- Equilibrio

$$\frac{dE_p}{dx} = -F_x = 0$$

$$\frac{d^2E_p}{dx^2}$$

1)

a) Eq stabile $\rightarrow \frac{d^2U}{dx^2} > 0$ (assumo σ e $\varepsilon > 0$)

Eq instabile $\rightarrow$ " < 0

Eq indiffer. $\rightarrow$ " $= 0$

⚠ bisogna cercare gli annullamenti di U'!!

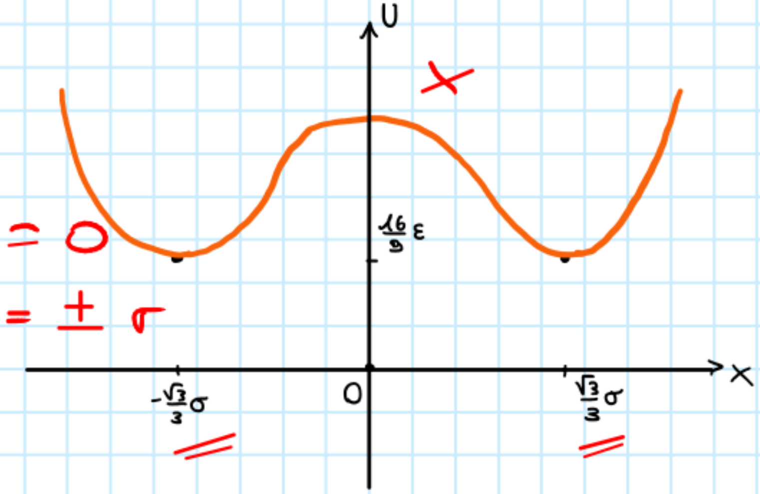
$$\rightarrow U(x) = \varepsilon\left(1-\left(\frac{x}{\sigma}\right)^2\right)^2 = \varepsilon\left(1+\left(\frac{x}{\sigma}\right)^4 - 2\left(\frac{x}{\sigma}\right)^2\right)$$

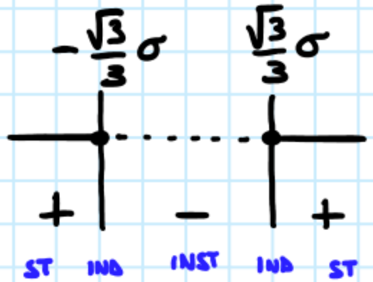
$$\frac{dU}{dx} = \varepsilon\left(\frac{4x^3}{\sigma^4} - \frac{4}{\sigma^2}x\right) = \frac{4\varepsilon}{\sigma^2}x\left(\frac{x}{\sigma^2} - 1\right) = 0$$

$$\Rightarrow x_1 = 0,\ x_{2,3} = \pm\sigma$$

$$\frac{d^2U}{dx^2} = \varepsilon\left(\frac{12}{\sigma^4}x^2 - \frac{4}{\sigma^2}\right)$$

$$\frac{\varepsilon}{\sigma^2}\left(\frac{12}{\sigma^2}x^2 - 4\right) \geq 0 \rightarrow \overbrace{\left(\frac{12}{\sigma^2}x^2 - 4 \geq 0\right.}^{>0}$$





$$\rightarrow x_{1,2} = \frac{0 \pm \sqrt{\frac{192}{\sigma^2}}}{\frac{24}{\sigma^2}} \begin{cases} +\frac{8\sqrt{3}}{24}\sigma = \frac{\sqrt{3}}{3}\sigma \\ -\frac{8\sqrt{3}}{24}\sigma = -\frac{\sqrt{3}}{3}\sigma \end{cases}$$

$$U(x_1) = \frac{16}{9}\varepsilon$$

$$U(x_2) = \frac{16}{9}\varepsilon$$

- II Newton

$$|\Sigma \vec{F}| = m|\vec{a}|$$

ERRORE

$$\Sigma|\vec{F}| = m \cdot a$$ ✗

$$\Sigma|\vec{F}| = ma$$

o che …

$\cancel{(\Sigma|\vec{F}| = m \cdot \vec{a})}$ ho ✗

- UNITÀ DI MISURA :

$$M = \frac{|\vec{g}||\vec{r}|^2}{G} = \frac{(9.81 \frac{m}{s^2})(6 \times 10^6)^2 m}{6.674 \times 10^{-11} \; N m^2/kg^2} = 5.29 \times 10^{24} \approx 10^{25}$$ ✗ UNITÀ DI MISURA

$$M \approx 10^{25}$$

• Vettori – scalari :

$$\vec{v}_1 = \frac{\rho_b \cdot V_b \cdot g}{\xi}$$

$$\vec{v} = \frac{\rho_b V_b g}{\xi}$$

$$-kx = m \cdot \vec{a} \rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

non puoi uguagliare un vettore e uno scalare x diagramma

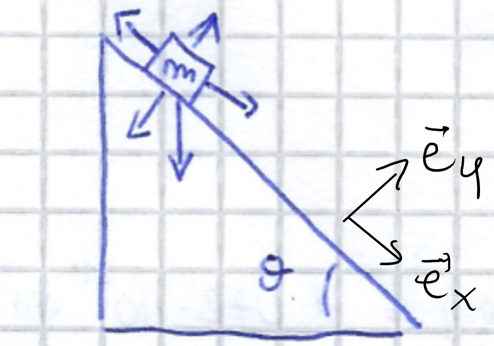
$$-kx = m\vec{a}$$

$$\frac{\vec{F}_d}{\vec{N}} \leq \mu_s$$

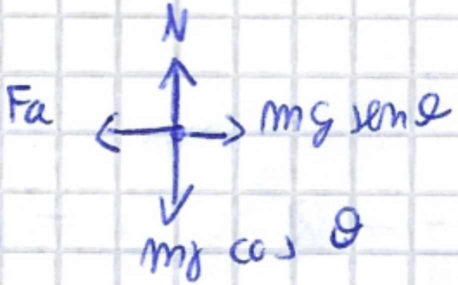
che cosa è il rapporto tra 2 vettori ??

$$\mu_s = \frac{\vec{F}_a}{\vec{N}}$$

- Definire e usare base $\vec{e}_x, \vec{e}_y$ per esprimere i vettori



diag. corpo libero



II Newton

segno?

Σ_x : $mg \operatorname{sen} \theta + Fa = m \cdot a$

segno?

Σ_y : $-mg \cos \theta + N = 0$

- Ipotesi ⇒ conclusioni

sapendo che $\frac{d^2x}{dt^2} = -\frac{k}{m} x$

da cosa?

delle x e delle y → da che cosa?

Noto che $\vec{n} = mg \cos \theta$; $\vec{f}_s = mg \operatorname{sen} \theta$

forza normale; forza di attrito statico

Devi partire da $\Sigma \vec{F} = m\vec{a} = \vec{0}$

So che affinché il blocco sia "fermo"

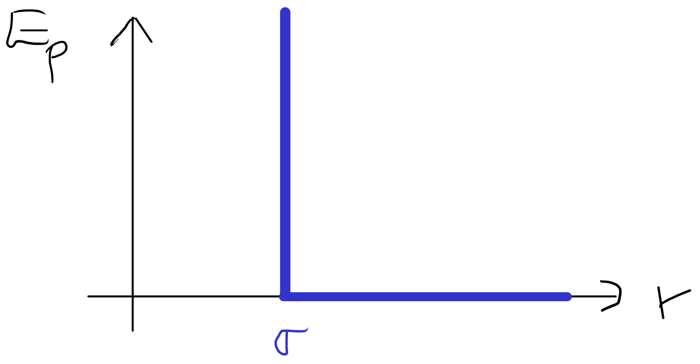
g

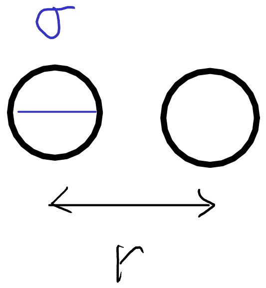
θ

- Archimede

INTERPRETAZIONE MICROSCOPICA DELL' ENTROPIA

Modello di sfere dure

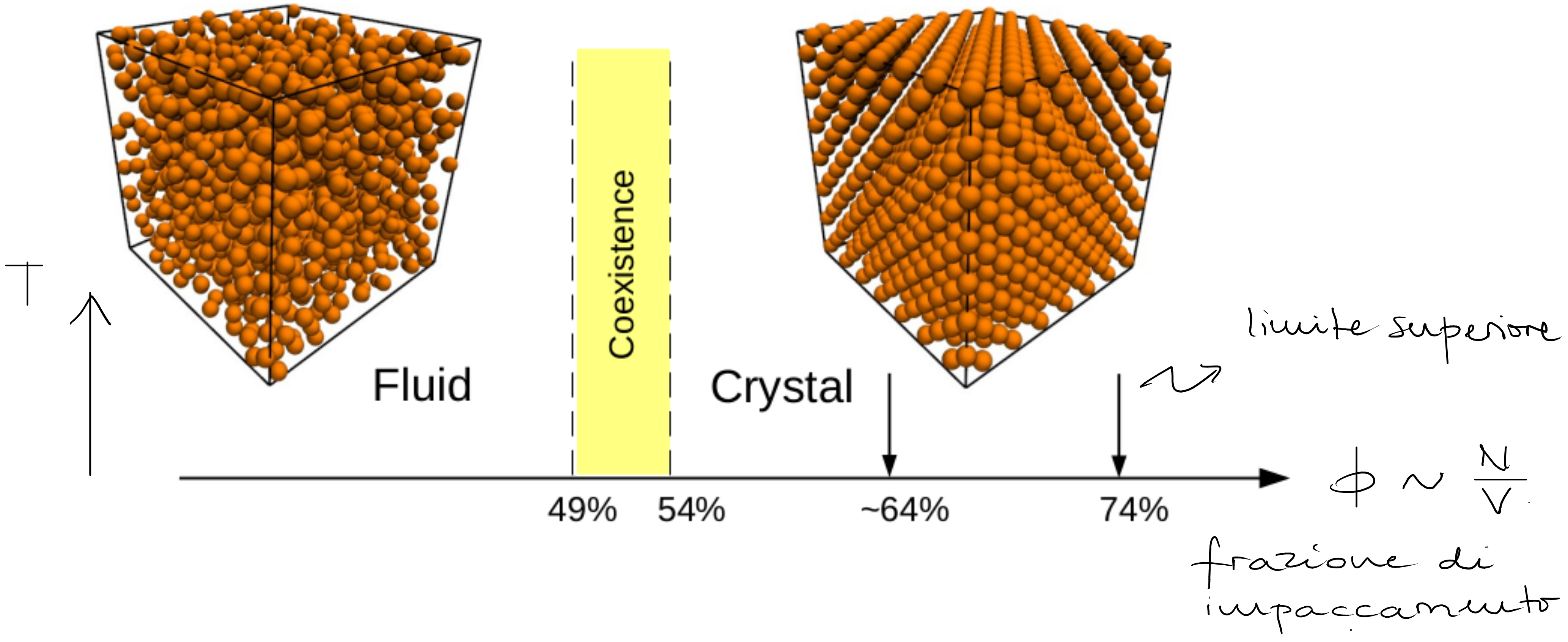




$$E_p = \begin{cases} 0 & r > \sigma \\ \infty & r \leq \sigma \end{cases}$$

Sospensioni colloidali

Diagramma di fase



Dinamica molecolare: integrazione numerica delle equazioni del moto

Phase Transition for a Hard Sphere System

B. J. ALDER AND T. E. WAINWRIGHT
University of California Radiation Laboratory, Livermore, California
(Received August 12, 1957)

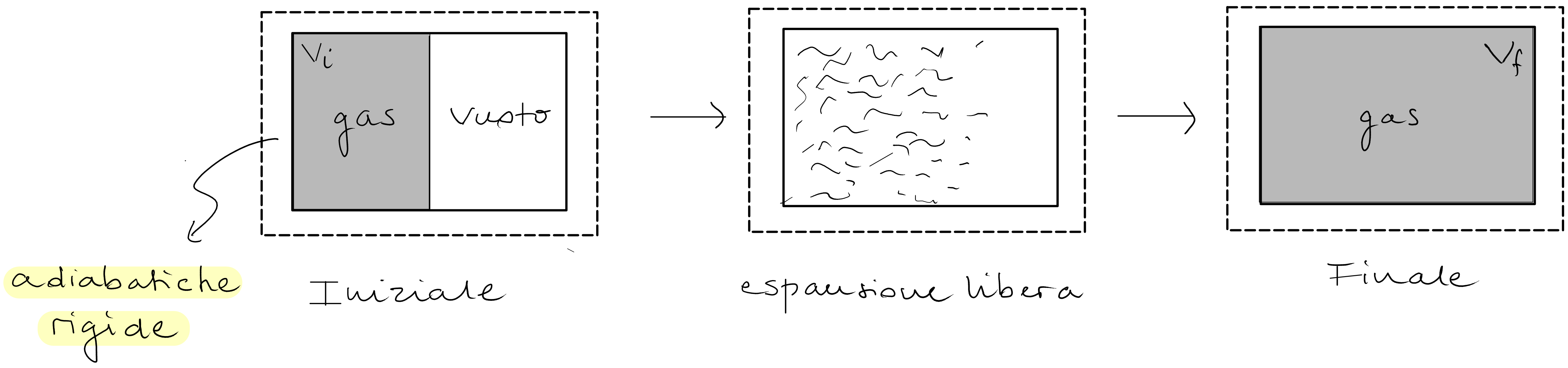
A CALCULATION of molecular dynamic motion has been designed principally to study the relaxations accompanying various nonequilibrium phenomena. The method consists of solving exactly (to the number of significant figures carried) the simultaneous classical equations of motion of several hundred particles by means of fast electronic computors. Some of the

Berni Alder '57

Se $\phi > 54\%$, $S_{cristallo} > S_{fluido}$

entropia $\not\Rightarrow$ disordine

Espansione libera di un gas perfetto



Trasf. adiabatica è reversibile? $\quad dS = \dfrac{\delta Q}{T}$ QS

Variazione di S tra i e f

$$U = C_v T$$

$$\Delta S = \int_i^f dS = \int_i^f \frac{\delta Q}{T} = \int_i^f \frac{dU}{T} + \int_i^f \frac{P}{T}\, dV = C_v \int_{U_i}^{U_f} \frac{dU}{U} + nR \int_{V_i}^{V_f} \frac{dV}{V}$$

trasf. immaginaria QS tra i e f

$$\delta Q = dU - \delta W = dU + PdV$$

$$= \underbrace{C_V \ln \frac{U_f}{U_i}}_{=0} + nR \ln \frac{V_f}{V_i} = nR \ln \frac{V_f}{V_i} = N K_B \ln \frac{V_f}{V_i}$$

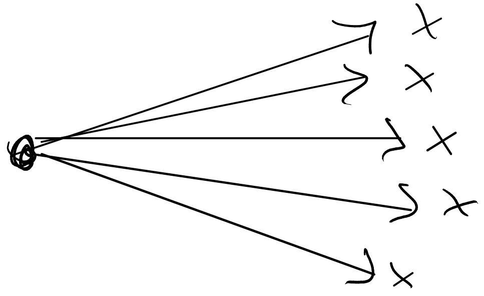
$$\text{I pr:} \quad \Delta U = U_f - U_i = \underbrace{W}_{=0} + \underbrace{Q}_{=0} = 0 \Rightarrow U_f = U_i$$

rigide

+ espansione nel vuoto

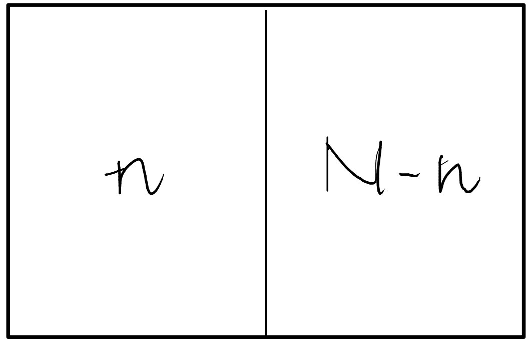
Es: $V_f = 2V_i \Rightarrow \Delta S = N K_B \ln 2 > 0$ ok per il II pr.

Macro	Micro
Termodinamica	Dinamica Newtoniana
$\{P, T, V, N, U, S\}$	$\{\vec{r}_1, \dots, \vec{r}_N, \vec{v}_1, \dots, \vec{v}_N\}$
macro-stato	micro-stato



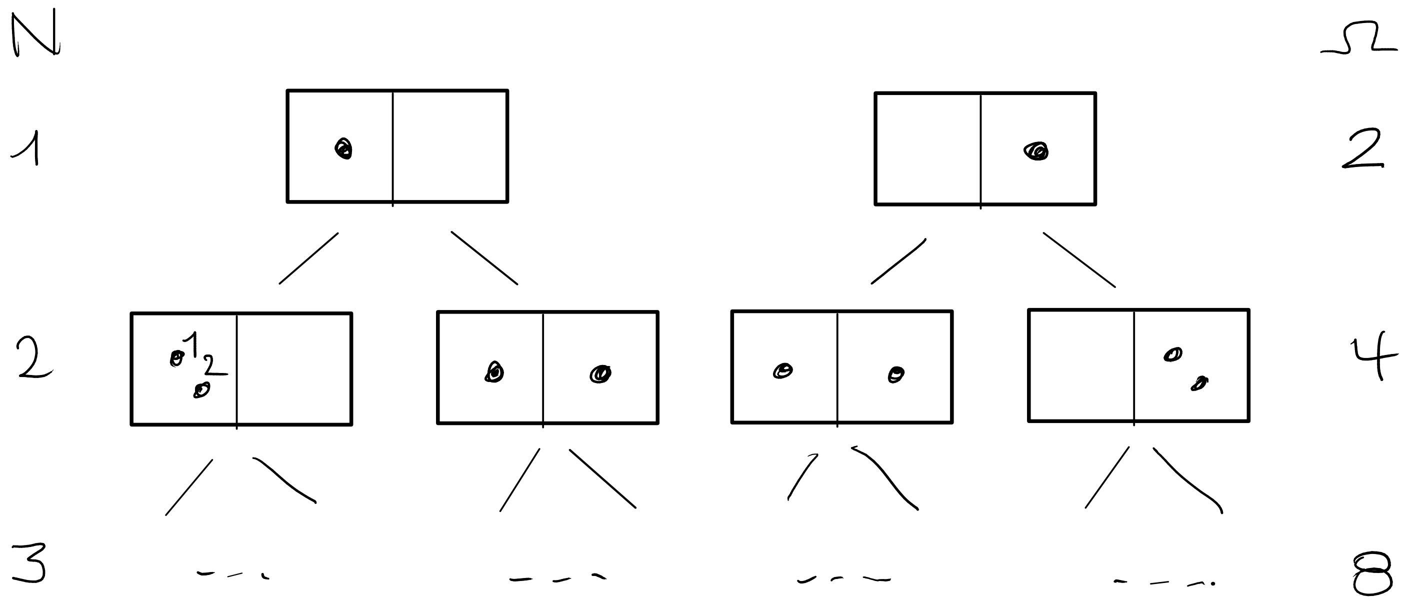
$S \rightarrow$ numero di micro-stati compatibili con un macro-stato

Modellizzazione schematica



Numero di realizzazioni totali

$$\Omega = 2^N$$



macro - stato : n

micro - stato : combinazione di n tra N

Combinazioni compatibili con macro-stato n

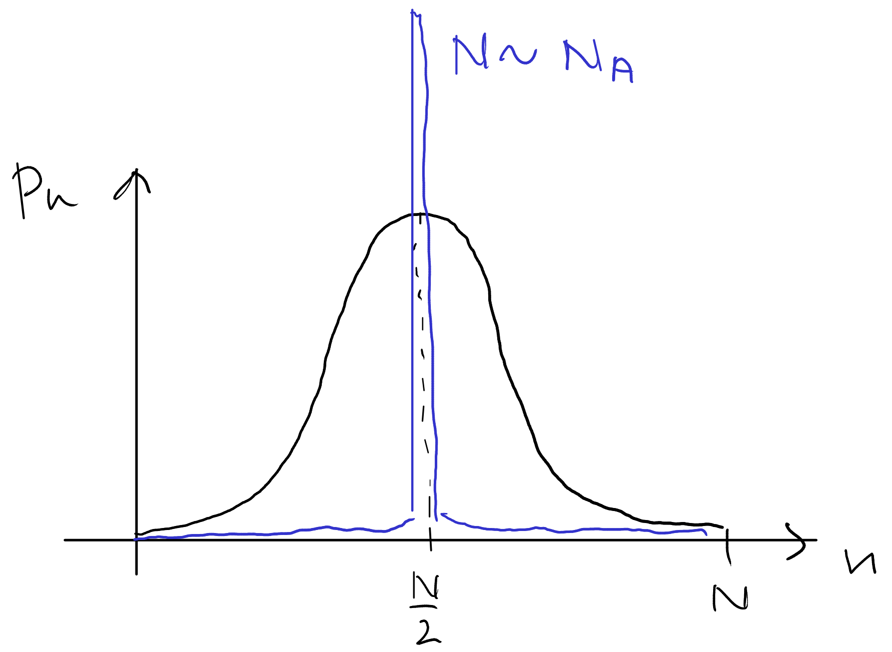
$$W_n = C_n^N = \binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

coefficiente binomiale

$$\left[\sum_{n=0}^{N} C_n^N = 2^N\right]$$

Probabilità del macro stato n

$$P_n = \frac{C_n^N}{\sum_{n=0}^{N} C_n^N} = \frac{W_n}{2^N}$$



$N = N_A$

Iniziale : $n = N$

Finale : $n = \frac{N}{2}$

$$p_N = \frac{1}{2^{10^{23}}} = 2^{-10^{23}} \approx 10^{-10^{22}}$$

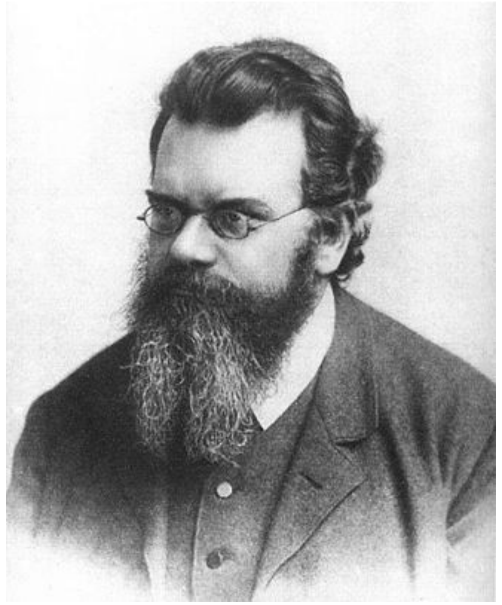
Entropia di Boltzmann $\rightarrow$ sistema isolato

Microstati hanno tutti uguale probabilità

$S \sim \log W \longrightarrow \boxed{S \equiv K_B \log W}$

W: numero di micro-stati compatibili con il macro-stato

K_B: costante di Boltzmann



Ludwig Boltzmann
1844 – 1906

Nota: $\log \equiv \ln$ in base e

S è additiva

$\boxed{1}\boxed{2}$

$S_1 \quad S_2$

$S_{\{1,2\}} = S_1 + S_2 \qquad W_1 \cdot W_2 = W \qquad \log(W_1 W_2) = \log W_1 + \log W_2$

Equilibrio $\rightarrow$ stato più probabile (n. massimo di realizzazioni micro)

S: misura della nostra ignoranza / incertezza riguardo allo stato microscopico del sistema

Es. espansione libera

Approssimazione di Stirling $N \gg 1$

$$\ln(N!) \approx N \ln N - N$$

$$S_n = K_B \ln W_n$$

$$\Delta S = S_{\frac{N}{2}} - S_N = K_B \ln\left[\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\right] - \underbrace{K_B \ln\left(\frac{\cancel{N!}}{\cancel{N!}\,1}\right)}_{=0}$$

$$= K_B \ln(N!) - 2K_B \ln\left[\left(\frac{N}{2}\right)!\right] = K_B \ln(N!) - 2K_B \ln\left[\left(\frac{N}{2}\right)!\right]$$

$$\approx K_B N \ln N - \underbrace{K_B N}_{\sim} - \cancel{2}\frac{N}{\cancel{2}} K_B \ln\left(\frac{N}{2}\right) + \underbrace{\cancel{2} K_B \frac{N}{\cancel{2}}}_{\sim}$$

$$= K_B \cancel{N \ln N} - K_B \cancel{N \ln N} + N K_B \ln 2$$

$$= K_B N \ln 2 \quad \square$$