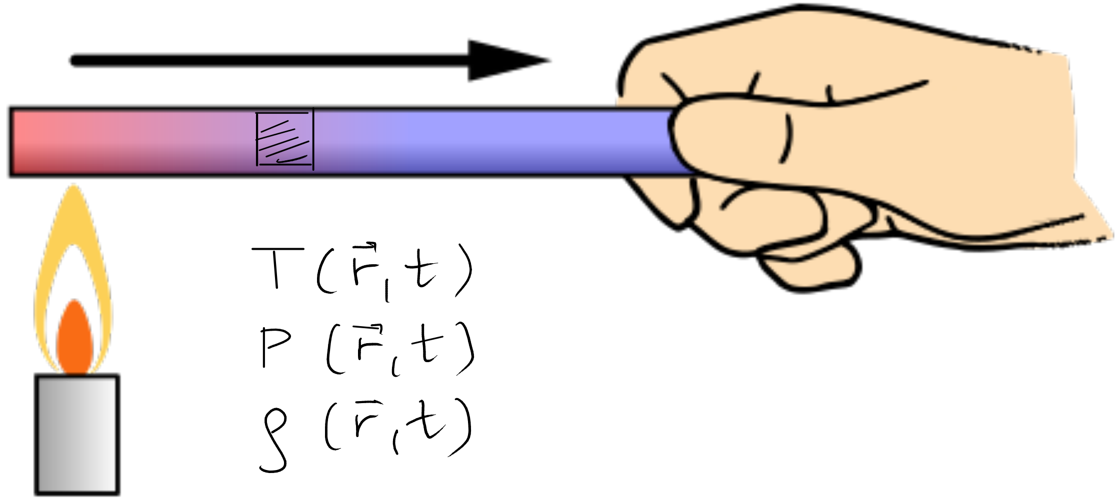
CONDUZIONE TERMICA

Termodinamica di non-equilibrio / fuori equilibrio → evoluzione temp.



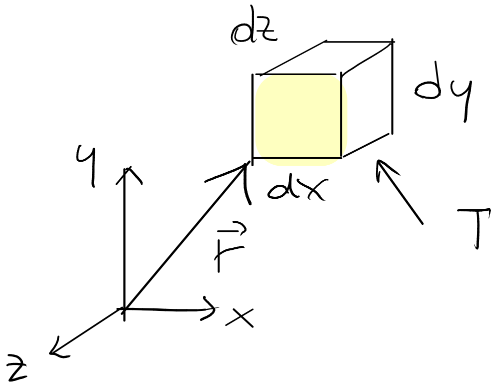
$T(\vec{r},t)$

$P(\vec{r},t)$

$\rho(\vec{r},t)$

Equilibrio locale:

tra t e $t+dt$, esiste nel punto $\vec{r}$ un sottosistema di lato dx, dy, dz in condizione di equilibrio termodinamico



$$l_0 \ll dx, dy, dz \ll l$$

$10^{-9}\,m$

micro — macro

$$\tau_0 \ll dt \ll \tau$$

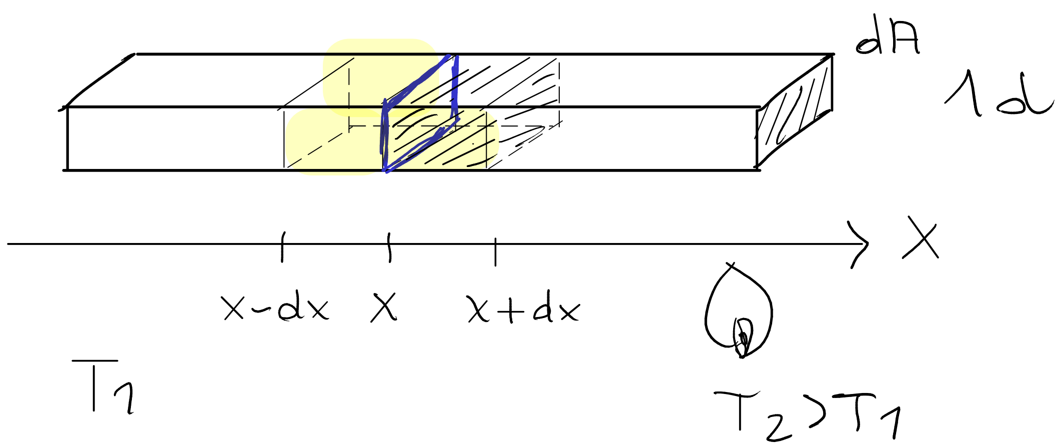
$10^{-12}\,s$

Conduzione termica:

scambi energetici sotto forma di calore in solidi o fluidi a riposo

(no convezione)

Legge di Fourier : empirica, non esatta



Calore scambiato dal sotto sistema in x con quello in x-dx

$$\delta Q \sim dA\, dt\, \frac{dT}{dx}$$

$$\delta Q = -\lambda\, dA\, dt\, \frac{dT}{dx}$$

$\uparrow$ conducibilità termica

Joseph Fourier
1768 – 1830

$$dT = T(x+dx) - T(x)$$

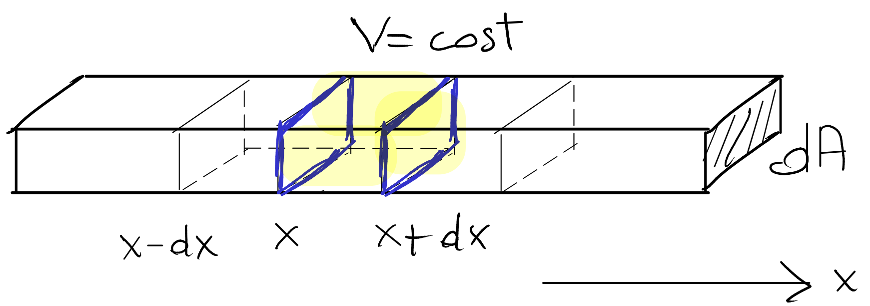
$$dT > 0 \Rightarrow \delta Q < 0$$

$\dfrac{\delta Q}{dt} \equiv I_t$ corrente termica SI : $\dfrac{J}{s} = W$ Watt

$\dfrac{\delta Q}{dA\, dt} \equiv J_t$ densità di corrente termica SI : $\dfrac{J}{s\, m^2} = \dfrac{W}{m^2}$

$J_t = -\lambda \dfrac{dT}{dx}$ legge di Fourier (1d) $\lambda \to$ SI : $\dfrac{W}{m \cdot K}$

Conservazione dell'energia



Per il sottosistema in x :

$$\underbrace{dE_c}_{=0} + dU = \underbrace{\delta W}_{=0} + \delta Q \Rightarrow dU = \delta Q$$

$$dU = \delta Q_-(x) + \delta Q_+(x) = \delta Q_-(x) - \underbrace{\delta Q_-(x+dx)}$$

calore scambiato dal sotto sistema in x+dx con quello in x

$$dU = J_t(x)\,dt\,dA - J_t(x+dx)\,dt\,dA = -\left[J_t(x+dx) - J_t(x)\right]dt\,dA$$

Esprimo dU come differenziale totale → V, T

$$dU = \left.\frac{\partial U}{\partial T}\right|_V dT + \left.\frac{\partial U}{\partial V}\right|_T \underbrace{dV}_{=0} = \left.\frac{\partial U}{\partial T}\right|_V dT = C_V\,dT \qquad V = cost$$

$$C_V\,dT = -\left[J_t(x+dx) - J_t(x)\right]dt\,dA$$

Capacità termica : $C_V = m c_V = V \rho c_V = dx\, dA\, \rho\, c_V$

$$dx\, \cancel{dA}\, \rho\, c_V\, dT = -\left[J_t(x+dx) - J_t(x) \right] dt\, \cancel{dA}$$

$$\frac{dT}{dt} = -\frac{1}{\rho c_V} \frac{J_t(x+dx) - J_t(x)}{dx}$$

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_V} \frac{\partial J_t}{\partial x}$$

Uso legge di Fourier : $J_t = -\lambda \frac{\partial T}{\partial x}$

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_V} \frac{\partial}{\partial x}\left(-\lambda \frac{\partial T}{\partial x}\right) = \frac{\lambda}{\rho c_V} \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\lambda}{\rho c_V} \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \underbrace{\frac{\lambda}{\rho c_V}}_{D_t} \frac{\partial^2 T}{\partial x^2}$$ <u>eq. del calore</u> $\rightarrow$ eq. del moto per T !

$D_t \equiv$ diffusività termica

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2}$$ eq. di diffusione $\rightsquigarrow$ RW

$\uparrow$
coefficiente di diffusione

Eq. differenziale alle derivate parziali

$$\Sigma \vec{F} = m \frac{d\vec{v}}{dt} = m \frac{d}{dt}\frac{d\vec{r}}{dt}$$ $t \to t' = -t \Rightarrow \Sigma \vec{F} = m \frac{d}{dt'}\frac{d\vec{r}}{dt'}$

$$\frac{\partial T}{\partial t} = D_t \frac{\partial^2 T}{\partial x^2}$$ $t \to t' = -t \Rightarrow -\frac{\partial T}{\partial t'} = D_t \frac{\partial^2 T}{\partial x^2}$

$\hookrightarrow$ irreversibilità

In 3d:

$\vec{J}_t = J_{tx}\vec{e}_x + J_{ty}\vec{e}_y + J_{tz}\vec{e}_z \longrightarrow$ legge di Fourier: $\vec{J}_t = -\lambda \vec{\nabla} T$

$$\frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z \quad \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

equazione del calore

$$\frac{\partial T}{\partial x} = D_T \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = D_T \vec{\nabla} \cdot \vec{\nabla} T = D_T \nabla^2 T$$

laplaciano