

Realizzazione:

da FIT di ordine 1

ad equazioni di stato

AB approccio relido
SOLO

per sistemi LTI SISO

di ordine $n=1$

$LTI, M=1, SISO, \text{Temp continuo}$

$$\begin{cases} \dot{x} = ax + bu \\ y = cx + du \end{cases}$$

$$a, b, c, d \in \mathbb{R}$$

$d=0 \iff$ sistema
strettamente proprio

$d \neq 0 \iff$ sistema non
strettamente proprio

FdT $D(s)$ grado 1
 $N(s)$ grado 0

FdT $D(s)$ grado 1
 $N(s)$ grado 1

$$\begin{cases} \dot{x} = ax + bu \\ y = cx + du \end{cases} \xRightarrow{d} \begin{cases} sX - x(0) = aX + bU \\ y = cX + dU \end{cases} \quad \overset{=0}{}$$

$$\begin{cases} X(s) = \frac{b}{s-a} U(s) \\ Y(s) = \frac{cb}{s-a} U(s) + d U(s) \end{cases}$$

$$T_{w,y}(s) = \frac{cb}{s-a} + d = \frac{ds + (cb - da)}{(s-a)}$$

$$T(s) = \frac{ds + (cb - da)}{(s - a)}$$

$d=0$
 sistema direttamente
 proprio

$$T(s) = \frac{cb}{s - a}$$

$M=0$
 $M=1$

$d \neq 0$

$$T(s) = \frac{ds + (cb - da)}{s - a}$$

$M=1$
 $M=1$

ordine $M=1$
 SISO

Voglio utilizzare queste informazioni per risolvere il

Problema inverso:

data una FdT di ordine 1, SISO, trovare una realizzabile in cf. di stato

$$F_{\text{OLT}} \quad n=1 \\ u=0$$

$$G(s) = \frac{f}{ps - q}$$

$$f, p, q \in \mathbb{R} \\ p \neq 0$$

Per confronto

$$\frac{f}{ps - q} \stackrel{f}{=} \frac{f/p}{s - (q/p)} \longleftrightarrow \frac{cb}{s - a}$$

divido Num
e Den per p

$$\begin{cases} cb = f/p \\ a = q/p \end{cases} \leftarrow \text{infinita solut.}$$

scelte comode

$$b=1 \rightarrow c = f/p$$

$$c=1 \rightarrow b = f/p$$

2 possibili soluzioni equivalenti

(2 tra le infinite possibili $\rightarrow$ scegliere $\kappa \in \mathbb{R}$) $\rightarrow$
 $\rightarrow$ calcolate b [equivalente a κ])

1° scelta

$$\begin{cases} \dot{x} = \left(\frac{g}{p}\right)x + u \\ y = \left(\frac{f}{p}\right)x \end{cases}$$

2° scelta

$$\begin{cases} \dot{x} = \left(\frac{g}{p}\right)x + \left(\frac{f}{p}\right)u \\ y = x \end{cases}$$

Fd/T $M=1$
 SISO $M=1$

$$G(s) = \frac{ps - q}{\eta s - w}$$

$p, q \in \mathbb{R}$
 $\eta, w \in \mathbb{R}$

Per confronto

$$G(s) = \frac{ps - q}{\eta s - w}$$

comporre
 la funzione in 2 parti
 1° parte algebrica
 2° parte Fd/T strict-propre

$p \neq 0$
 $\eta \neq 0$

$ps - q$	$\eta s - w$
$-ps + \frac{w p}{\eta}$	p/η
$\hline \frac{wp - q\eta}{\eta} \triangleq M$	

$$G(s) = \left(\frac{p}{\eta} \right) + \frac{M}{\eta s - w}$$

$$G(s) = \left(\frac{P}{\pi} \right) + \frac{M}{\pi s - \omega}$$

$$\left\{ d = \frac{P}{\pi} \right.$$

$$G(s) = d + \frac{cb}{s - \alpha}$$

come il caso
precedente

$$\left\{ \begin{array}{l} c = 1 \\ b = 14/\pi \\ \alpha = 15/\pi \end{array} \right.$$

$$\left\{ \begin{array}{l} b = 1 \\ c = 14/\pi \\ \alpha = 15/\pi \end{array} \right.$$

Esami

①

$$T(s) = \frac{2}{s+1}$$

$\longleftrightarrow$

$$\frac{cb}{s-a}$$

$$G(s) = \frac{cb}{s-a}$$

$$G(s) = d + \frac{\tau b}{s-a}$$

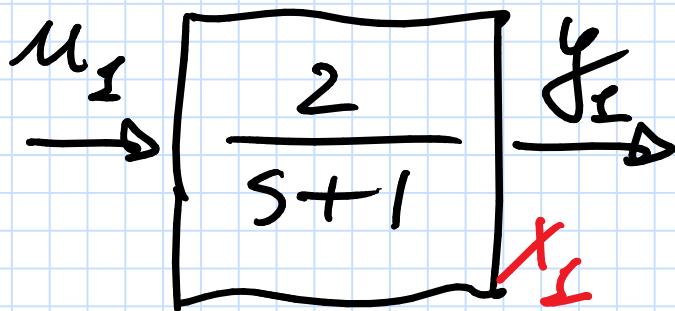
$$a = -1$$

$$b=1$$

$$c=2$$

$$\tau=1$$

$$b=2$$



$$\begin{cases} \dot{x}_1(t) = -x_1(t) + 2u_1(t) \\ y_1(t) = x_1(t) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u_1(t) \\ y_1(t) = 2x_1(t) \end{cases}$$

$$\textcircled{2} T(s) = \frac{2s+3}{s+5}$$

$$M=1$$

$$N=1$$

$$\begin{array}{r|l} 2s+3 & s+5 \\ \hline -2 & -10 \\ \hline 1 & -7 \end{array}$$

$$T(s) = 2 - \frac{7}{s+5}$$

$$G(s) = d + \frac{cb}{s-a}$$

$$d=2$$

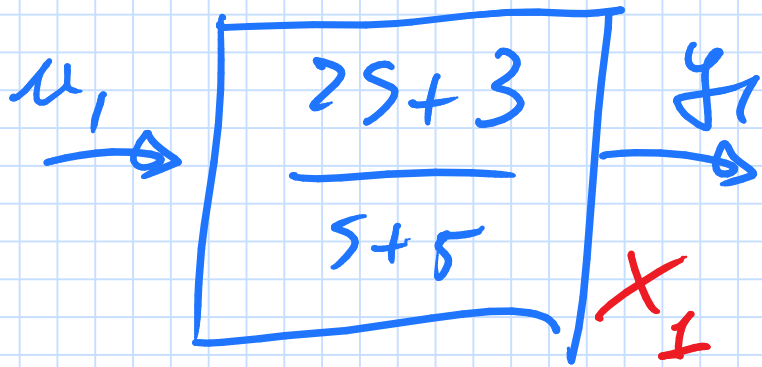
$$a=-5$$

$$b=1$$

$$c=-7$$

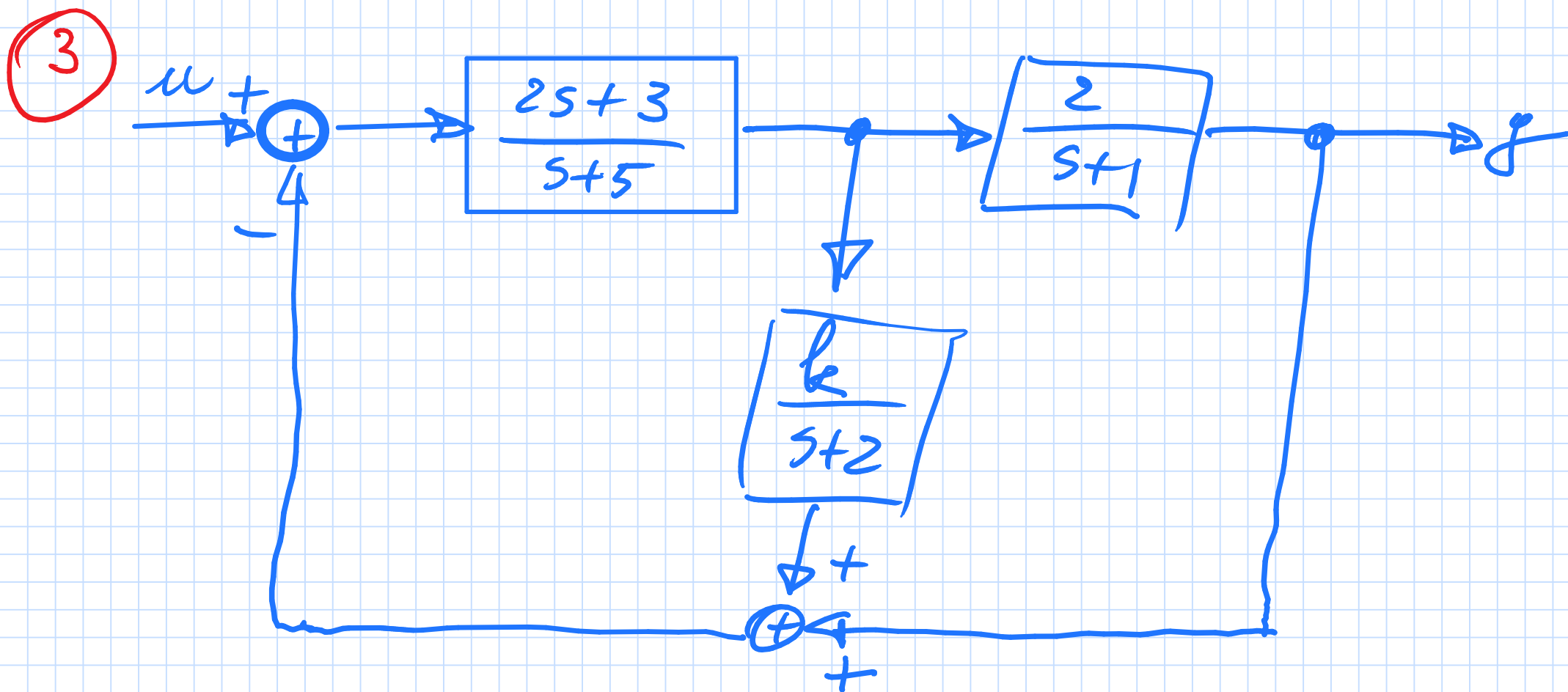
$$c=1$$

$$b=-7$$

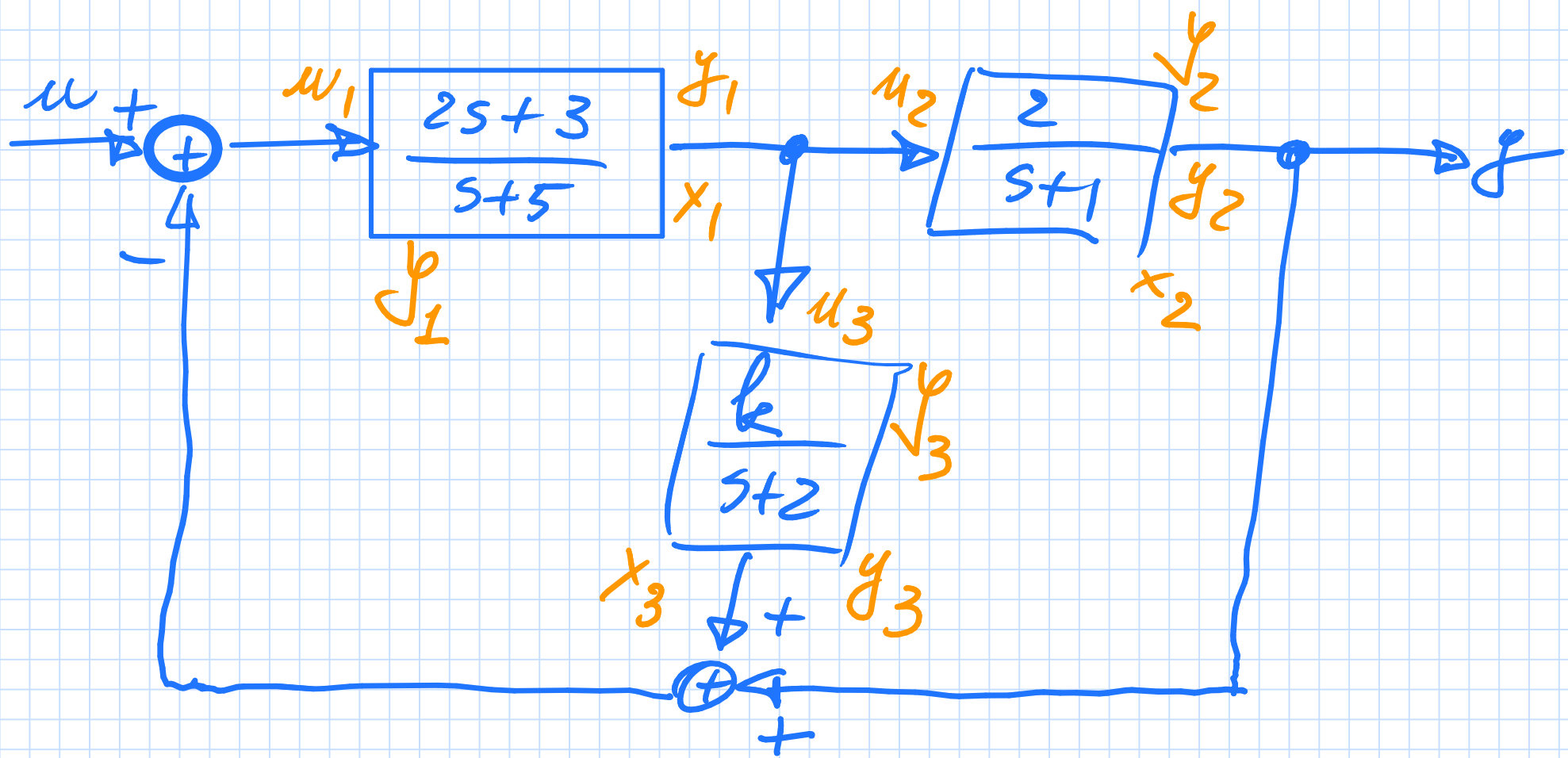


$$\begin{cases} \dot{x}_1(t) = -5x_1(t) + u_1(t) \\ y_1(t) = -7x_1(t) + 2u_1(t) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = -5x_1(t) - 7u_1(t) \\ y(t) = x_1(t) + 2u_1(t) \end{cases}$$



Trovare una realizzazione in cf. di stato
NON modificare lo schema!



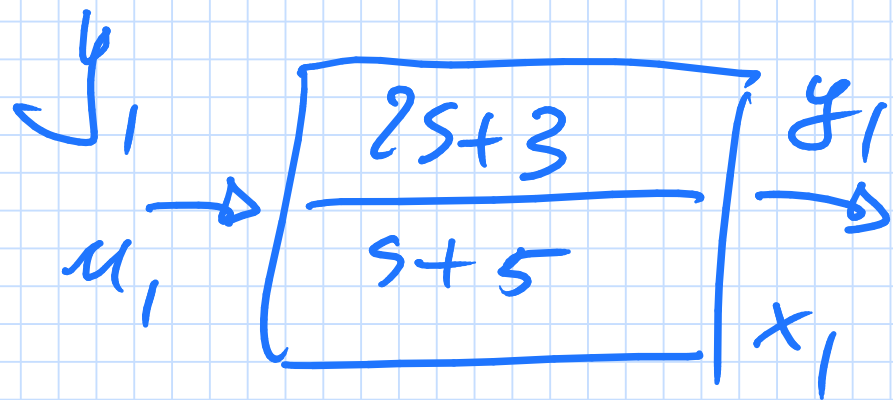
(•) etichettare i blocchi $y_1, y_2, y_3, \dots$

(•) etichettare ingressi, uscite e variabili di stato di ogni blocco

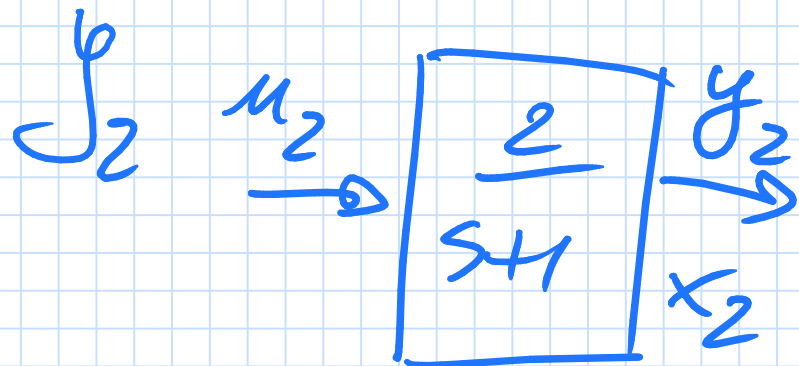
(•) trovare una soluz. in sp. dato $\forall y_i$

(•) derivare con eq. costitutive le interconnessioni tra i blocchi y_i

(•) eliminare variabili ausiliarie $\begin{pmatrix} u_1, y_1, u_2, u_3 \\ y_2, y_3 \end{pmatrix}$



$$\begin{cases} \dot{x}_1 = -5x_1 - 7u_1 \\ y_1 = x_1 + 2u_1 \end{cases}$$



$$\begin{cases} \dot{x}_2 = -x_2 + 2u_2 \\ y_2 = x_2 \end{cases}$$

$$\begin{array}{c} \dot{y}_3 \\ \mu_3 \end{array} \left[\frac{k}{s+2} \right] \begin{array}{c} y_3 \\ x_3 \end{array}$$

$$\begin{cases} \dot{x}_3 = -2x_3 + k\mu_3 \\ y_3 = x_3 \end{cases}$$

$$\dot{x}_1 = -5x_1 - 7\mu_1$$

$$\dot{x}_2 = -x_2 + 2\mu_2$$

$$\dot{x}_3 = -2x_3 + k\mu_3$$

$$\mu_1 = u - y_2 - y_3 = u - x_2 - x_3$$

$$\mu_2 = y_1 = x_1 + 2\mu_1 = x_1 - 2x_2 - 2x_3 + 2u$$

$$\mu_3 = y_1 = x_1 + 2\mu_1 = x_1 - 2x_2 - 2x_3 + 2u$$

$$y_1 = x_1 + 2\mu_1$$

$$y_2 = x_2$$

$$y_3 = x_3$$

$$y = y_2 = x_2$$

Dopo aver sostituito le variabili e cessorie sfruttando le relazioni algebriche che legano le var. ausiliarie alle variabili di stato (x_1, x_2, x_3) di ingresso e uscita del sistema totale (u, y)

$$\begin{cases} \dot{x}_1 = -5x_1 + 7x_2 + 7x_3 - 7u & \text{ordine 3} \\ \dot{x}_2 = +2x_1 - 5x_2 - 4x_3 + 4u \\ \dot{x}_3 = kx_1 - 2kx_2 - 2(1+k)x_3 + 2ku \\ y = x_2 \end{cases}$$

LTI, SISO, $n=1$, Tiempo discreto

$$\begin{cases} x(k+1) = a x(k) + b u(k) \\ y(k) = c x(k) + d u(k) \end{cases} \quad \begin{array}{l} a, b, c, d \in \mathbb{R} \\ a \neq 0 \end{array}$$

$$T(z) = C (zI - A)^{-1} B + D$$

$$= \frac{a b}{z - a} + d$$

← estructura
análogo al
caso "a Tiempo
continuo"

$$G_1(z) = \frac{f}{pz - q}$$

$$m = 0$$

$$n = 1$$

$$p \neq 0$$

système
strictement
propre

Par comparaison

$$G_1(z) = \frac{f/p}{z - q/p}$$

$$\longleftrightarrow T(z) = \frac{cb}{z - a}$$

$$a = q/p$$

$$\begin{cases} x(k+1) = (q/p)x(k) + u(k) \\ y(k) = (f/p)x(k) \end{cases}$$

$$b = 1$$

$$c = f/p$$

$$a = 1$$

$$b = f/p$$

$$\begin{cases} x(k+1) = q/p x(k) + f/p u(k) \\ y(k) = x(k) \end{cases}$$

$$G_1(z) = \frac{ez + f}{pz + q}$$

$$e, f, p, q \in \mathbb{R}$$

$$e \neq 0$$

$$p \neq 0$$

$$M=1$$

$$M=1$$

En modo analógico al caso "2 Tenso outdoors"

$$\begin{array}{r|l} ez + f & pz + q \\ \hline -ez - \frac{eq}{p} & \frac{e}{p} \\ \hline 1 & f - \frac{eq}{p} \end{array}$$

$$G(z) = \frac{e}{p} + \frac{(f - \frac{eq}{p})/p}{z + q/p}$$

$$T(z) = 1 + \frac{cb}{z - a}$$