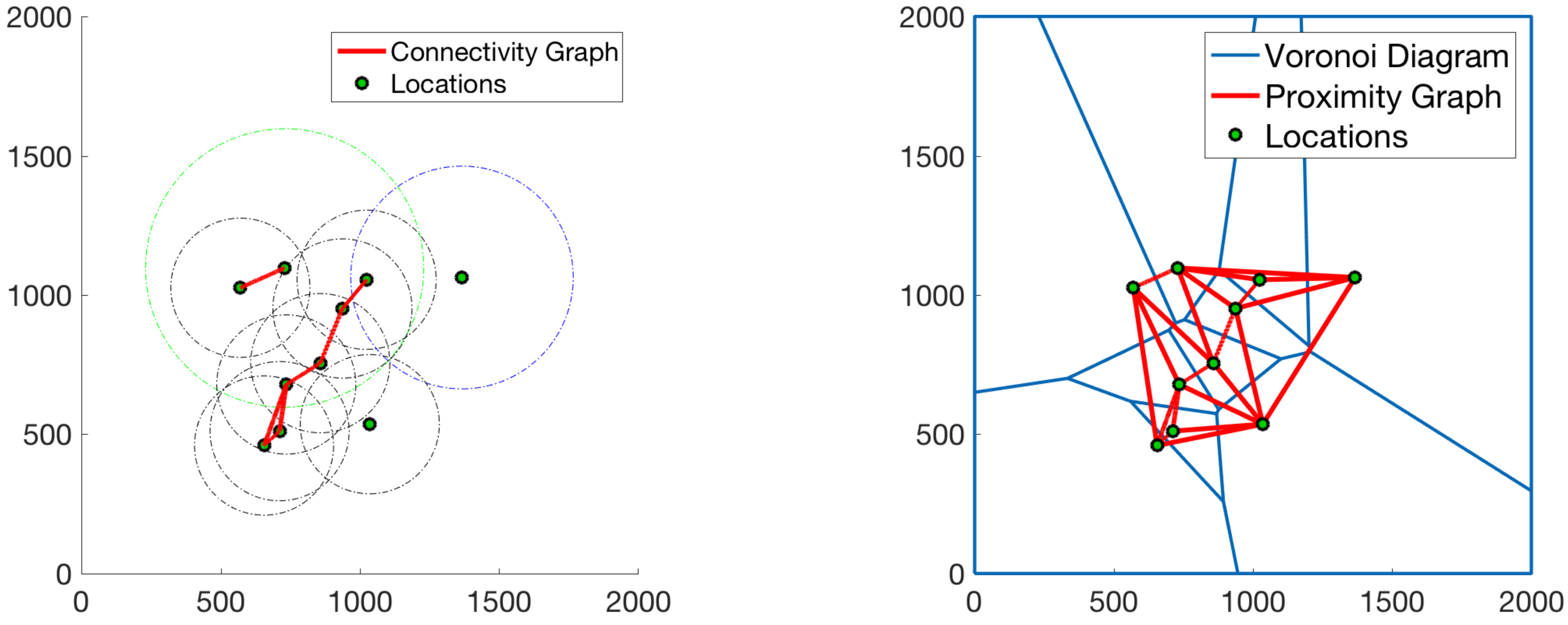
Space Model $S = \langle L, W \rangle$

– L is a set of nodes that we call locations;

– $W \subseteq L \times \mathbb{R} \times L$ is a proximity function associating a label $w \in \mathbb{R}$ to distinct pair $\ell_1, \ell_2 \in L$. If $(\ell_1, w, \ell_2) \in W$, it means that there is an edge from ℓ_1 to ℓ_2 with weight $w \in \mathbb{R}$



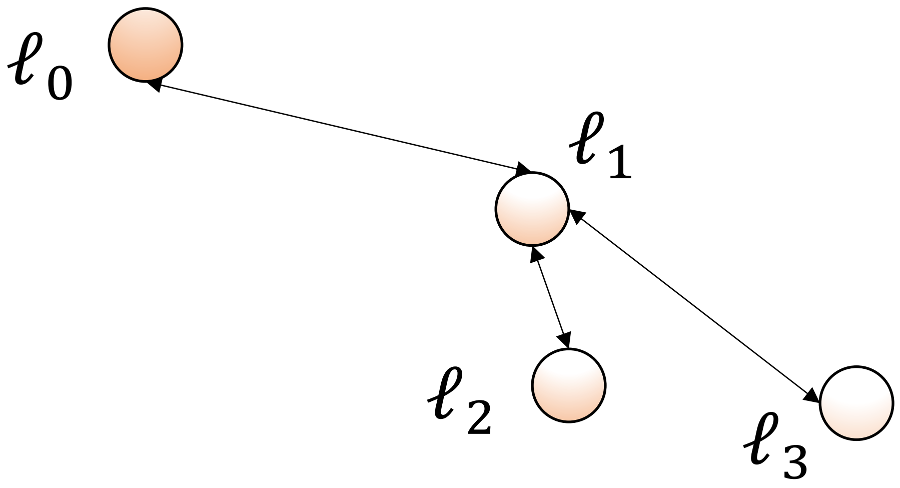
Example



Connectivity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000
Voronoi Diagram
Proximity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000

Route $\tau = \ell_0 \ell_1 \ell_2 \ldots$

It is a infinite sequence s.t. $\forall i \geq 0 \; \exists \, w \; s.t. (\ell_i, w, \ell_{i+1}) \in W$



$\ell_0 \ell_1 \ell_2 \ell_1 \ldots$ is a route

$\ell_0 \ell_1 \ell_2 \ell_3 \ldots$ is a not route

$\tau[i]$ to denote the $i - th$ node τ

$\tau(\ell)$ to denote the first occurrence of $\ell \in \tau$

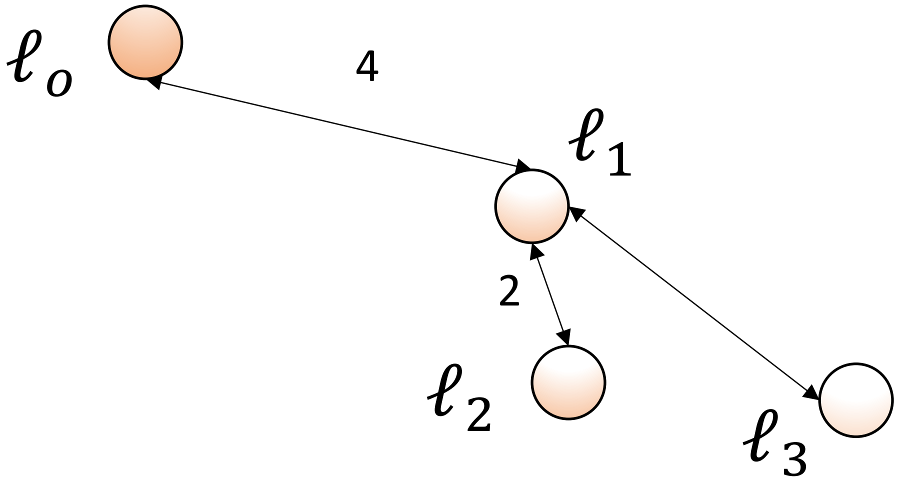
Route Distance $d_\tau^f[i]$

The distance $d_\tau^f[i]$ up to index i is:

$$d_\tau^f[i] = \begin{cases} 0 & i = 0 \\ f(d_{\tau[1..]}^f[i-1], w) & (i > 0) \ and \ \tau[0] \overset{w}{\mapsto} \tau[1] \end{cases}$$

$$d_\tau^f(\ell) = d_\tau^f[\tau(\ell)]$$

Route Distance $d_{\tau}^{f}[i]$



$$weight(x, y) = x + y$$

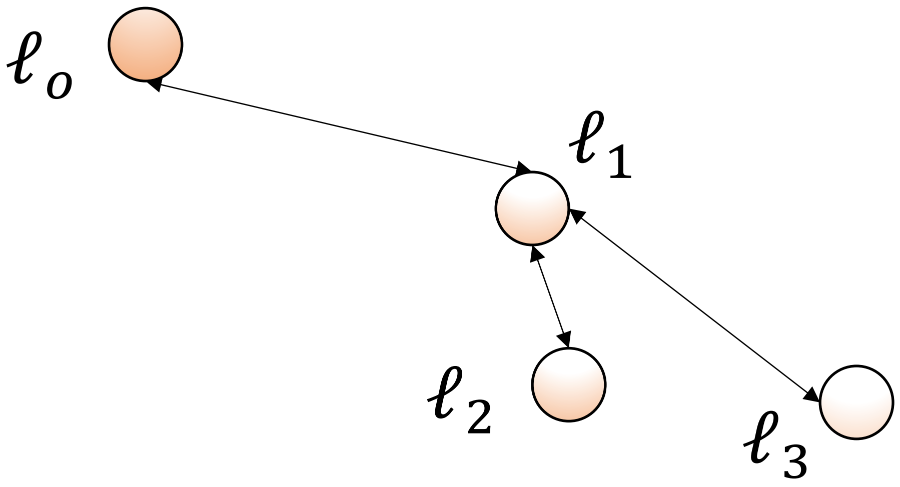
$$hops(x, y) = x + 1$$

$d_{\ell_0\ell_1\ell_2..}^{weight}[2] =$ weight($d_{\ell_1\ell_2..}^{weight}[1]$, 4) = $d_{\ell_1\ell_2}^{weight}[1]$ + 4 = …

= weight($d_{\ell_2..}^{weight}[0]$, 2) + 4 = 6

Location Distance $d_S^f[\ell_i, \ell_j]$

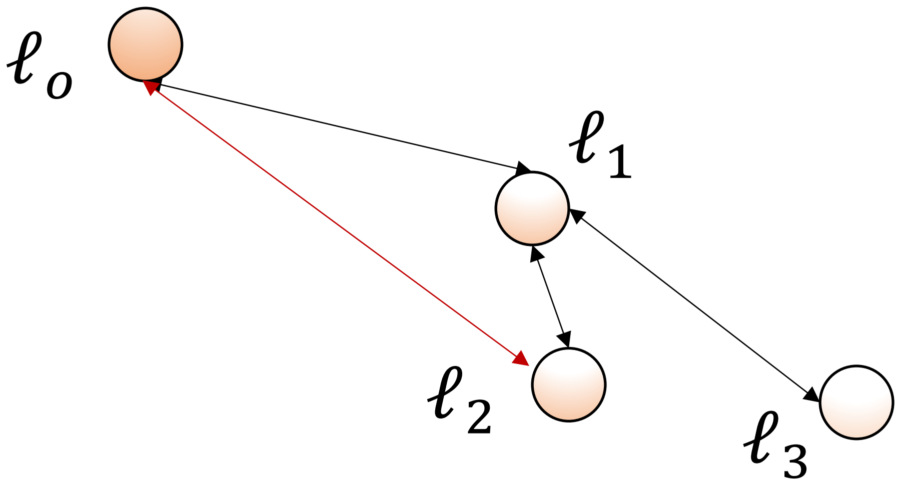
$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] \mid \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 2**

Location Distance

$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] | \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 1**

Signal and Trace

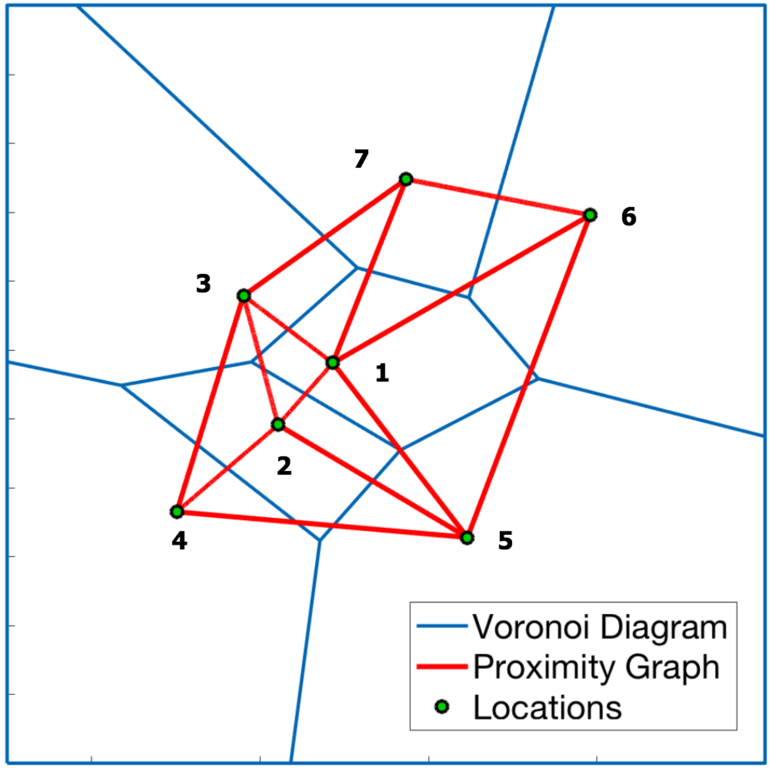
Spatio-Temporal Signals $\sigma: L \rightarrow \mathbb{T} \rightarrow D$

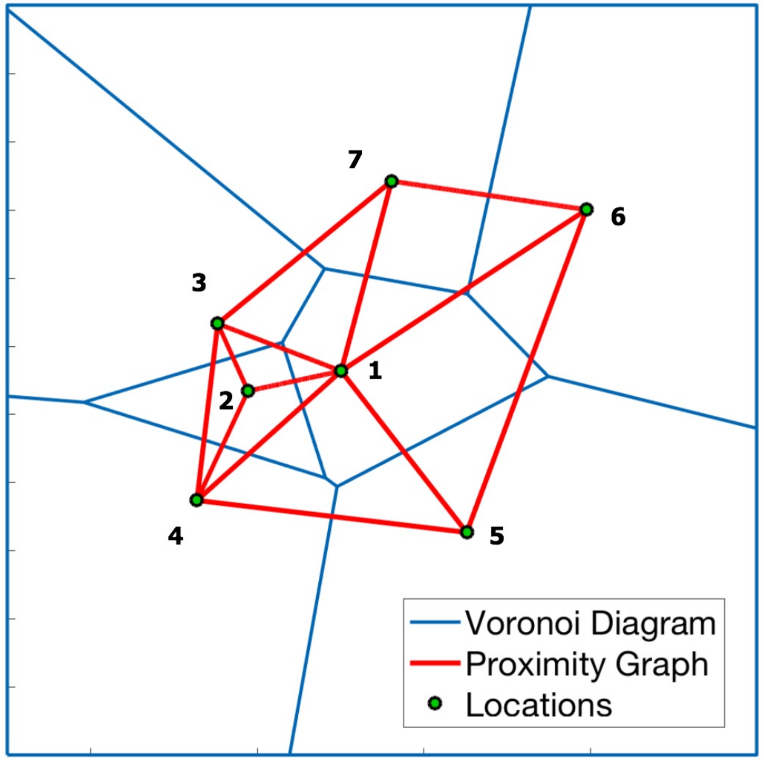
Spatio-Temporal Trace $\vec{\boldsymbol{x}}: L \rightarrow \mathbb{T} \rightarrow D^n$

$$x(\ell) \;=\; (\nu_B \,, \nu_T \,)$$
$$x(\ell, t) \;=\; (\nu_B(t)\,, \nu_T(t)\,)$$

Dynamic Spatial Model

(t_i, S_i) for $i = 1, \dots, n$ and $S(t) = S_i \forall t \in [t_i, t_{i+1})$





STREL

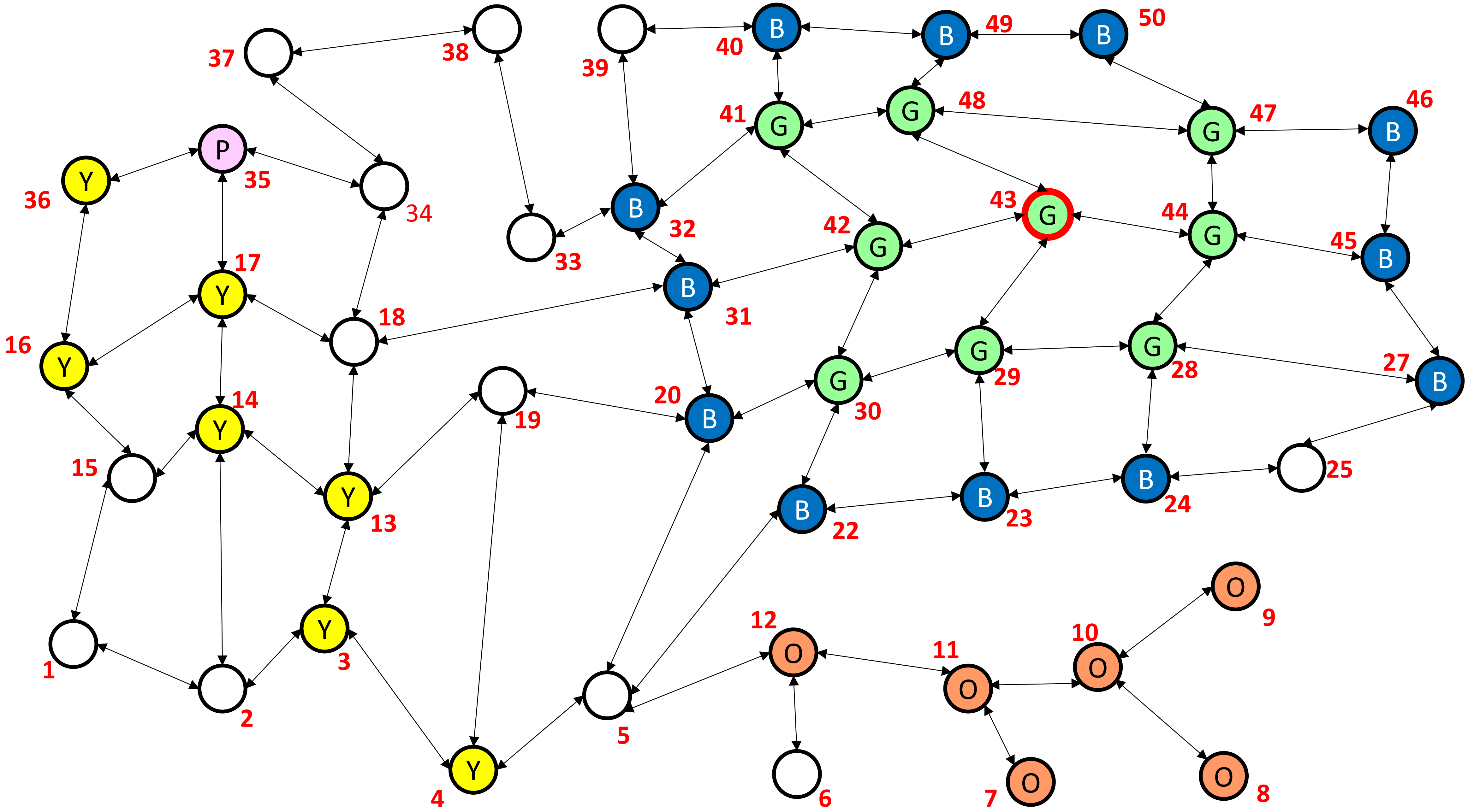
Spatio- Temporal Reach and Escape Logic (STREL)

It is an extension of the Signal Temporal Logic with a number of spatial modal operators

STREL Syntax
$\varphi := true \mid \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \, \mathrm{U}_I \, \varphi_2 \mid \varphi_1 \, \mathrm{S}_I \, \varphi_2 \mid \varphi_1 \, \mathcal{R}_d^f \, \varphi_2 \mid \mathcal{E}_d^f \, \varphi$

In addition, we can derive:

- The disjunction operator: $\vee$
- the temporal operators: $F_I, G_I, O_I, \mathrm{H_I}$
- the spatial operators: somewhere, everywhere and surround

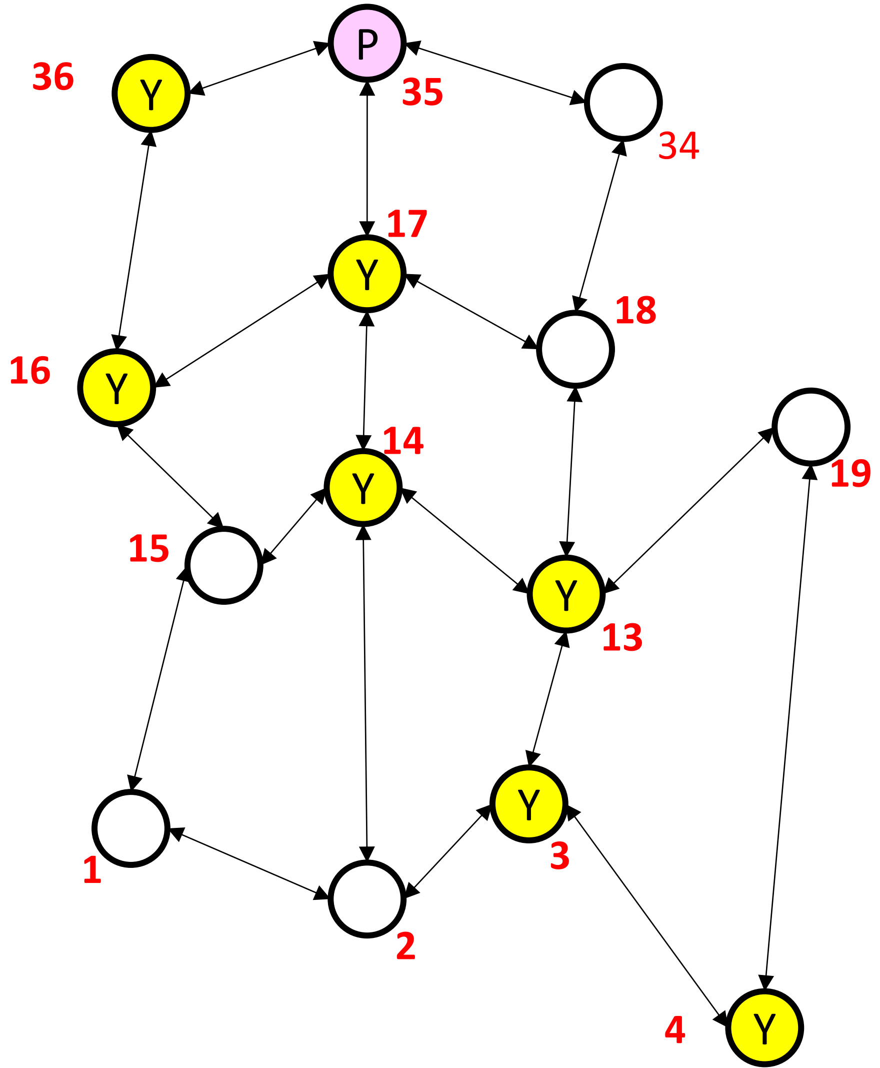


37
38
39
40
B
B
49
B
50
41
G
G
48
47
G
46
B
P
35
Y
36
34
B
32
33
42
G
43
G
44
G
45
B
17
Y
18
B
31
16
Y
G
29
G
28
27
B
14
Y
19
20
B
G
30
15
Y
13
B
22
B
23
B
24
25
Y
3
1
2
12
O
11
O
10
O
O
9
Y
4
5
6
O
7
O
8

Reach: $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$

$(S, \vec{x}, \ell, t)$ satisfies $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$ iff it satisfies φ_2 in a location ℓ' reachable from ℓ through a route τ, with a length $d_\tau^f[\ell'] \in [d_1, d_2]$ and such that $\tau[0] = \ell$ and all its elements with index less than $\tau(\ell')$ satisfy φ_1

Reach $\quad yellow\,\mathcal{R}^{hops}_{[1,4]}\,pink$



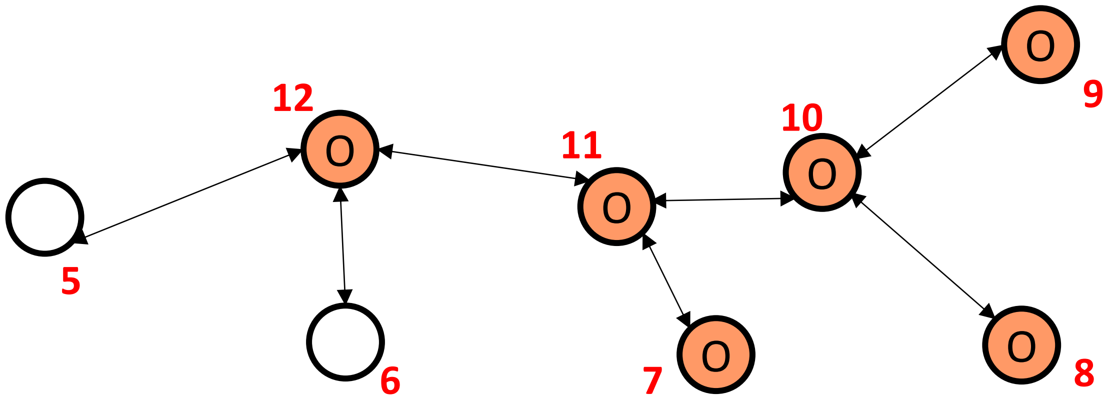
$$\tau = \ell_3 \ell_{13} \ell_{14} \ell_{17} \ell_{35}$$

$$d_{\tau}^{hops}[\ell_{35}] = 4, \text{ where } \tau[0] = \ell_3$$

Escape: $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$

$(S, \vec{x}, \ell, t)$ satisfies $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$ if and only there exists a route τ and a location $\ell' \in \tau$ such that $\tau[0] = \ell$, $d_S^f\ [\ell, \ell'] \in [d_1, d_2]$ and all elements $\tau[0], \ldots \tau[k]$ (with $\tau(l') = k$) satisfy φ

Escape: $\mathcal{E}^{hops}_{[3,\infty]} orange$

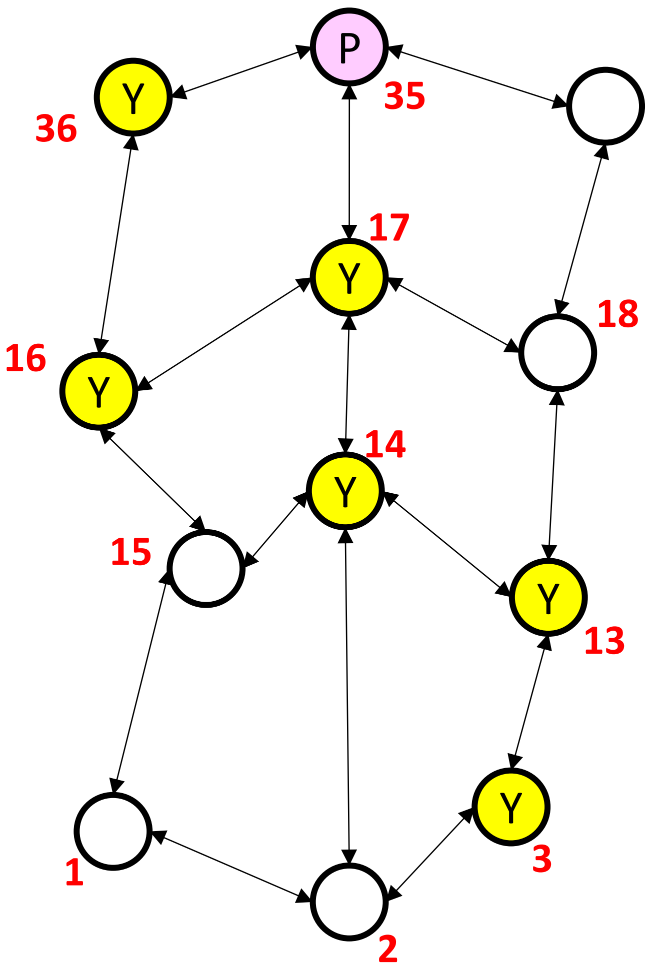


$$\tau = \ell_9 \ell_{10} \ell_{11} \ell_{12}$$

$$\tau[0] = \ell_9, \ \tau[3] = \ell_{12}$$

$$d^{hops}_S[\ell_9, \ell_{12}] = 3$$

Somewhere: $\diamondsuit^{f}_{[d_1,d_2]}\varphi$



$(S, \vec{x}, \ell, t)$ satisfies $\diamondsuit^{f}_{[d_1,d_2]}\varphi$ iff there exists a location ℓ' reachable from ℓ, via a path with length $d^f_\tau[\ell'] \in [d_1, d_2]$, that satisfies φ

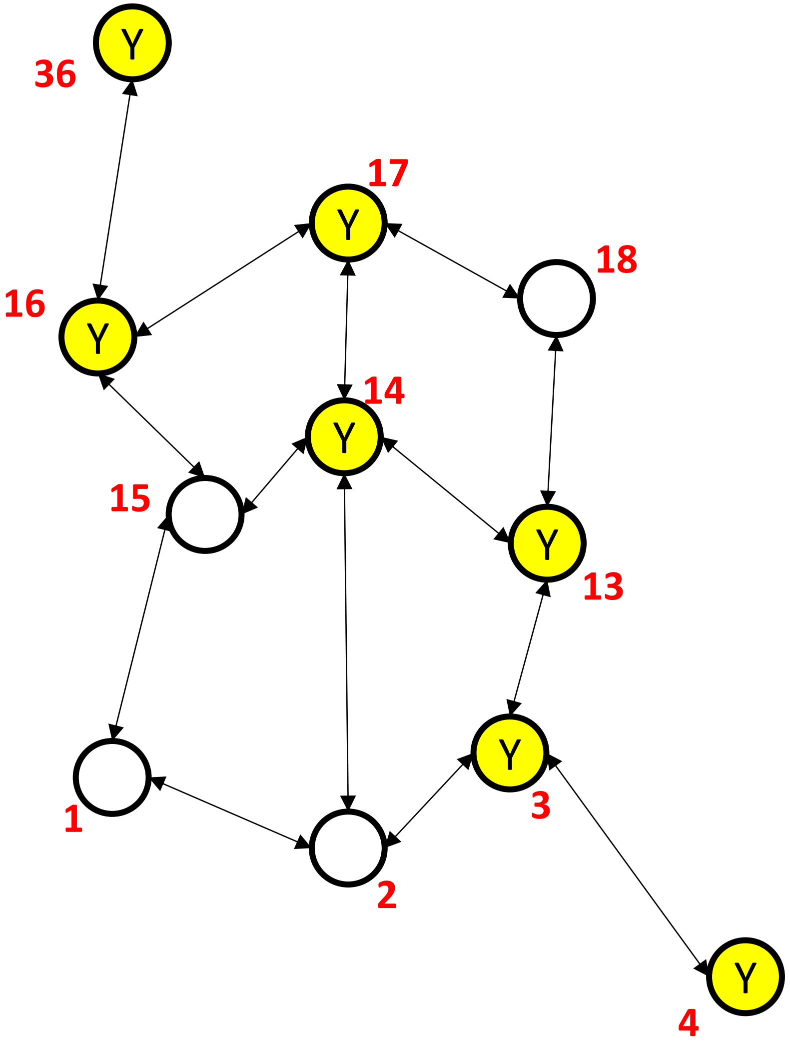
$$\diamondsuit^{hops}_{[3,5]} pink$$

$$\tau[0] = \ell_1, \ \tau[k] = \ell_{35}$$

$$\tau = \ell_1 \ldots \ell_{35}$$

$$d^{hops}_\tau(k) \in [3, 5]$$

Everywhere: $\boxdot^{f}_{[d_1,d_2]}\varphi$



$(S, \vec{x}, \ell, t)$ satisfies $\boxdot^{f}_{[d_1,d_2]}\varphi$ iff all the locations ℓ' reachable from ℓ via a path with length $d^{f}_{\tau}[\ell'] \in [d_1, d_2]$, satisfy φ

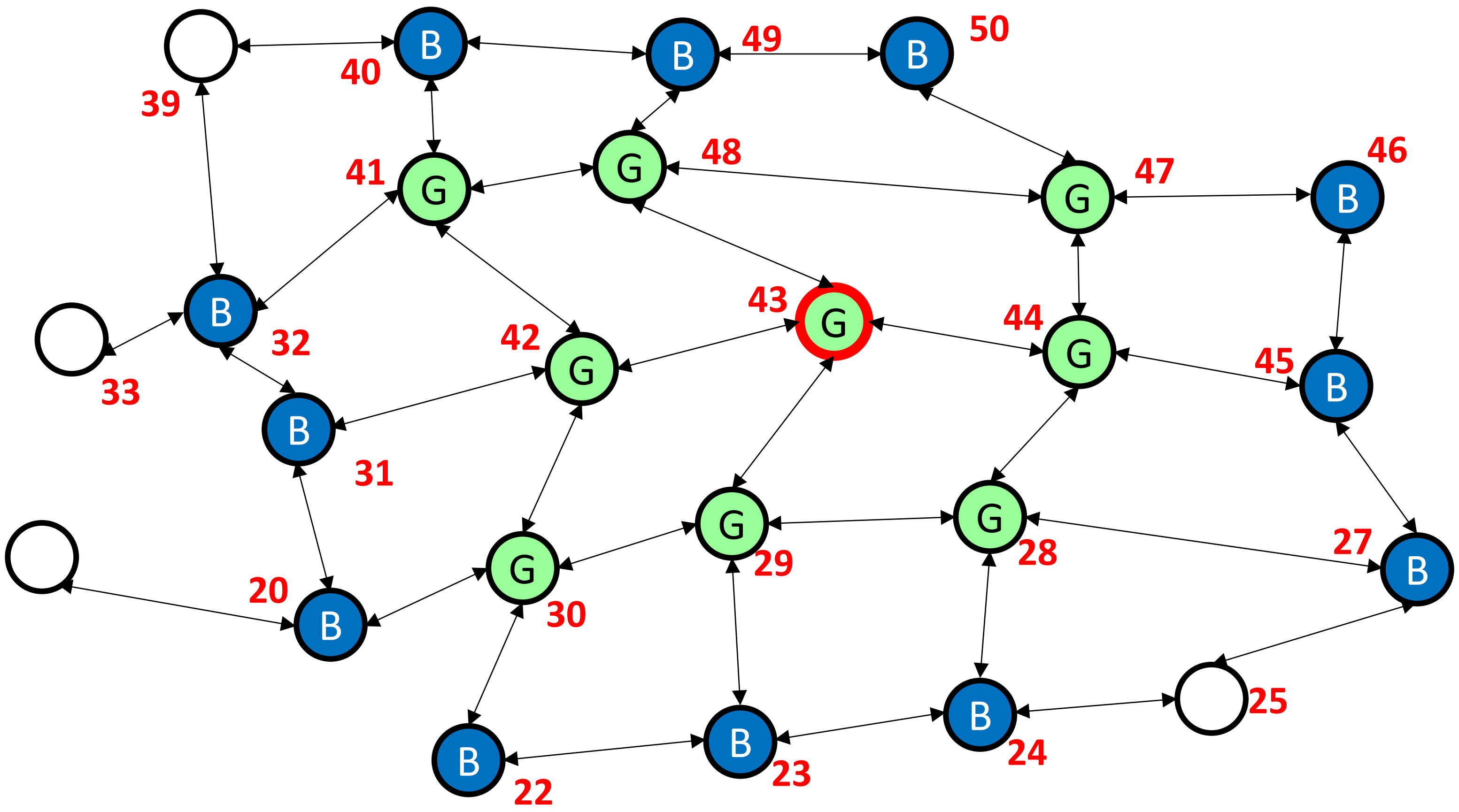
$$\boxdot^{hops}_{[2,3]} yellow$$

Surround: $\varphi_1 \circledcirc^{f}_{[d_1,d_2]} \varphi_2$

$(S, \vec{x}, \ell, t)$ iff there exists a φ_1-region that contains ℓ, all locations in that region satisfies φ_1 and are reachable from ℓ via a path with length less than d_2 .

All the locations that do not belong to the φ_1-region but are directly connected to a location of that region must satisfy φ_2 and be reached from ℓ via a path with length in the interval $[\mathrm{d}_1, d_2]$.

Surround: $green \circledcirc^{hops}_{[0,100]} blue$



Surround: $green \circledcirc^{hops}_{[2,3]} blue$