

Tecniche di programmazione in chimica computazionale

Examples

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Dipole moment and center of charge in molecules

For any molecule/cluster the dipole moment $\vec{\mu}$:

$$\vec{\mu} = \sum_{\alpha} q_{\alpha} \vec{r}$$

with α running on the atoms, q_{α} are the **charges** on atoms (Mulliken, Lowdin etc.) and \vec{r} are the **atomic coordinates**

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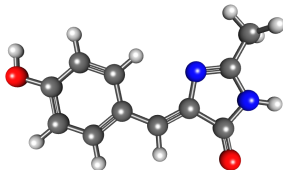
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For charged systems, the center of charge \vec{c} is:

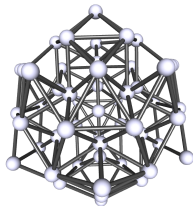
$$\vec{c} = \frac{\sum_{\alpha} q_{\alpha} \vec{r}}{\sum_{\alpha} q_{\alpha}}$$

Dipole moment and center of charge in molecules

- Copy files from </home/tpcc/2022/CC> to your work space
- HBDI (chromophore of GFP)



- Ag_{55}^{5+}



- Compute center of charge and dipole of the two systems (`cc_dip.f90`)
- Compute center of mass of the two systems (`com.f90`)

- Square matrix \mathbf{A} ($N \times N$)

Matrix diagonalization

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- Matrix diagonalization: $\mathbf{A} = \mathbf{U} \mathbf{D} \mathbf{U}^{-1}$
- $\mathbf{D} = \text{diag}(a_1, a_2 \dots a_N)$, a_i eigenvalues of \mathbf{A}
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- Example **diag.f90**

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- Transpose conjugated of a matrix: example `tconjug.f90`

Franck-Condon factors

Born-Oppenheimer approximation

$$\Psi_{e\nu}(q_e, q_N) = \psi_e(q_e; q_N) \chi_\nu^e(q_N)$$

$$\mu_{e,\nu;e',\nu'} = \int dq_N \chi_\nu^{e,*}(q_N) M_{ee'} \chi_{\nu'}^{e'}(q_N)$$

$$M_{ee'} = \int dq_e \psi_e^*(q_e; q_N) \hat{\mu} \psi_{e'}(q_e; q_N)$$

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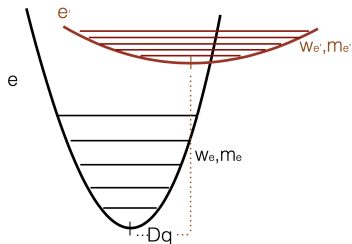
$$\mu_{e,\nu;e',\nu'} = M_{ee'}(\bar{q}_N) \int dq_N \chi_\nu^{e,*}(q_N) \chi_{\nu'}^{e'}(q_N)$$

$$S_{\nu,\nu'}^{e,e'} = \int dq_N \chi_\nu^{e,*}(q_N) \chi_{\nu'}^{e'}(q_N)$$

$$FC_{\nu,\nu'}^{e,e'} = |S_{\nu,\nu'}^{e,e'}|^2$$

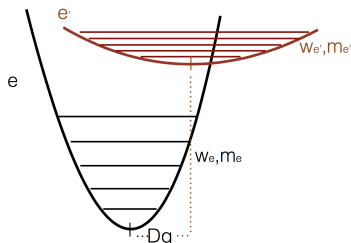
Franck-Condon factors

- Harmonic oscillator: $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega\hat{x}^2$ ($\omega = \sqrt{\frac{k}{m}}$)



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- **Harmonic** eigenfunctions

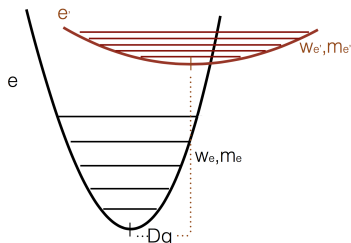
$$\chi_0^e(q_N) = \left(\frac{m_e\omega_e}{\pi}\right)^{1/4} \exp[-(m_e\omega_e)q_N^2/2]$$

$$\chi_1^e(q_N) = \sqrt{2} \left(\frac{m_e\omega_e}{\pi}\right)^{1/4} (\sqrt{m_e\omega_e}q_N) \exp[-(m_e\omega_e)q_N^2/2]$$

$$\chi_2^e(q_N) = \frac{1}{\sqrt{2}} \left(\frac{m_e\omega_e}{\pi}\right)^{1/4} [2m_e\omega_e q_N^2 - 1] \exp[-(m_e\omega_e)q_N^2/2]$$

Franck-Condon factors

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- **Example** [fc.f90](#)

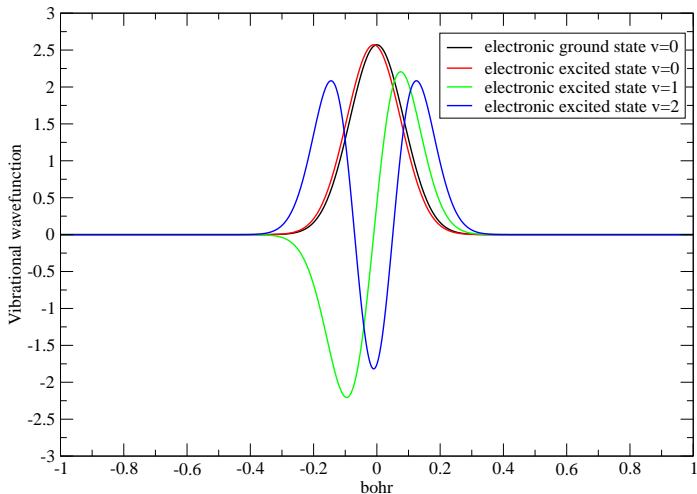
- Given the same frequencies, displacement, reduced masses, how the FC factor changes with the vibrational quantum number ν (0, 1 or 2) of the electronic excited state?

Franck-Condon factors

- Given the same frequencies, displacement, reduced masses, how the FC factor changes with the **vibrational quantum number ν (0, 1 or 2)** of the electronic excited state?
- Given the same frequencies and reduced masses, how the FC factor changes with the **displacement Δq** (for a chosen ν)?

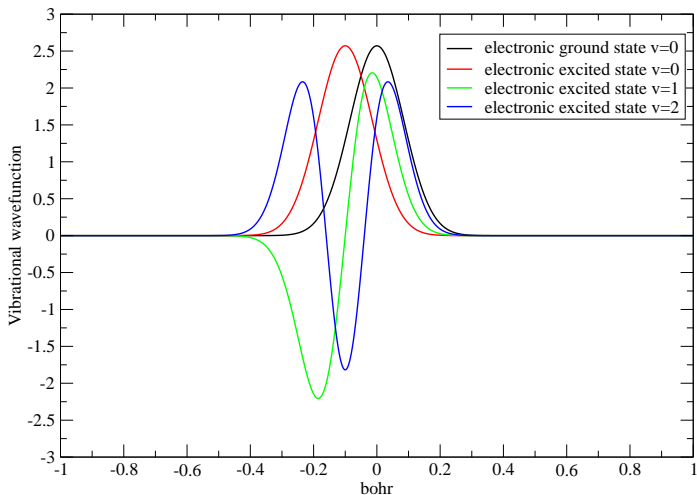
"Pseudo" Br₂ results

Displacement 0.01 bohr



"Pseudo" Br₂ results

Displacement 0.1 bohr



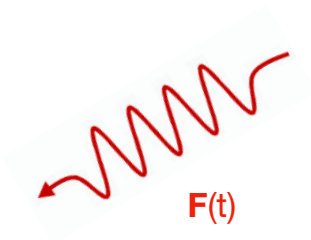
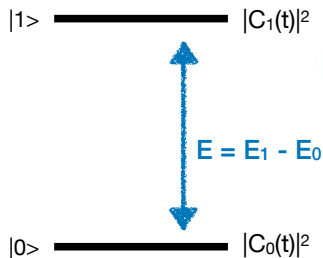
Simulating real-time dynamics

- Light-matter interaction, simulating time-resolved spectroscopies

Simulating real-time dynamics

- Light-matter interaction, simulating time-resolved spectroscopies
- Two-level system

$$|\psi(t)\rangle = C_0(t)|0\rangle + C_1(t)|1\rangle$$



Time-dependent Schrödinger equation (TDSE)

$$i\frac{\partial|\Psi(t)\rangle}{\partial t} = \hat{H}(t)|\Psi(t)\rangle$$

$$\hat{H}(t) = \hat{H}_0 - \hat{\mu} \cdot \mathbf{F}(t)$$

$$i\frac{\partial C_0(t)}{\partial t} = C_0(t)E_0 - \mathbf{F}(t) (\langle 0|\hat{\mu}|0\rangle + \langle 0|\hat{\mu}|1\rangle)$$

$$i\frac{\partial C_1(t)}{\partial t} = C_1(t)E_1 - \mathbf{F}(t) (\langle 1|\hat{\mu}|1\rangle + \langle 1|\hat{\mu}|0\rangle)$$

$$\mathbf{F}(t) = \mathbf{F}_{\max} \exp\left(-\frac{(t - t_{mid})^2}{2\sigma^2}\right) \sin(\omega t),$$

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- Executable `wavet.x` in `/home/tpcc/2022/WaveT`

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- input file, `ci_ini.inp` `ci_energy.inp` and `ci_mut.inp` in `/home/tpcc/2022/WaveT`
 - `input`: parameters for propagating TDSE
 - `ci_ini.inp`: initial populations
 - `ci_energy.inp` contains excitation energy E
 - `ci_mut.inp` contains dipoles and transition dipoles

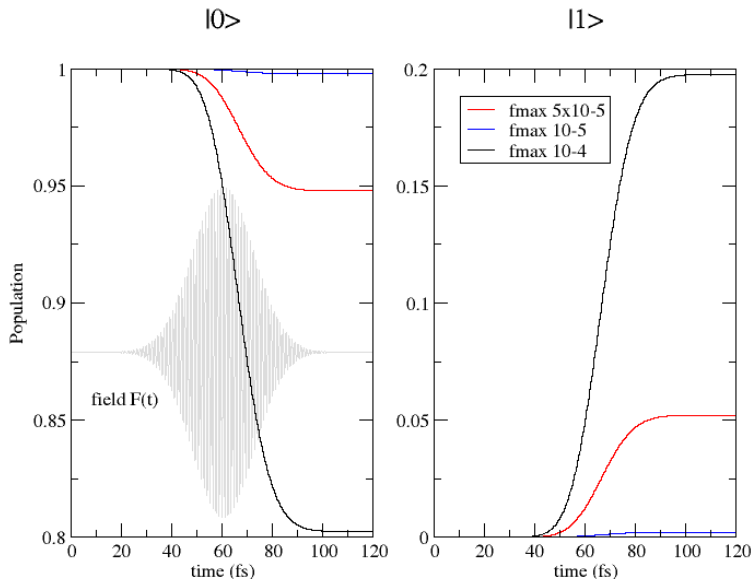
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 - `ci_energy.inp` contains excitation energy E
 - `ci_mut.inp` contains dipoles and transition dipoles
- To run the simulation, copy the files in your working directory, and type `./wavet.x < input > output`
- Many files are produced, focus on `c_t_1.dat`
 - `c_t_1.dat`: time evolution of $|C_0(t)|^2$ and $|C_1(t)|^2$
step time Population $|0\rangle, |1\rangle$

Simulating real-time dynamics

- 1 Check the last value of $|0\rangle$ and $|1\rangle$ populations by changing the **amplitude** of the pulse ($f_{\max} = 10^{-4}, 5 \times 10^{-5}, 10^{-5}$, resonant frequency)
- 2 Check the last value of $|0\rangle$ and $|1\rangle$ populations by changing the **frequency** of the pulse ($f_{\max} = 5 \times 10^{-5}$, $\omega = 0.11, 0.13$ and resonant)

Simulating real-time dynamics



Simulating real-time dynamics

